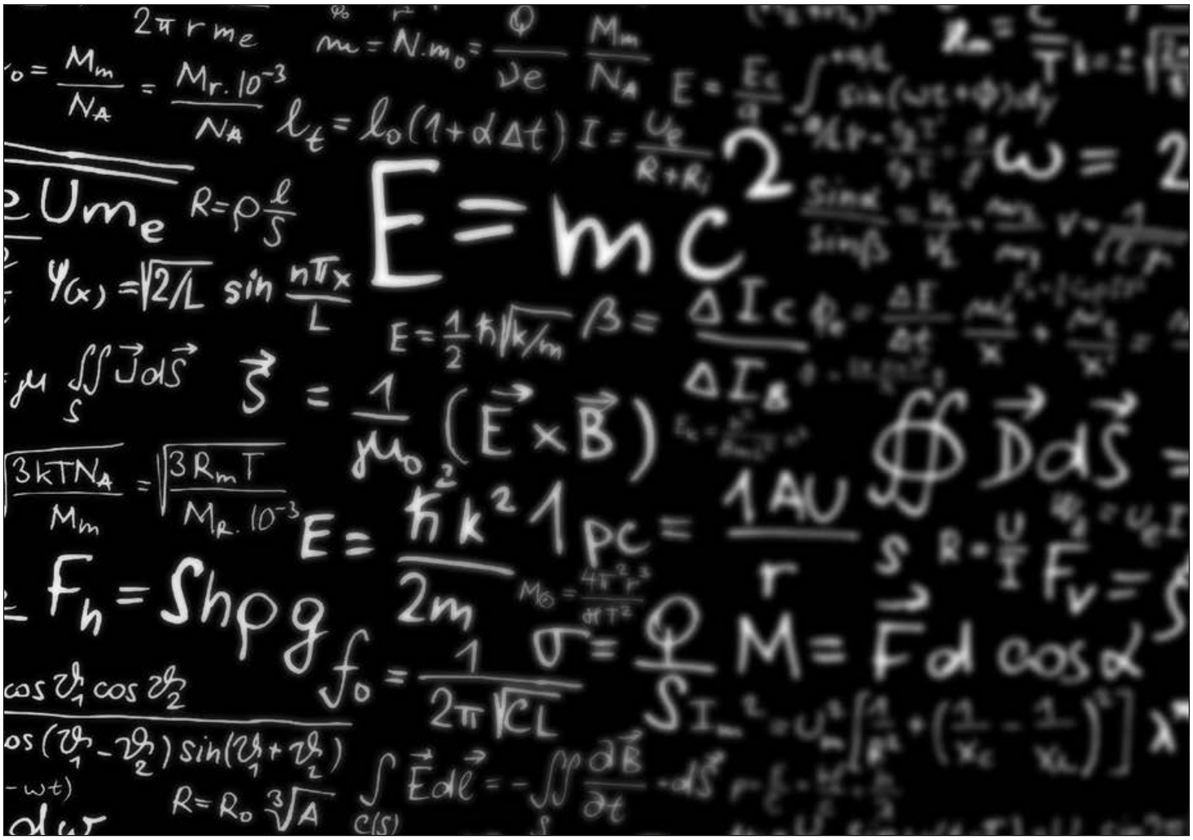

# Asian Journal of Mathematics & Statistics

ISSN 1994-5418

Asian Journal of Mathematics and Statistics 6 (1): 43-51, 2013
ISSN 1994-5418 / DOI: 10.3923/ajms.2013.43.51


# A Qualitative Study of Biological Pest Control System

[1]Y.O. Aderinto, [1]O.M. Bamibgola, [2]F.M. Jimoh, [2]M.A. Ganiyu and [3]T. Aliu
[1]Department of Mathematics, Faculty of Science, University of Ilorin, Ilorin, Nigeria
[2]Department of Physical Sciences, Al-Hikmah University, PMB 1601, Ilorin, Nigeria
[3]Department of Statistics and Mathematical Sciences, College of Pure and Applied Sciences, Kwara State University, PMB 1530, Ilorin, Nigeria

