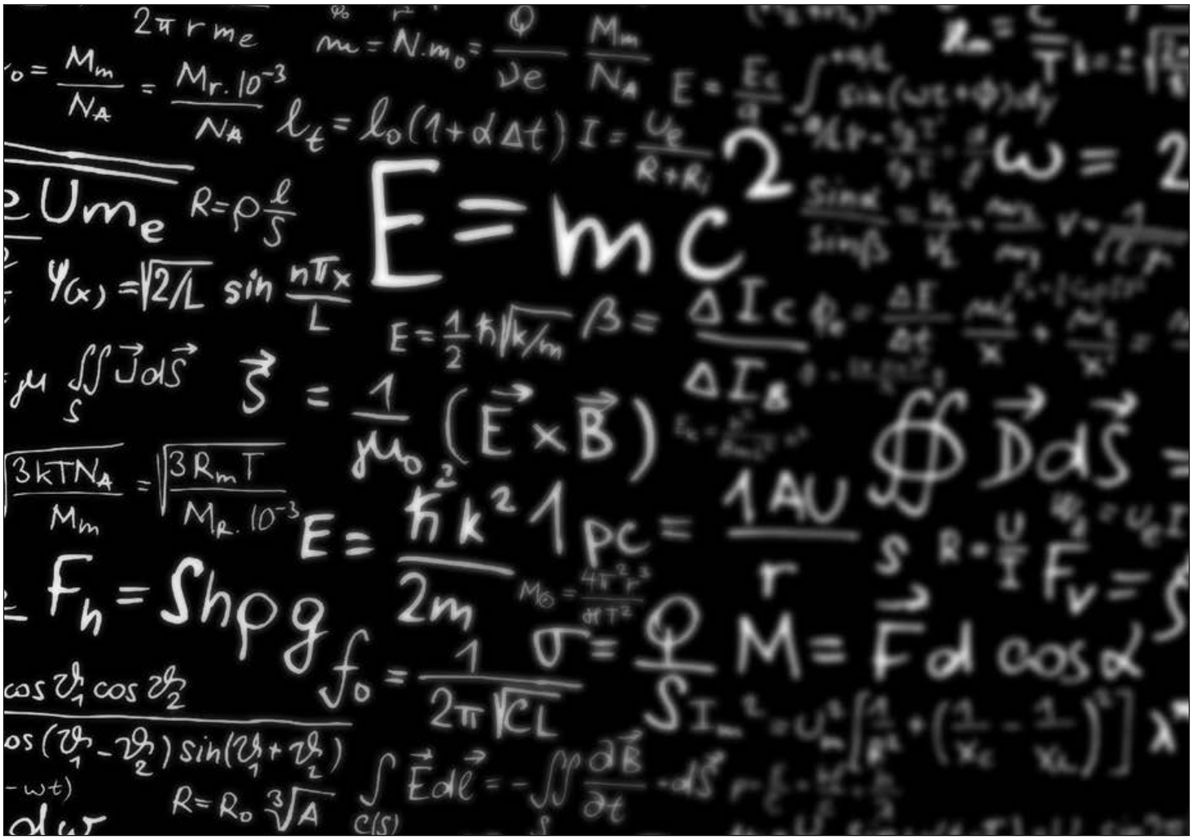

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Contribution to 1-quasi-total Graphs

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ABSTRACT
This study illustrate some narrative and remarkable properties of 1-quasi-total graphs. It has been acquired results of various graphs such as regular graphs, complete graphs, complete bipartite, cycles and chromatic number of 1-quasi-total graphs.

Key words: 1-quasi-total graph, regular, chromatic number, bi-partite graph, complete graphs

INTRODUCTION
Harary (1972) made an aperture on the concept of total graphs. Later on Bondy and Murty (1976) studied the applications of graph theory. Afterward Devadass (1977) made his contribution on regular graphs. Michalak (1981) made an attempt on middle and total graphs with coarseness number equal to 1. Further, Kulli (1989) investigated on planar graphs. Deo (2001) made an extensive study on this concept. After a while West (2002) developed the concept of quasi-total graphs. Srinivasulu (2006) derived some results on total graphs. Further, Mishra and Chandrasekaran (2007) made a contribution on total graphs.

This paper develops some of the properties of 1-quasi-total graph. A graph G is k-regular of $d(v) = k$ for all $v \in V$; a regular graph is one that is k-regular for some k (Bondy and Murty, 1976). The chromatic number, $\chi(G)$, of G is the minimum k for which G is k- colourable; if $\chi(G) = k$, G is said to be k- chromatic (Bondy and Murty, 1976; Deo, 2001). Unless otherwise mentioned all the graphs that we consider are finite graphs.

