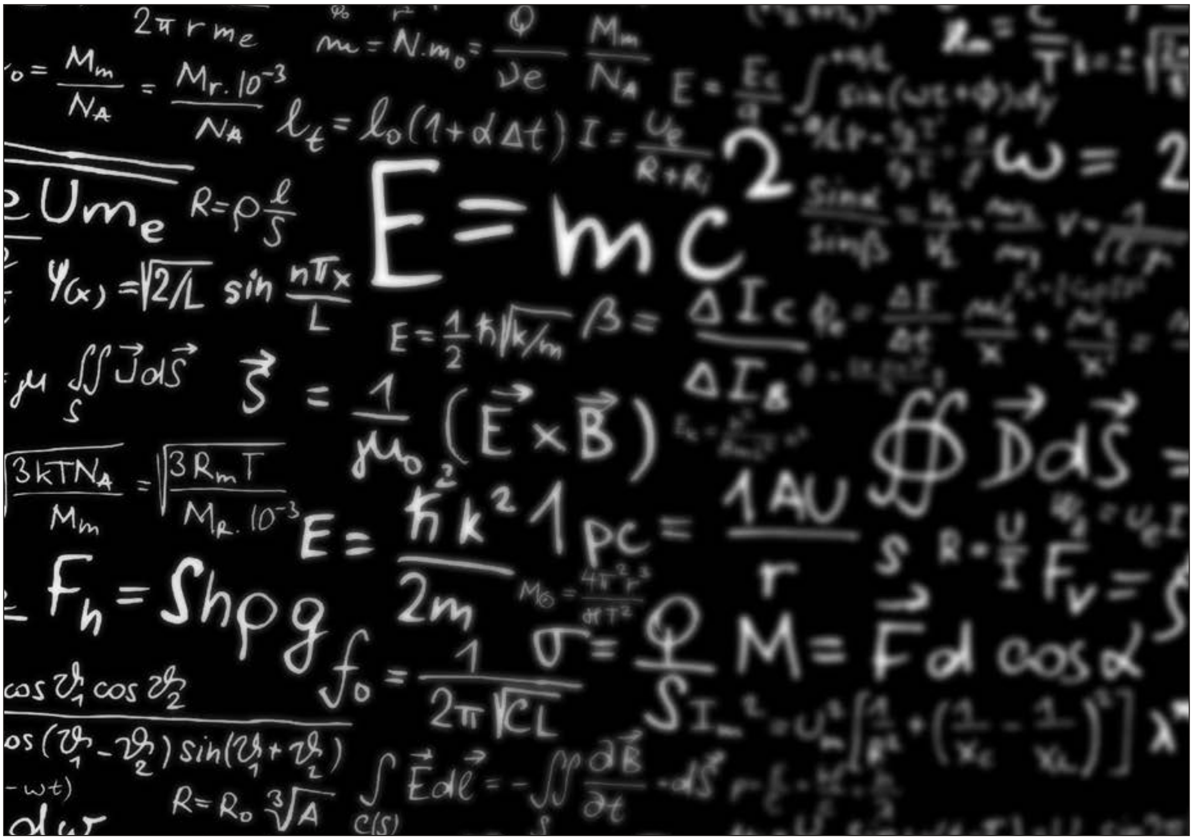

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A Class of Product-cum-dual to Product Estimators of the Population Mean in Survey Sampling Using Auxiliary Information

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ABSTRACT

In this study, a class of product-cum-dual to product estimators have been proposed for estimating finite population mean of the study variate. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Theoretical and empirical studies have been done to demonstrate the superiority of the proposed estimator over the other estimators.

Key words: Finite population mean, auxiliary variate, product and dual to product estimators, mean square error, efficiency

INTRODUCTION

The literature on survey sampling describes a great variety of techniques for using auxiliary information in order to obtain improved estimators for estimating some most common population parameters such as, population total, population mean, population proportion, population ratio, etc. More often, we are interested in the estimation of the mean of a certain characteristic of a finite population on the basis of a sample taken from the population.