*Corresponding Author: Y.O. Aderinto, Department of Mathematics, Faculty of Science, University of Ilorin, Ilorin, Nigeria*

## ABSTRACT

Agricultural pests are the insects that feed on crops and damaged them. Most current agricultural pest control methods focused on chemical insecticides. Research works have shown that these chemicals have many disastrous consequences. However, effective control of these pests can be obtained through the use of living organisms to reduce the density of pest below economic damaging level and this is refers to as biological pest control. If a mathematical model for the biological system is provided, then the effects of such pests can be controlled by the methods of optimal control theory. In this study, the biological control of agricultural pest system via optimal control theory approach was qualitatively studied. In an attempt to minimize the pest population below injury level and stabilize the natural enemies' population. The system was analysed, equilibrium point for the system was determined, stability and economic loss free equilibrium was equally established. Numerical values were employed to check for the validity of the method and the result was found to be effective.

**Key words:** Biological control, stability, optimal control, pest population

## INTRODUCTION

Pests according to English dictionary can be defined as any organism that is damaging to livestock, crops, humans or land fertility (somebody or something that disturb the activities of others). In order words, pests are damaging organisms that affect both human activities and crops production. Rafikov and Balthazar (2005) affirmed that agricultural pest is the species that causes damage to crops or interfere with crops.

The application of chemical insecticides according to Uboh *et al.* (2011), is an attempt to control pests directly at low cost. However, the application of these chemical insecticides has many disastrous consequences classified into three categories. Firstly, the efficiency is decreasing due to progressive resistance by insects. Second, they are not selective; they have negative impact over the beneficial insect population which leads to reduction of natural pests control due to destruction of pests as well as pest natural enemies, application results to more and new different types of pests as well as destructive surges. Thirdly, the chemicals also have many environmental effects that include chemical resides in crops and in the agricultural ecosystems. And finally, high number of accidents that intoxicates human beings and in some cases leads to their deaths do happen. In line with these, the U.S. government has created restrictive legislation to reduce the use of chemical insecticides and have increased research for the development of new safer method (Rafikov and Balthazar, 2005).

Many researchers have worked on agricultural pests and pesticides, Sharma *et al.* (2010) studied the fluctuation of pest population of *Earias vittella* (Fab.) and their relation with prevailing weather conditions. Asare-Bediako *et al.* (2010) studied the effectiveness of intercropping cabbage with non-host crops in reducing the effect of the diamondback moth pest on cabbage. The assessment of organochlorine and organophosphate pesticides in sediments from nine major municipal drains in India was carried out by Kumar *et al.* (2011). In the same way, Ismail *et al.* (2009) worked on the adsorption-desorption and mobility of the herbicide in two ricefield soils. Ram *et al.* (2011) worked on toxicity of a combination pesticide which is dangerous to human health. Others are, Biswas *et al.* (2010), Alle *et al.* (2009) and Okafor and Omodamiro (2006), to mention a few. Most current pest control methods focus mainly on chemical insecticides.

Biological control is the use of living organisms to control pest populations, making them less abundant and thus less damage to the environment. In general, biological control is the use of parasitoid, predators or pathogens to maintain the density of pest population in equilibrium level below economic damages than would occur without these natural enemies, Rafikov and Balthazar (2005), Barbu (1994), Andres *et al.* (1979) and Rafikov *et al.* (2008). According to Vanden *et al.* (1982), the application of biological pest control is the introduction of pest natural enemies (prey-predator) to reduce the density of pest population without human intervention. Biological control can be categorized into three primary methods. First is the preservation and periodic release of the natural enemy, either seasonal introduction of a small population of natural enemies or a massive release. Second: Classical control which includes importing and establishing new natural enemies to the pest and establishes a permanent population. And lastly, is environmental manipulation where alternative prey, attracts, ants and pest itself are introduced to the system (Rafikov *et al.*, 2008). For instance, the importation of parasites which biologically controls bean pest alfalfa weevil, also, the control of some vegetable pests by augmentative release of their natural enemies which have been applied in greenhouses in Europe. However, there are also many cases whose effective natural enemies have not been found or have not been successfully established (Thomas and Martin, 2002; Parrella *et al.*, 1992). For effective, successful biological control application, the dynamics or analysis of the pest and its natural enemy populations must be understood.