1-quasi-TOTAL GRAPHS
Definition 1: Let G be a graph with vertex set V(G) and edge set E(G). The 1-quasi-total graph (denoted by $Q_1(G)$) of G is defined as follows: The vertex set of $Q_1(G)$ that is:

$$V(Q_1(G)) = V(G) \cup E(G)$$

Two vertices x, y in $V(Q_1(G))$ are adjacent if they satisfy one of the following conditions:

- x, y are in V(G) and $\overline{xy} \in G$
- x, y are in E(G)
- x, y are incident in G

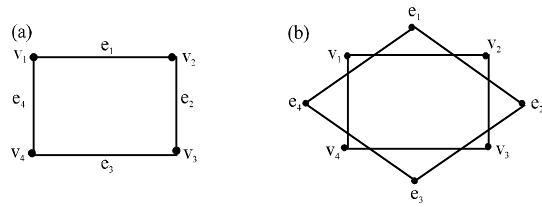

Fig. 1(a-b): 1-quasi-total graph $Q_1(G)$ of graph G

where, G is a subgraph of $Q_1(G)$ and $Q_1(G)$ is a subgraph of T(G).

Example: Consider the graph G given in Fig. 1a. Let us construct the 1-quasi-total graph $Q_1(G)$ of the Fig. 1b:

$$V(Q_1(G)) = \{V(G)\cup E(G)\} = \{v_1, v_2, v_3, v_4, e_1, e_2, e_3, e_4\}$$

We know that:

$$E(G)\subseteq E(Q_1(G))$$

So, $\overline{v_1v_4}$, $\overline{v_4v_3}$, $\overline{v_3v_2}$ and $\overline{v_2v_1} \in E(Q_1(G)$:

- Since, e_1 and e_2 are incident in G, there is an edge $\overline{e_1e_2} \in E(Q_1(G))$
- Since, e_1 and e_4 are incident in G, there is an edge $\overline{e_1e_4} \in E(Q_1(G))$
- Since, e_2 and e_3 are incident in G, there is an edge $\overline{e_2e_3} \in E(Q_1(G))$
- Since, e_3 and e_4 are incident in G, there is an edge $\overline{e_3e_4} \in E(Q_1(G))$

Therefore:

$$E(Q_1(G)) = \{\overline{v_1v_4}, \overline{v_4v_3}, \overline{v_3v_2}, \overline{v_2v_1}, \overline{e_1e_2}, \overline{e_1e_4}, \overline{e_2e_3}, \overline{e_3e_4}\}$$

The 1-quasi-total graph $Q_1(G)$ is given by the Fig. 1b.

Theorem 1 : If G = (V, E) is a k- regular graph, then $Q_1(G)$ is a ring sum of exactly one k- regular graph G with $|V|$ vertices and one 2(k-1) regular line graph with $|E|$ vertices.

Proof: By the definition of $Q_1(G)$:

$$V(Q_1(G)) = V(G)\cup E(G) = V(G)\cup V(L(G)) \text{ (since, } V(L(G)) = E(G))$$

Let $s\in E(G)$. If s is an edge in G, then $s\in E(G)$. If $s\notin E(G)$, then (by the definition of $Q_1(G)$), $s = \overline{e_1e_2}$ where e_1, $e_2\in E(G)$ and e_1, e_2 are adjacent edges in G. By the definition of L(G) it follows that $s\in E(L(G))$. Therefore:

$$E(Q_1(G))\subseteq E(G)\cup E(L(G))$$

By Note 1, $E(G) \cup E(L(G)) \subseteq E(Q_1(G))$. Now we proved that $Q_1(G) = G \cup L(G)$, the union of the two graphs G and L(G). Since, $V(G) \cap V(L(G)) = V(G) \cap E(G) = \phi$, there exists no common edge in G and L(G). This means that $E(G) \cap E(L(G)) = \phi$. This implies that $G \cup L(G) = G \oplus L(G)$.

Hence, $Q_1(G) = G \cup L(G) = G \oplus L(G)$. We show that $Q_1(G)$ is a ring sum of K- regular graph G and one 2(k-1) regular line graph. By the above result it is enough to prove that G is regular and L(G) is 2(k-1) regular. By hypothesis G is k-regular. We have to show L(G) is 2(k-1) regular graph.
We have to prove degree of $e \in V(L(G))$ is 2(K-1).

Let $e \in V(L(G))$, then e is $e = \overline{uv}$ for some u, $v \in V(G)$. Since, e is an edge of G, then e is adjacent with 2(K-1) edges of G. Hence, the degree $e \in V(L(G))$ is 2(K-1). Hence, the result.

Corollary 1: If G is a cycle of length n, then $Q_1(G)$ is the ring sum of exactly two disjoint cycles of length n.

Proof: Suppose G is a cycle of length n. From Mishra and Chandrasekaran (2007), we have that, if G is a cycle of length 'n', then L(G) is a cycle of length n. Since, $Q_1(G) = G \oplus L(G)$ (by above theorem 1) Q_1 (G) is equal to the ring sum of two disjoint cycles of length 'n'. The proof is complete.

Corollary 2: For all n, $Q_1(K_n)$ is a ring sum of r-regular K_n and 2(r-1) regular LK_n graph, where 'r' denotes the degree of the vertices in complete graph K_n on n vertices.