When the auxiliary information is utilized at the estimation stage; and the study and the auxiliary variables are positively correlated, the classical ratio strategy is considered to be most practicable in many situations. However, it has the limitation of having at the most same efficiency as that of linear regression strategy. The use of auxiliary information at the estimation stage appears to have started with the work of Cochran (1940). He developed the ratio estimator to estimate the population mean or total of the study variate y by using supplementary information on an auxiliary variate x, positively correlated with y. On the other hand, if the correlation between study variate and auxiliary variate is negative, the product method of estimation is used. Robson (1957) and Murthy (1964) envisaged product estimator, Searls (1964) used coefficient of variation of the study variate, motivated by Searls (1964), Sisodia and Dwivedi (1981) utilized coefficient of variation of auxiliary variate. Srivenkataramana (1980) first proposed dual to ratio estimator, Bandyopadhyay (1980) suggested dual to product estimator, Singh and Espejo (2003), Singh and Tailor (2005) and Tailor and Sharma (2009) worked on ratio-cum-product estimators. Sharma and Tailor (2010) worked on ratio-cum-dual to ratio estimator for estimating finite population mean. These motivate authors to propose a new product-cum-dual to product estimators of finite population mean.

For more applications of ratio, linear regression estimators etc. (Sahai *et al*., 2006; Uddin *et al*., 2006; Adewara, 2007; Sodipo and Obisesan, 2007; Salha and Ahmed, 2009; Mohamed *et al*., 2008; Olaomi and Ifederu, 2008; Gali *et al*., 2008; Al-Khasawneh, 2010; Abd El-Salam, 2011; Gholizadeh *et al*., 2011; Okereke, 2011; Oyamakin, 2012).

Consider a finite population U = (u_1, u_2, ...u_N) of size N units. Let y and x denotes the study and auxiliary variates, respectively. A sample of size n(n<N) is drawn using Simple Random Sampling Without Replacement (SRSWOR) scheme to estimate the population mean

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

of the study variate y. Let the sample mean $(\bar{x}, \bar{y})$ are the unbiased estimator of $(\bar{X}, \bar{Y})$ based on n observations.

Then the usual ratio and product estimators for $\bar{y}$ are

$$\bar{Y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \text{ and } \bar{Y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$$

Where,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ and } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Let us consider a transformation $x_i^* = (1+g)\bar{X} - gx_i$, i = 1,2,..., N, where $g = \frac{n}{N-n}$.

Then $\bar{x}^* = (1+g)\bar{X} - g\bar{x}$ is also unbiased estimator for $\bar{X}$ and correlation of $(\bar{y}, \bar{x}^*)$ is negative.

Using the transformation $x_i^* = (1+g)\bar{X} - gx_i$, Srivenkataramana (1980) obtained dual to ratio estimator as

$$\bar{y}_R^* = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right)$$

and Bandyopadhyay (1980) obtained dual to product estimator as

$$\bar{y}_P^* = \bar{y}\left(\frac{\bar{X}}{\bar{x}^*}\right)$$

In this paper, we have proposed a class of estimators based on the usual product estimator and estimator given by Bandyopadhyay (1980) for estimating finite population mean of the study variate. Numerical illustrations are given in the support of the present study.

THE PROPOSED CLASS OF ESTIMATORS

The proposed 'product-cum-dual to product' estimators is given by

$$\bar{y}_{PdP} = \bar{y}\left[\alpha\left(\frac{\bar{x}}{\bar{X}}\right) + (1-\alpha)\left(\frac{\bar{X}}{\bar{x}^*}\right)\right] \quad (1)$$

where α is a suitably constant.