The purpose of this study was to use mathematical modelling tool to qualitatively and quantitative study and analyse crop pest and its natural enemy population basically prey-predator model population in an attempt to control the menace of agricultural pest.

## MATHEMATICAL FORMULATION OF THE MODEL

Biological pest control can be modelled as a dynamics of biological systems in which two species interact one a predator and one its prey (Volterra, 1926; Rafikov *et al.*, 2008; Mills and Getz, 1996). Some works have been done on the pest control (Shoemaker, 1973; Goh, 1980; Parrella *et al.*, 1992), where optimal feedback policies model was formulated via optimal control theory. In these cases, some of the important factors in the interaction between the prey-predator or host-parasitoid model and some other species of the same ecosystem and the environment, influence of control on reproduction and competition were less considered. Rafikov *et al.* (2008)

emphasizes on models that consider more than two species has been made for more clarifications. For the purpose of this work, emphasises is on models that consider three different species with different environmental condition.

Consider the system of interactive population given by:

$$\left.\begin{aligned}
\frac{dP_1}{dt} &= aP_1 - b_1P_1Q_1 - \alpha P_2 - \beta P_3 \\
\frac{dP_2}{dt} &= \alpha P_2 - b_2P_2Q_2 - aP_1 - \beta P_3 \\
\frac{dP_3}{dt} &= \beta P_3 - b_3P_3Q_3 - aP_1 - \alpha P_2 \\
\frac{dQ_1}{dt} &= -CQ_1 + d_1P_1Q_1 - d_2Q_2 - d_3Q_3 + u_1a \\
\frac{dQ_2}{dt} &= -rQ_2 + d_2P_2Q_2 - d_1Q_1 - d_3Q_3 + u_2\alpha \\
\frac{dQ_3}{dt} &= -qQ_3 + d_3P_3Q_3 - d_1Q_1 - d_2Q_2 + u_3\beta
\end{aligned}\right\} \quad (1)$$

where, Q is the predator population density, P is the prey population density a, $\alpha$, $\boldsymbol{\beta}$ are the prey's birth rate for the three species, $b_i$ is the predator's attack rate (the rate of growth of predator). c, r, q are the predator's death rate, $d_i$ is the development rate of the predator or the efficiency which prey are converted to predators, $u_i$ are the controls. The subscript implies that different areas have different environmental conditions.

The following assumptions were made about environment and evolution of prey-predator population. First, the prey population finds food all the time but the food supply of the predator population depends entirely on the prey populations. Second, the rate of change in population is proportional to its size and the environment does not change in favour of any species.

The objective function to be minimize is given by:

$$\begin{aligned}
\text{Min } J(u_i) &= \int_0^{t_f} (CP_1 + Au^2 + Bu^2)dt \\
&= \int_0^{t_f} [CP_i + (A+B)u_i^2]dt, \quad i = 1,2,3,\ 0 \le u_i \le 1
\end{aligned} \quad (2)$$

where, A is the cost associated with using biological control, B is the cost associated with the presence of prey on the crop and with using insecticides (this include social cost. u is magnitude of biological control effort/magnitudes of insecticides control effort, C is appropriate constant associated with the number of prey. For effective introduction of predators at a given rate we choose u such that $0 \le u \le 1$.

The problem is to minimize Eq. 2 subject to (1), that is to minimize the pest population below injury level and stabilize the natural enemies' population within the level appropriate to pest control.

## ANALYSIS OF THE MODEL

Here, the optimal control model was qualitatively and quantitatively analysed. The equilibrium point was derived, using the control u so that the optimal pest control strategy is obtains through biological control that is to keep the density of the pest population in an equilibrium level below economic damages.

**Economic loss-free equilibrium and stability:** A good strategy for achieving the objective of attaining the density of the pest population below economic damages is to minimize the pest population as well a s the cost associated with biological control. In line with this, we are to determine the equilibrium point for the system and establish whether or not the system is both stable and economic loss free at this point.

