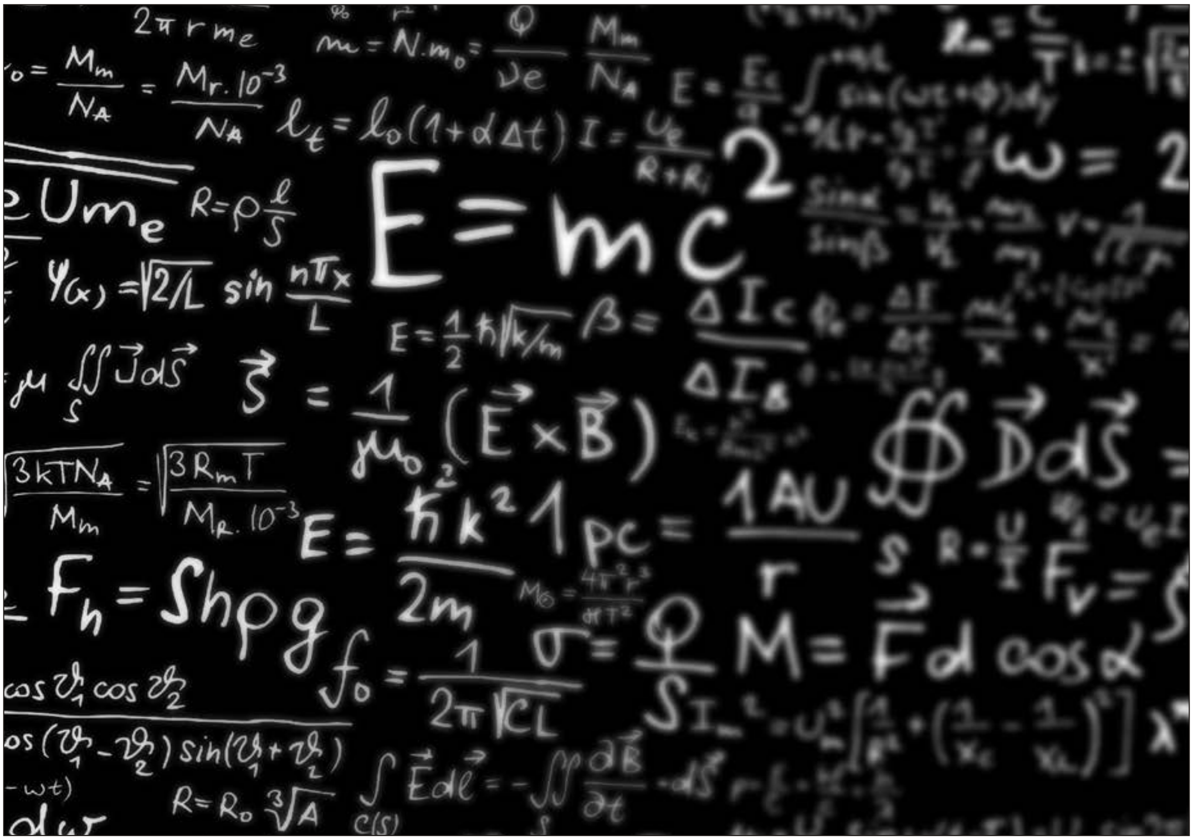

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On Fuzzy Subgroups with Operators

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ABSTRACT

In this study, we further study the theory of fuzzy subgroups and discuss some new concepts such as fuzzy subgroups with operator, normal fuzzy subgroups with operator, homomorphism with operator, etc. While some elementary properties are discussed, such as the intersection operation, the image and inverse image of fuzzy subgroups with operator.

Key words: Fuzzy sets, fuzzy subgroups, normal fuzzy subgroups, m-groups, m-fuzzy subgroups, m-normal fuzzy subgroups, m-homomorphism

INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965), there have been a number of generalizations of this fundamental concept. Rosenfeld (1971) introduced the concept of fuzzy group. The concept was discussed further by many researchers, as Naraghi (2009) and Sulaiman and Ahmad (2010). On the other hand many scholars have studied the anti-homomorphism in fuzzy groups and obtained many results. The concept of anti-homomorphism was discussed further by many researchers and obtained many results, such that Abdullah and Jeyaraman (2010). The aim of this study is to define the fuzzy subgroups with operators "m-fuzzy subgroups", m-normal fuzzy subgroups and study some of its properties.

PRELIMINARY DEFINITIONS

Definition (Pandiammal *et al.*, 2010): Let G be a group, M be a set, if:

- $mx \in G \ \forall x \in G, m \in M$
- $m(xy) = (mx)y = x(my) \ \forall x, y \in G, m \in M$

Then m is said to be a left operator of G, M is said to be a left operator set of G. G is said to be a group with operators. We use phrase " G is an M-group" in stead of a group with operators. If a subgroup of M-group G is also M-group, then it is said to be an M-subgroup of G.

Definition (Zadeh, 1965): Let X be a non empty set, a fuzzy set μ is just a function from X onto [0, 1].

Definition (Rosenfeld, 1971): Let G be a group and μ be a fuzzy set on G. μ is said to be a fuzzy subgroup of G, if for x, y ∈ G:

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}$$
$$\mu(x^{-1}) = \mu(x)$$

Definition (Wu, 1981): A fuzzy subgroup μ of a group G is called normal fuzzy subgroup if $\mu(x^{-1}yx) \geq \mu(y)\ \forall\ x, y \in G$.

Definition (Kim, 2003): If Φ is a homomorphism from a group G_1 to the group G_2 and let μ be a fuzzy subgroup of G_2, then the inverse image of μ under:

- Φ is $(\Phi^{-1}(\mu)(x) = \mu(\Phi(x))\ \forall x \in G$.