To obtain the bias (B) and Mean Square Error (MSE) of $\bar{y}_{PdP}$ to the first degree of approximation, we write $e_0 = (\bar{y}-\bar{Y})/\bar{Y}$ and $e_1 = (\bar{x}-\bar{X})/\bar{X}$, such that

$$\left.\begin{array}{l} E(e_0)=E(e_1)=0,\ \ E(e_0^2)=\dfrac{1-f}{n}C_y^2, \\ E(e_1^2)=\dfrac{1-f}{n}C_x^2,\ \ E(e_0e_1)=\dfrac{1-f}{n}CC_x^2 \end{array}\right\} \tag{2}$$

where, $f = n/N$ is the sampling fraction, $C_y^2 = S_y^2/\bar{Y}^2$, $C_x^2 = S_x^2/\bar{X}^2$, $C = \rho C_y/C_x$ and defined as $C_y = S_y/\bar{Y}$ and $C_x = S_x/\bar{X}$ are the coefficient of variations of the study variate y and auxiliary variate x, respectively.

$\rho = S_{xy}/(S_xS_y)$ is the correlation coefficient between y and x.

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2 \text{ and } S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$$

are the population variances of study variate y, auxiliary variate x, respectively and

$$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right)$$

is the covariance between y and x.

Expressing $\bar{y}_{PdP}$ in terms of e's and neglecting terms of e's having power greater than two, we have

$$\bar{y}_{PdP}-\bar{Y} \cong \bar{Y}[e_0+(e_1+e_0e_1)\{g-\alpha(g-1)\}+(1-\alpha)g^2e_1^2] \tag{3}$$

Taking expectation on both sides in Eq. 3 and using the results of Eq. 2, we get the bias of $\bar{y}_{PdP}$ as

$$B(\bar{y}_{PdP}) = \frac{1-f}{n}\bar{Y}C_x^2\{(1-\alpha)(C+g)g+\alpha C\} \tag{4}$$

The bias, $B(\bar{y}_{PdP})$ in Eq. 4 is 'zero' if

$$\alpha = \frac{g(C+g)}{g(C+g)-C}$$

Thus, the estimator $\bar{y}_{PdP}$ with

$$\alpha = \frac{g(C+g)}{g(C+g)-C}$$

is almost unbiased.

Squaring both sides of Eq. 3, taking the expectation and using the results of Eq. 2, we get the mean square error of $\bar{y}_{PdP}$ to the first degree of approximation as

$$M(\bar{y}_{PdP})=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+C_x^2\{g-\alpha(g-1)\}\{2C+g-\alpha(g-1)\}\right] \tag{5}$$

which is minimized when

$$\alpha=\frac{1}{g-1}(g+C)=\alpha_{opt}\text{ (say)} \tag{6}$$

Substituting the value of α from Eq. 6 in Eq. 1 yield the 'asymptotically optimum estimator' (AOE) as

$$\bar{y}_{PdP}^{opt.}=\bar{y}\left[\left(\frac{g+C}{g-1}\right)\frac{\bar{x}}{\bar{X}}-\left(\frac{1+C}{g-1}\right)\frac{\bar{X}}{\bar{x}^*}\right]$$

Thus the resulting bias and MSE of $\bar{y}_{PdP}^{opt.}$ are, respectively given by

$$B\left(\bar{y}_{PdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}C_x^2\left\{g^2\frac{(1+C)}{1-g}-C^2\right\} \tag{7}$$

and

$$M\left(\bar{y}_{PdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_y^2\left(1-\rho^2\right) \tag{8}$$

Eq. 8 shows that mean square error of $\bar{y}_{PdP}^{opt.}$ is same as that of the approximate mean square error of the usual linear regression estimator $\bar{y}_{reg.}=\bar{y}+b_{yx}(\bar{X}-\bar{x})$, where b_{xy} is the sample regression coefficient of y on x.

Remark 1: For α = 1, the estimator $\bar{y}_{PdP}$ in Eq. 1 reduces to the usual product estimator $\bar{y}_P$. The bias and MSE of $\bar{y}_P$ can be obtained by putting α = 1 in Eq's. 4 and 5, respectively as

$$B(\bar{y}_P)=\bar{Y}\frac{1-f}{n}CC_x^2$$

and

$$M(\bar{y}_P)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+C_x^2(1+2C)\right\} \tag{9}$$

Remark 2: For α = 0, the estimator $\bar{y}_{PdP}$ in Eq. 1 boils down to dual to product estimator $\bar{y}_P^*$ of Bandyopadhyay (1980). The bias and MSE of $\bar{y}_P^*$ can be obtained by putting α = 0 in Eq's. 4 and 5, respectively as

$$B\left(\bar{y}_P^*\right)=\frac{1-f}{n}\bar{Y}C_x^2g(g+C)$$

and

$$M\left(\bar{y}_p^*\right)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+gC_x^2\left(g+2C\right)\right\} \tag{10}$$

Thus, we see that this study provides unified treatment towards the properties of different estimators.