**Definition 1:** Equilibrium point: Consider the system:

$$\frac{dx_1}{dt} = P(x_1, x_2) \quad \text{and} \quad \frac{dx_2}{dt} = Q(x_1, x_2)$$

A point $(x_1^*, x_2^*)$, for which $P(x_1^*, x_2^*) = 0 = Q(x_1^*, x_2^*)$ is called an equilibrium point of the system. The point $(x_1^*, x_2^*)$ is a trajectory point, i.e., the solutions starting at this point always remain within reasonable distance of it. The equilibrium point according to Al-Shabi and Abo-Zeid (2010) is called locally asymptotically stable if it is locally stable, global attractor (i.e., if every solution converges to the point as $n\to\infty$) and globally asymptotically stable if it is locally asymptotically stable and global attractor.

The global asymptotic stability of an equilibrium point of a differential system can be expressed according to an elementary result in stability theory which stated that if the Jacobian matrix of function f i.e., Jf (x), has eigenvalues with negative real part at a singular point, then the point is asymptotically stable. In order words if Jf(x) has eigenvalues with negative real part at any critical point in R, then the critical point is globally asymptotically stable (Sabatini, 1990).

Economic Loss-free Equilibrium (ELFE) of the model is obtain by setting the right hand side of the equation to zero and taking all the predator and prey terms in the equation to be zero. Thus, there is a steady stable (equilibrium point) of the system called economic loss free equilibrium i.e., a state where there is no economic loss as t tends to infinity (after a long term has passed). For more on loss free equilibrium (Bhunu *et al.*, 2008; Castilio-Chavez *et al.*, 2008).

## UNIQUENESS OF EQUILIBRIUM POINT

**Assumption 1:** If $f_i$ and $g_i$ (i = 1, 2, n) are Lipschitz continuous, then there exist positive constants $k_i$, $l_i$ such that:

$$|f_i(u)-f_i(v)|\le k_i|u-v|,\ |g_i(u)-g_i(v)|\le l_i|u-v|$$

For all u, v$\in$R and i = 1, 2,..., n. For proof see Cao and Wang (2003).

**Theorem 1:** Given that assumption (1) is satisfied, then Eq. 2 has a unique equilibrium point.

**Proof:** Let $E_1 = (P_i, D_i)^T$ and $E_2 = (Q_i, F_i)^T$ denote the two equilibrium points of the model (2), i = 1, 2, 3, where, $P_i = [p_1, p_2, p_3]^T$, $D_i = [D_1, D_2, D_3]^T$, $Q_i = [Q_1, Q_2, Q_3]^T$ and $F_i = [F_1, F_2, F_3]^T$, Then, we have:

$$\left.\begin{array}{l} a_1 p_{1i} - b_{1i} p_{1i} Q_i - \alpha_i p_{2i} - \beta_i p_{3i} = 0 \\ \alpha_1 p_{2i} - b_{1i} p_{2i} Q_{2i} - a_i p_{1i} - \beta_i p_{3i} = 0 \\ \beta_1 p_{3i} - b_{3i} p_{3i} Q_{3i} - a_i p_i - \alpha_i p_{2i} = 0 \\ a_i D_{1i} - b_{1i} D_{1i} Q_{1i} - \alpha_i D_{2i} - \beta_i D_{3i} = 0 \\ \alpha_i D_{2i} - b_{2i} D_{2i} Q_{2i} - a_i D_{1i} - \beta_i D_{3i} = 0 \\ \beta_i D_{3i} - b_{3i} D_{3i} Q_{3i} - \alpha_i D_i - \alpha_i D_{2i} = 0 \end{array}\right\} \tag{3}$$