M-fuzzy subgroups

Definition (Muthuraj *et al.*, 2010): Let G be a group and μ be a fuzzy group of G, if $\mu(mx) \geq \mu(x)$ hold for any $x \in G$, $m \in M$ then μ is said to be a fuzzy group with operator of G. We use the phrase "μ is an m-fuzzy subgroup of G" instead of a fuzzy subgroup with operator of G.

Example: Let μ be a fuzzy subset of an m-group G and μ defined by:

$$\mu(x) = \begin{cases} 0.1; \text{Otherwise} \\ 0.8;\ x \in G \end{cases}$$

μ is an m-fuzzy subgroup of G.

Definition (Muthuraj *et al.*, 2010): Let G_1, G_2 be an m-group, Φ be a homomorphism from G_1 into G_2. If $\Phi(m\,x) = m.\ \Phi(x)\ \forall\ x \in G$, $m \in M$, then Φ is called m-homomorphism .

Proposition: Let G be an m-group and μ, δ both be m-fuzzy subgroups of G, then, $\mu \cap \delta$ is an m-fuzzy subgroup of G.

Proof: Its clear that $\mu \cap \delta$ is a fuzzy subgroup of G. For any $x \in G$, $m \in M$.

$$(\mu \cap \delta)(mx) = \min\{\mu(mx), \delta(mx)\} \geq \min\{\mu(x), \delta(x)\} = (\mu \cap \delta)(x)$$

Hence $\mu \cap \delta$ is an m-fuzzy subgroup of G.

Corollary: The intersection of any family of m-fuzzy subgroups is an m-fuzzy subgroup.

Definition (Pandiammal *et al.*, 2010): Let G be an m-group, μ is said to be an m-normal fuzzy subgroup of G, if μ is not only an m-fuzzy subgroup of G but also a normal fuzzy subgroup of G.

Proposition: Let G be an m-group, μ, δ both be m-normal fuzzy subgroups of G, then $\mu \cap \delta$ is an m-normal fuzzy subgroup of G.

Proof: It easy to know by above proposition $\mu \cap \delta$ is an m-fuzzy subgroup of G, also we know $\mu \cap \delta$ is a normal fuzzy subgroup of G, hence $\mu \cap \delta$ is an m-normal fuzzy subgroup of G.

Corollary: The intersection of any family of m-normal fuzzy subgroups is an m-normal fuzzy subgroup.

Theorem: Let G_1, G_2 be an m-group and μ, δ be two m-fuzzy subgroup of G_2 and μ be m-normal fuzzy subgroup of δ . Let Φ be an m-homomorphism G_1 into G_2, then $\Phi^{-1}(\mu)$ is an m-normal fuzzy subgroup of $\Phi^{-1}(\delta)$.

Proof: Clearly $\Phi^{-1}(\delta)$, $\Phi^{-1}(\mu)$ is an m-fuzzy subgroups of G. It follows easily that:

$$\Phi^{-1}(\mu)\subseteq\Phi^{-1}(\delta)$$

Now:

$$\Phi^{-1}(\mu)(xyx^{-1}) = \mu(\Phi(xyx^{-1}) = \mu(\Phi(x)\,\Phi(y)\,(\Phi(x)^{-1})\geq\min\{\mu(\Phi(y)), \delta(\Phi(x))\} = \min\{\Phi^{-1}(\mu)(y), \Phi^{-1}(\delta)(x)\};\ \forall x, y\in G$$

Hence $\Phi^{-1}(\mu)$ is normal fuzzy subgroup of $\Phi^{-1}(\delta)$.
Thus $\Phi^{-1}(\mu)$ is an m-normal fuzzy subgroup of $\Phi^{-1}(\delta)$.

Theorem: Let Φ be an m-homomorphism from the m-group G_1 to the m-group G_2, then:

- If μ is an m-fuzzy subgroup of G_2, then $\Phi^{-1}(\mu)$ is an m-fuzzy subgroup of G_1
- If μ is an m-normal fuzzy subgroup of G_2, then $\Phi^{-1}(\mu)$ is an m-normal fuzzy subgroup of G_1

Proof:

- Since $\Phi^{-1}(\mu)$ is an fuzzy subgroup of G_1:

$$(\Phi^{-1}(\mu))(mx) = \mu(\Phi(mx)) = \mu(m\Phi(x))\geq\mu(\Phi(x)) = (\Phi^{-1}(\mu))(x)$$

- Since we know $\Phi^{-1}(\mu)$ is normal fuzzy subgroup of G_1 and by above $\Phi^{-1}(\mu)$ is an m-fuzzy subgroup, hence $\Phi^{-1}(\mu)$ is an m-normal fuzzy subgroup of G_1

Theorem: Let Φ be a homomorphism from the m-group G_1 into m-group G_2 and λ is an m-fuzzy subgroup of G_2. Then λ **o** Φ is an mfuzzy subgroup of G_1.

Proof: Since λ is an m –fuzzy subgroup of G_2. Thus:

$$\lambda(m\,\Phi(x))\geq\lambda(\Phi(x)) = (\lambda o\Phi)(x)$$

Therefore λ**o**Φ is an m-fuzzy subgroup.

Corollary: If μ is an m-normal fuzzy subgroup of G_2, then μ**o**Φ is an m-normal fuzzy subgroup of G_1.

CONCLUSION

In this study, we studied the concept of fuzzy subgroup with operator and we used it to study some properties on the subgroup with operator like to introduce the concept normal fuzzy subgroup with operator and discuss the intersection operation. After that, we have studied on this topic the m- homomorphism and the effect on the image and inverse image of fuzzy and normal fuzzy subgroups with operator.

In the next studies we will formulize the concept of fuzzy subgroups with two operators and applied it to some properties such as fuzzy cosets, conjugate, etc.

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