EFFICIENCY COMPARISONS

Comparisons of the proposed estimator $\bar{y}_{PdP}$

With sample mean per unit estimator $\bar{y}$: The MSE of sample mean $\bar{y}$ under SRSWOR sampling scheme is given by

$$M\left(\bar{y}\right)=\frac{1-f}{n}\bar{Y}^2C_y^2 \tag{11}$$

From Eq's. 5 and 11, it is envisaged that the proposed class of estimators $\bar{y}_{PdP}$ is better than $\bar{y}$ if

$$\{g-\alpha(g-1)\}\{-(2C+g)+\alpha(g-1)\}>0$$

This condition holds if

$$\text{either}\quad \frac{g}{g-1}>\alpha \text{ and } \frac{1}{g-1}\left(g+2C\right)<\alpha$$

$$\text{or}\quad \frac{g}{g-1}<\alpha \text{ and } \frac{1}{g-1}\left(g+2C\right)>\alpha$$

or equivalently

$$\min\left\{\frac{g}{g-1},\ \frac{1}{g-1}\left(g+2C\right)\right\}<\alpha<\max\left\{\frac{g}{g-1},\ \frac{1}{g-1}\left(g+2C\right)\right\}$$

With usual ratio estimator: To compare with the usual ratio estimator $\bar{y}_R$, the MSE of $\bar{y}_R$ to the first degree of approximation is given by

$$M\left(\bar{y}_R\right)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+C_x^2\left(1-2C\right)\right\} \tag{12}$$

From Eq's. 5 and 12, we note that the estimator $\bar{y}_{PdP}$ has smaller MSE than that of the usual ratio estimator $\bar{y}_R$ if

$$\{g+1-\alpha\ (g-1)\}\ \{-2C+(\alpha-1)\ (g-1)\}>0$$

While establishing the above condition, we have

$$\text{either } 1+\frac{2C}{g-1}<\alpha<\frac{g+1}{g-1}$$
$$\text{or} \quad \frac{g+1}{g-1}<\alpha<1+\frac{2C}{g-1}$$

or equivalently

$$\min\left(\frac{g+1}{g-1},\ 1+\frac{2C}{g-1}\right)<\alpha<\max\left(\frac{g+1}{g-1},\ 1+\frac{2C}{g-1}\right)$$

With usual product estimator: From Eq's. 5 and 9, we found that the estimator $\bar{y}_{PdP}$ will dominate over the usual product estimator $\bar{y}_P$ if

$$\{-(g-1)+\alpha\,(g-1)\}\ \{2C+1+g-\alpha\,(g-1)\}>0$$

This condition holds if

$$\text{either } \frac{1}{g-1}(2C+g+1)<\alpha<1$$
$$\text{or} \quad 1<\alpha<\frac{1}{g-1}(2C+g+1)$$

or equivalently

$$\min\left\{1,\ \frac{1}{g-1}(2C+g+1)\right\}<\alpha<\max\left\{1,\ \frac{1}{g-1}(2C+g+1)\right\}$$

With dual to ratio estimator: The bias and MSE of $\bar{y}_R^*$ to the first degree of approximation, respectively are as

$$B\left(\bar{y}_R^*\right)=-\bar{Y}\frac{1-f}{n}gCC_x^2$$

and

$$M\left(\bar{y}_R^*\right)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+gC_x^2\,(g-2C)\right\} \qquad (13)$$