and

$$\left.\begin{array}{l} -c_i Q_{1i} + d_{1i} P_{1i} Q_{1i} - d_{2i} Q_{2i} - d_{3i} Q_{3i} + u_{1i} a_i = 0 \\ -r_i Q_{2i} + d_{2i} p_{2i} Q_{2i} - d_{1i} Q_{1i} - d_{3i} Q_{3i} + u_{2i} \alpha_i = 0 \\ -q_i Q_{3i} + d_{3i} p_{3i} Q_{3i} - d_{1i} Q_{1i} - d_{2i} Q_{2i} + u_{3i} \beta_i = 0 \\ -c_i F_{1i} + d_{1i} P_{1i} F_{1i} - d_{2i} F_{2i} - d_{3i} F_{3i} + u_{1i} a_i = 0 \\ -r_i F_{2i} + d_{2i} P_{2i} F_{2i} - d_{1i} F_{1i} - d_{3i} F_{3i} + u_2 \alpha_i = 0 \\ -q_i F_{3i} + d_{3i} P_{3i} F_{3i} - d_{1i} F_{1i} - d_{2i} F_{2i} + u_{3i} \beta_i = 0 \end{array}\right\} \tag{4}$$

These implies that:

$$a_i(p_{1i}-D_{1i}) = b_{1i}Q_{1i}(p_{1i}-D_{1i})+\alpha_i(p_{2i}-D_{2i})+\boldsymbol{\beta}_i(p_{3i}-D_{3i}) \tag{5a}$$

$$\alpha_i(p_{2i}-D_{2i}) = b_{2i}Q_{2i}(p_{2i}-D_{2i})+a_i(p_{1i}-D_{1i})+\boldsymbol{\beta}_i(p_{3i}-D_{3i}) \tag{5b}$$

$$\boldsymbol{\beta}_i(p_{3i}-D_{3i}) = b_{3i}Q_{3i}(p_{3i}-D_{3i})+a_i(p_{1i}-D_{1i})+\alpha_i(p_{2i}-D_{2i}) \tag{5c}$$

and

$$d_{1i}p_{1i}(Q_{1i}-F_{1i}) = c_i(Q_{1i}-F_{1i})+d_{2i}(Q_{2i}-F_{2i})+d_{3i}(Q_{3i}-F_{3i}) \tag{6a}$$

$$d_{2i}p_{2i}(Q_{2i}-F_{2i}) = r_i(Q_{2i}-F_{2i})+d_{1i}(Q_{1i}-F_{1i})+d_{3i}(Q_{3i}-F_{3i}) \tag{6b}$$

$$d_{3i}p_{3i}(Q_{3i}-F_{3i}) = q_i(Q_{3i}-F_{3i})+d_{1i}(Q_{1i}-F_{1i})+d_{2i}(Q_{2i}-F_{2i}) \tag{6c}$$

Using the assumption above, we obtain from Eq. 5a:

$$\begin{gathered} a_i|p_{1i}-D_{1i}| \le L_1|b_1Q_{1i}||p_{1i}-D_{1i}|+L_2\alpha_i|p_{2i}-D_{2i}|+L_3\boldsymbol{\beta}_i|p_{3i}-D_{3i} \\ \Rightarrow (a-|L_1|-L_2\, L_3|\alpha_1\boldsymbol{\beta}_1|)(|p_{1i}-D_{1i}|, |p_{2i}-D_{2i}|, |p_{3i}-D_{3i}|)^T \le (0, 0, 0)^T \end{gathered}$$

Multiply by $(a_i - | L_1 | - L_2 L_3 | \alpha_i \boldsymbol{\beta}_i |)^{-1}$, we obtain:

$$(|p_{1i}-D_{1i}|, |p_{2i}-D_{2i}|, |p_{3i}-D_{3i}|)^T \le (0, 0, 0)^T$$

Which implies that for i = 1, 2, 3:

$$\begin{gathered} p_{1i} = D_{1i},\ p_{2i} = D_{2i},\ p_{3i} = D_{3i} \\ \text{i.e.,}\quad p_i = D_i \end{gathered} \tag{7}$$

The same thing is applicable to Eq. 5b and 5c.
Also from Eq. 6a:

$$D_{1i}p_{1i}|Q_{1i}\text{-}F_{1i}| \le |k_1|c_1|Q_{1i}\text{-}F_{1i}| + k_2|d_{2i}\text{-}F_{2i}| + k_3|d_{3i}||Q_{3i}\text{-}F_{3i}|$$
$$\Rightarrow (d_{1i}p_{1i}\text{-}k_2|d_{2i}|\text{-}k_3|d_{3i}|)(|Q_{1i}\text{-}F_{1i}|, |Q_{2i}\text{-}F_{2i}|, |Q_{3i}\text{-}F_{3i}|)^T \le (0, 0, 0)^T$$