Proof: Consider r- regular K_n, $n \in N$, then d(v) = r for all $v \in V(K_n)$. We have to prove $Q_1(K_n)$ contains exactly one r-regular K_n and 2(r-1)-regular $L(K_n)$. Since, $Q_1(K_n) = K_n \oplus L(K_n)$. Clearly by hypothesis K_n is r-regular. We have to prove $L(K_n)$ is 2(r-1) regular. Let $e \in V(L(K_n))$ then $e = \overline{uv}$ for some, $v \in V(K_n)$, that implies $e \in E(K_n)$, then 'e' is adjacent with 2(r-1) edges of G. Hence, the degree of $e \in V(L(K_n))$ is 2(r-1), d(e) = 2(r-1), for all $e \in V(L(K_n))$. Therefore, L (K_n) is 2(r-1) regular.

Theorem 2: For any $K_{m,n}$($m \neq n$) graph with the bipartition X, Y, the degree of the vertices in the X-Set is n, the degree of the vertices in Y-set is m and the remaining vertices have degree m+n-2 in $Q_1(K_{m,n})$ of $K_{m,n}$.

Proof: Let X, Y be a partition of $K_{m,n}$. X contains 'm' vertices and Y contains 'n' vertices. Let $v \in X$, then, v is adjacent with 'n' vertices in Y. Therefore, v has degree 'n' in $Q_1(K_{m,n})$. Hence, the degree of the vertices in X is 'n' in $Q_1(K_{m,n})$. Similarly $u \in Y$ then, u is adjacent with 'm' vertices in X. Therefore, u has degree 'm' in $Q_1(K_{m,n})$. Hence, the degree of every vertex in Y is 'm' in $Q_1(K_{m,n})$. Let 'e' be an edge of $K_{m,n}$. Clearly 'e' is incident with two vertices $e = \overline{uv}$, for some, $u \in X$, $v \in Y$. Since, 'u' is incident with 'n' edges including 'e'. Similarly 'v' is incident with 'm' edges including 'e'. Therefore, 'e' is adjacent with (n-1) edges through 'u' and (m-1) edges through 'v'. Hence, the degree of 'e' in $Q_1(K_{m,n})$ is(n-1)+(m-1) = m+n-2. This completes the proof.

Example 2: The following example illustrates the Theorem 2 (Fig. 2).

Theorem 3: Let G be a graph with v>1, e≥1, then $\chi(Q_1(G)) \geq 2$.

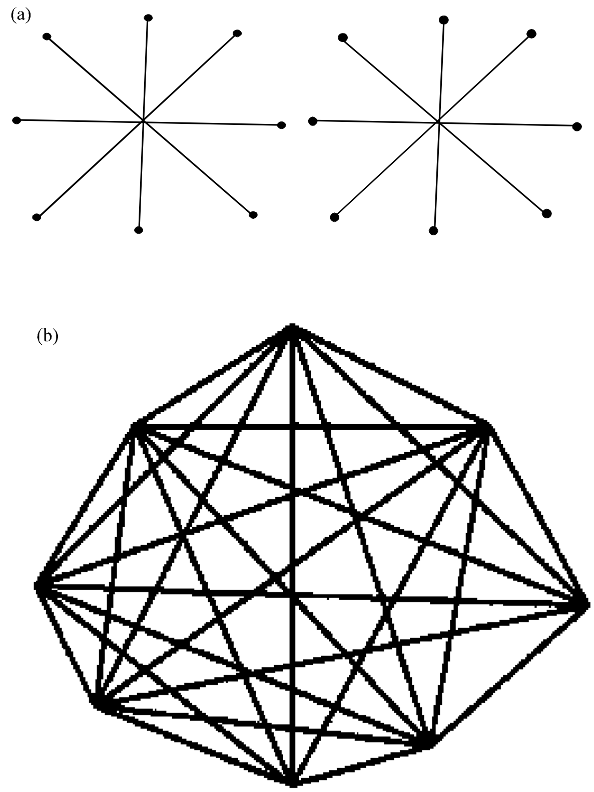

Fig. 2(a-b): The Graph of (a) $K_{1,8}$ (b) $Q_1(K_{1,8})$

Proof: Let 'e' be an edge of G:

$$e = \overline{uv}$$

Therefore, the vertex $e \in V(Q_1(G))$ is adjacent with vertices 'u' and 'v' in G.

$$\chi(Q_1(G)) \geq 2$$

Corollary 3: The Chromatic number (Mishra and Chandrasekaran, 2007) of $k_{1,n}$ of $Q_1(k_{1,n})$ is 2.

Corollary 4: The chromatic number of line graph of $Q_1(k_{1,n})$ is n-1.

CONCLUSION

This study illustrates some remarkable properties of 1-quasi-total graphs. It has been obtained certain results on graphs such as regular graphs, complete graphs, complete bipartite, cycles and chromatic number of 1-quasi-total graphs. Also the above discussion investigates the concepts of 1-quasi-total graph in regular and complete graphs. Also it has been deliberated the chromatic number of 1-quasi-total graph and 1-quasi-total graph of a complete bi- partite graphs.

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