From Eq's. 5 and 13, it is found that the proposed class of estimators $\bar{y}_{PdP}$ will dominate over Srivenkataramana (1980) estimators $\bar{y}_R^*$ if

$$\{2g-\alpha\,(g-1)\}\ \{-2C+\alpha(g-1)\}>0$$

This condition holds if

$$\text{either}\ \frac{2g}{g-1} > \alpha \ \text{and} \ \frac{2C}{g-1} < \alpha$$
$$\text{or}\ \frac{2g}{g-1} > \alpha \ \text{and} \ \frac{2C}{g-1} > \alpha$$

or equivalently

$$\min\left(\frac{2g}{g-1}, \ \frac{2C}{g-1}\right) < \alpha < \max\left(\frac{2g}{g-1}, \ \frac{2C}{g-1}\right)$$

With dual to product estimator: From Eq's. 5 and 10, we have

$$M\left(\bar{y}_p^*\right) > M\left(\bar{y}_{PdP}\right) \text{ if}$$

$$\alpha(g-1)\ \{2C+2g-\alpha(g-1)\}>0$$

This condition holds if

$$\text{either}\ \ 0 > \alpha \ \text{and} \ \frac{2}{g-1}(C+g) < \alpha, \ \text{when } 2n > N$$

$$\text{or}\ \ 0 < \alpha \ \text{and} \ \frac{2}{g-1}(C+g) > \alpha, \ \text{when } 2n > N$$

Thus, it seems from the above results that the proposed estimator $\bar{y}_{PdP}$ may be made better than other estimators by making a suitable choice of the values of α within the ranges.

Comparison of 'AOE' of $\bar{y}_{PdP}^{opt.}$: From Eqs. 8-13, it is found that the 'AOE' $\bar{y}_{PdP}^{opt.}$ is more efficient than the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$ since

$$M\left(\bar{y}\right) - M\left(\bar{y}_{PdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}^2\rho^2 C_y^2 > 0$$

$$M\left(\bar{y}_R\right) - M\left(\bar{y}_{PdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}^2 C_x^2\left(1-C\right)^2 > 0$$

$$M\left(\bar{y}_P\right) - M\left(\bar{y}_{PdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}^2 C_x^2\left(1+C\right)^2 > 0$$

$$M\left(\bar{y}_R^*\right) - M\left(\bar{y}_{PdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}^2 C_x^2\left(C-g\right)^2 > 0 \qquad \text{and}$$

$$M\left(\bar{y}_p^*\right) - M\left(\bar{y}_{PdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}^2 C_x^2 (C+g)^2 > 0$$

Hence, we conclude that the proposed estimator '$\bar{y}_{PdP}$' is the best (in the sense of having optimum MSE). Now we state the following theorem.

Theorem: To the first degree of approximation, the proposed strategy under optimal condition (6) is always more efficient than $M(\bar{y})$, $M(\bar{y}_R)$,$M(\bar{y}_P)$, $M(\bar{y}_R^*)$, $M(\bar{y}_P^*)$ and equally efficient with linear regression estimator $M(\bar{y}_{reg})$.

NUMERICAL ILLUSTRATIONS

To examine the merits of the constructed estimator over its competitors numerically, we have considered seven natural population data sets. The sources of the population, the nature of the variates y and x and the values of the various parameters are listed in Table 1.

To reflect the gain in the efficiency of the proposed estimator $\bar{y}_{PdP}$ over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$ the effective ranges of α along with its optimum values with respect to the above population data sets are presented in Table 2.

To observe the relative performance of different estimators of $\bar{y}$ we have computed the percentage relative efficiencies (PREs) of the different estimators with respect to the usual unbiased estimator $\bar{y}$ and are presented in Table 3.