Multiply by $(d_{1i}p_{1i}\text{-}k_2 \mid d_{2i} \mid \text{-}k_3 \mid d_{3i})^{-1}$, we obtain:

$$(|Q_{1i}\text{-}F_{1i}|, |_{2i}\text{-}F_{2i}|, |Q_{3i}\text{-}F_{3i}|)^T \le (0, 0, 0)^T$$

This implies that for i = 1, 2, 3:

$$Q_{1i} = F_{1i},\ Q_{2i} = F_{2i},\ Q_{3i} = F_{3i}$$
$$\text{i.e.,}\quad Q_i = F_{1i} \tag{8}$$

The same approach hold for Eq. 6b and 6c. From Eq. 7 and 8, we conclude that the model has a unique equilibrium point.

## STABILITY ANALYSIS

Using the information earlier, from Eq. 1 we obtain:

$$\left.\begin{array}{l} 0 = aP_1 - b_1P_1Q_1 - \alpha P_2 - \beta P_3 \\ 0 = \alpha P_2 - b_2P_2Q_2 - aP_1 - \beta P_3 \\ 0 = \beta P_3 - b_3P_3Q_3 - aP_1 - \alpha P_2 \\ 0 = -cQ_1 + d_1P_1Q_1 - d_2Q_2 - d_3Q_3 + u_1a \\ 0 = -rQ_2 + d_2P_2Q_2 - d_1Q_1 - d_3Q_3 + u_2\alpha \\ 0 = -qQ_3 + d_3P_3Q_3 - d_1Q_1 - d_2Q_2 + u_3\beta \end{array}\right\} \tag{9}$$

There is need to compute the linearization of the system in order to determine the behaviour of the different populations near the equilibrium solutions.

For the system of Eq. 9, the Jacobian, matrices' J, is given:

$$J(P_1,P_2,P_3,Q_1,Q_2,Q_3) = \begin{bmatrix} a-b_1Q_1 & -\alpha & -\beta & -b_1p_1 & 0 & 0 \\ -a & \alpha-b_2Q_2 & -\beta & 0 & -b_2p_2 & 0 \\ -a & -\alpha & \beta-b_3Q_3 & 0 & 0 & -b_3p_3 \\ d_1Q_1 & 0 & 0 & -c+d_1p_1 & -d_2 & -d_3 \\ 0 & d_2Q_2 & 0 & -d_1 & -r+d_2p_2 & -d_3 \\ 0 & 0 & d_3Q_3 & -d_1 & -d_2 & -q+d_3p_3 \end{bmatrix} \tag{10}$$

The economic loss free equilibrium points is given by $(P_1^*, P_2^*, P_3^*, Q_1^*, Q_2^*, Q_3^*) = (0, 0, 0, u_1, a, u_2\alpha, u_3\beta)$, At the point $(P_1^*, P_2^*, P_3^*, Q_1^*, Q_2^*, Q_3^*)$, the Jacobian of the systems is given by:

$$\begin{pmatrix} a-b_1u_1a & -\alpha & -\beta & 0 & 0 & 0 \\ -a & \alpha-b_2\alpha u_2 & -\beta & 0 & 0 & 0 \\ -a & -\alpha & \beta-b_3\beta u_3 & 0 & 0 & 0 \\ d_1u_1a & 0 & 0 & -c & -d_2 & -d_3 \\ 0 & d_2\alpha u_2 & 0 & -d_1 & -r & -d_3 \\ 0 & 0 & d_3\beta u_3 & -d_1 & -d_2 & -q \end{pmatrix} \tag{11}$$

Table 1: Ecological data for numerical analysis of the model

| Parameter | Meaning | Value |
|---|---|---|
| a | Prey birth rate (species 1) | 0.17 |
| $\alpha$ | Prey birth rate (species 2) | 0.116 |
| $\beta$ | Prey birth rate (species 3) | 0.18 |
| C | Predator death rate (species 1) | 0.00017 |
| r | Predator death rate species 2) | 0.00017 |
| q | Predator death rate (species 3) | 0.00012 |
| $b_1$ | Predator attack rate (species 1) | 0.20 |
| $B_2$ | Predator attack rate (species 2) | 0.20 |
| $b_3$ | Predator attack rate (species 3) | 0.20 |
| $d_1$ | development rate of the predator (species 1) | 0.0085 |
| $d_2$ | development rate of the predator (species 2) | 0.0085 |
| $d_3$ | development rate of the predator (species 3) | 0.00085 |
| $p_1$ | Prey population (species 1) | 1000 |
| $p_2$ | Prey population (species 2) | 1000 |
| $p_3$ | Prey population (species 3) | 1000 |
| $Q_1$ | Predator population species 1 | 250 |
| $Q_2$ | Predator population density species 2 | 250 |
| $Q_3$ | Predator population density species 3 | 260 |
| $u_1$ | Magnitude of biological (control effort) species 1 | 0.5 |
| $u_2$ | Magnitude of biological (control effort) species 2 | 0.6 |
| $u_3$ | Magnitude of biological (control effort) species 3 | 0.4 |

The data are used for numerical analysis, Some of these parameter values were from Rafikov *et al*. (2008)

For the economic loss free to be stable, all the eigenvalues of Jacobian Eq. 11 must be negative (i.e., the value of $\lambda_i$ determine the stability).

Hence, by the application of numerical data in Table 1, eigenvalues was computed using R statistical package (available at www.cran.org). The following values were obtained for eigenvalues $\lambda_i$: $\lambda_1 = 0.334704$, $\lambda_2 = 0.252666$, $\lambda_3 = -0.166690$, $\lambda_4 = -0.016909$, $\lambda_5 = 0.008646$ and $\lambda_6 = 0.008434$.

This implies that the economic loss free equilibrium can be stable, though it is not all the eigenvalues that are negative but less than 1.

The mathematical analysis revealed that biological pest control can be modelled as a dynamics system in which two species interact. One a predator and the other its prey. The work emphasized on models that consider three different species with different environmental conditions. The results showed that pest population can be minimized below damaged level and natural enemies' population can be stabilized within the level appropriate to pest control. It was established that equilibrium point for the system is unique. Economies loss free equilibrium is stable when all the eigenvalues are negative; two of the eigenvalues are negative while the remaining three is less than 1 which implies stability in future. Hence, the study of stability is essential in predicting the future course so that biological pest control can be effectively designed. Since the use of chemical insecticides has so many disastrous consequences, therefore biological pest control should be vigorously focused, in order to combat the economics damage cause by agricultural pest.

## CONCLUSIONS

In this study, the mathematical model of biological pest control system was presented. Three different species with different environmental conditions were considered. The equilibrium point was qualitatively analysed and found to be unique. The stability point for economic loss free equilibrium of the system was determined in an attempt to keep the density of pest population below economics damage level. Numerical values were employed to check for the validity of the method and from the result obtained, economic loss free equilibrium can be stable.

## REFERENCES

Al-Shabi, M.A. and R. Abo-Zeid, 2010. Global asymptotic stability of a higher order difference equation. J. Applied Math. Sci., 4: 839-847.

Alle, A., A. Dembelle, B. Yao and G. Ado, 2009. Distribution of organochlorine pesticides in human breast milk and adipose tissue from two locations in Cote d'Ivoire. Asian J. Applied Sci., 2: 456-463.

Andres, L.A., E.R. Oatman and R.G. Simpson, 1979. Re-Examination of Pest Control Practices. In: Biological Control and Insectpest Management, Davis, D.W., S.C. Hoyt, J.A. McMurtry and M.T. AliNiazee (Eds.). University of California, Oakland, USA.

Asare-Bediako, E., A.A. Addo-Quaye and A. Mohammed, 2010. Control of diamondback moth (*Plutella xylostella*) on cabbage (*Brassica oleracea* var *capitata*) using intercropping with non-host crops. Am. J. Food Technol., 5: 269-274.