Table 1: Description of the populations

Population	Study variate y	Auxiliary variate x	N	n	ρ	C_y	C_x	$\bar{Y}$	References
1	Consumption per capita	Deflated prices of veal	30	6	-0.6823	0.2278	0.0986	7.6375	Maddala (1977)
2	Log of leaf burn in sec	Chlorine percentage	30	6	-0.4996	0.7001	0.7493	0.6860	Steel and Torrie (1960)
3	---	--	106	20	0.82	4.18	2.02	15.37	Kadilar and Cingi (2006)
4	---	---	20	8	-0.9199	0.3552	0.3943	19.55	Pandey and Dubey (1988)
5	Output	No. of workers	80	20	0.9150	0.3542	0.9484	51.8264	Murthy (1967)
6	No. of villages in the circles	A circle consisting more than five villages	89	12	0.766	0.604	2.1901	3.36	Sukhatme and Sukhatme (1970)
7	Output	Fixed capital	80	20	0.9413	0.3542	0.7507	51.8264	Murthy (1967)

Table 2: Effective ranges of α and its optimum values of the estimator $\bar{y}_{PdP}$

	Ranges of α under which the proposed estimator $\bar{y}_{PdP}$ is better than					Optimum value
Population	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_R^*$	$\bar{y}_P^*$	α_{opt}
1	(-0.33, 3.87)	(-1.67, 5.20)	(1.00, 2.54)	(-0.67, 4.20)	(0.00, 3.54)	1.7685
2	(-0.33, 0.91)	(-1.67, 2.24)	(-0.42, 1.00)	(-0.67, 1.24)	(0.00, 0.58)	0.2891
3	(-4.73, -0.30)	(-3.42, -1.61)	(-6.03, 1.00)	(-4.42, -0.61)	(-5.03, 0.00)	-2.5141
4	(-2.00, 2.97)	(-5.00, 5.97)	(-0.03, 1.00)	(-4.00, 4.97)	(0.00, 0.97)	0.4861
5	(-1.53, -0.50)	(-2.00, -0.03)	(-3.03, 1.00)	(-1.03, -1.00)	(-2.03, 0.00)	-1.0126
6	(-0.69, -0.18)	(-1.37, 0.50)	(-1.87, 1.00)	(-0.50, -0.37)	(-0.87, 0.00)	-0.4349
7	(-1.83, -0.50)	(-2.00, -0.33)	(-3.33, 1.00)	(-1.33, -1.00)	(-2.33, 0.00)	-1.1662

Table 3: Percentage relative efficiencies of different estimators with respect to $\bar{y}$

Population	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}^*_R$	$\bar{y}^*_P$	$\bar{y}_{PdP}$ or $\bar{y}^{opt.}_{PdP}$
1	100.00	56.24	167.59	86.25	115.73	187.10
2	100.00	31.11	92.93	74.69	124.34	133.26
3	100.00	226.76	49.36	120.73	83.55	305.25
4	100.00	23.40	526.50	34.37	537.23	650.26
5	100.00	30.59	7.65	612.44	29.16	614.34
6	100.00	11.64	5.08	220.46	45.77	241.99
7	100.00	66.58	10.55	591.38	35.35	877.54

RESULTS AND DISCUSSION

We have proposed an efficient class of estimators of the combination of usual product estimator and dual to product estimator of Bandyopadhyay (1980) as in Eq. 1 and obtain 'AOE' for the proposed estimator. Theoretically, we have demonstrated that the proposed class of estimators is more efficient than conventional estimators and estimators given by Srivenkataramana (1980) and Bandyopadhyay (1980) under the effective ranges of α along with its optimum values. In addition to it, these theoretical results have been presented through numerical investigations.

Table 2 provides the wide ranges of α and its optimum values for which the proposed class of estimators $\bar{y}_{PdP}$ or $\bar{y}^{opt.}_{PdP}$ are more efficient than all other estimators. It is also observed from Table 2 that there is a scope for choosing the values of α to obtain better estimators than $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}^*_R$ and $\bar{y}^*_P$.

From Table 3, it is quite evident that there is a substantial gain in efficiency by using the proposed estimators $\bar{y}_{PdP}$ or $\bar{y}^{opt.}_{PdP}$ over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}^*_R$ and $\bar{y}^*_P$. This shows that even if the scalar α deviates from its optimum value ($\alpha_{opt.}$), the suggested estimator $\bar{y}_{PdP}$ will yields better estimates than $\bar{y}_{PdP}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}^*_R$ and $\bar{y}^*_P$. Thus, the use of proposed class of estimators is preferable over others.

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