Barbu, V., 1994. Mathematical Methods in Optimization of Differential Systems. Kluwer Academe Publishers, Netherlands, pp: 33-38.

Bhunu, C.P., W. Ganra, Z. Mukandavire and M. Zimba, 2008. Tuberculosis transmission model with chemoprophylaxis and treatment. Bull. Math. Biol., 70: 1163-1191.

Biswas, A.K., N. Kondaiah, A.S.R. Anjaneyulu and P.K. Mandal, 2010. Food safety concerns of pesticides, veterinary drug residues and mycotoxins in meat and meat products. Asian J. Anim. Sci., 4: 46-55.

Cao, J. and J. Wang, 2003. Global asymptotic stability of a general class of recurrent neural networks with time-varying delays. Proc. IEEE Trans. Circuits Syst. I: Fundam. Theory Appl., 50: 34-44.

Castilio-Chavez, C., Z. Feng and A.F. Capurro, 2008. A Model for TB with Exogenous Reinfection. IMA Print, USA., pp: 1-23.

Goh, B.S., 1980. Management and Analysis of Biological Populations. Elsevier Scientific Publishing Company, Amsterdam, The Netherlands.

Ismail, B.S., M. Sameni and M. Halimah, 2009. Adsorption, desorption and mobility of 2,4-D in two Malaysian agricultural soils. Asian J. Agric. Res., 3: 67-77.

Kumar, B., R. Gaur, G. Goel, M. Mishra and D. Prakash *et al*., 2011. Distribution of pesticides in sediments from municipal drains in Delhi, India. Asian J. Sci. Res., 4: 271-280.

Mills, N.J. and W.M. Getz, 1996. Modelling the biological control of insect pest: Review of host-parasitoid models. Ecolog. Modelling, 92: 121-143.

Okafor, P.N. and O.D. Omodamiro, 2006. Assessment of chemical/phytotoxin and microbial contamination of pasta foods marketed in Nigeria. Am. J. Food Technol., 1: 190-195.

Parrella, M.P., K.M. Heinz and L. Nunney, 1992. Biological control through augmentative releases of natural enemies: A strategy whose time has come. Am. Entomologist, 38: 172-180.

Rafikov, M. and J.M. Balthazar, 2005. Optimal pest control problem in population dynamics. Applied Math. Comput., 24: 65-81.

Rafikov, M., J.M. Balthazar and H.F.V. Bremen, 2008. Mathematical modeling and control of population systems: Applications in biological pest control. Applied Math. Comput., 200: 557-573.

Ram, S.B.H., C.U. Devi, C. Susma, V.R. Jasti, T.M.V. Kumar and G. Thirumurugan, 2011. Effect of polytrin C (combination pesticide) on the ach ease inhibition in plasma and brain of Wistar rats. Am. J. Biochem. Mol. Biol., 1: 101-105.

Sabatini, M., 1990. Global asymptotic stability of critical points in the plane. J. Dyn. Syst. Ordinary Differ. Equ., 48: 97-103.

Sharma, R.P., R. Swaminathan and K.K. Bhati, 2010. Seasonal incidence of fruit and shoot borer of okra along with climatic factors in udaipur region of India. Asian J. Agric. Res., 4: 232-236.

Shoemaker, C., 1973. Optimization of agricultural pest management III: Results and extensions of a model. Math. Biosci., 18: 1-22.

Thomas, R.C. and J.C. Martin, 2002. Optimizing future heat and power generation. IEEE Transaction on Power Systems, pp. 1-12.

Uboh, F.E., E.N. Asuquo, M.U. Eteng and E.O. Akpanyung, 2011. Endosulfan-induces renal toxicity independent of the route of exposure in rats. Am. J. Biochem. Mol. Biol., 1: 359-367.

Vanden, B., P.S. Messenger and A.P. Gutierrez, 1982. An Introduction to Biological Control. Plenum Press, New York, USA.

Volterra, V., 1926. Fluctuations in the abundance of a species considered mathematically. Nature, 118: 558-560.