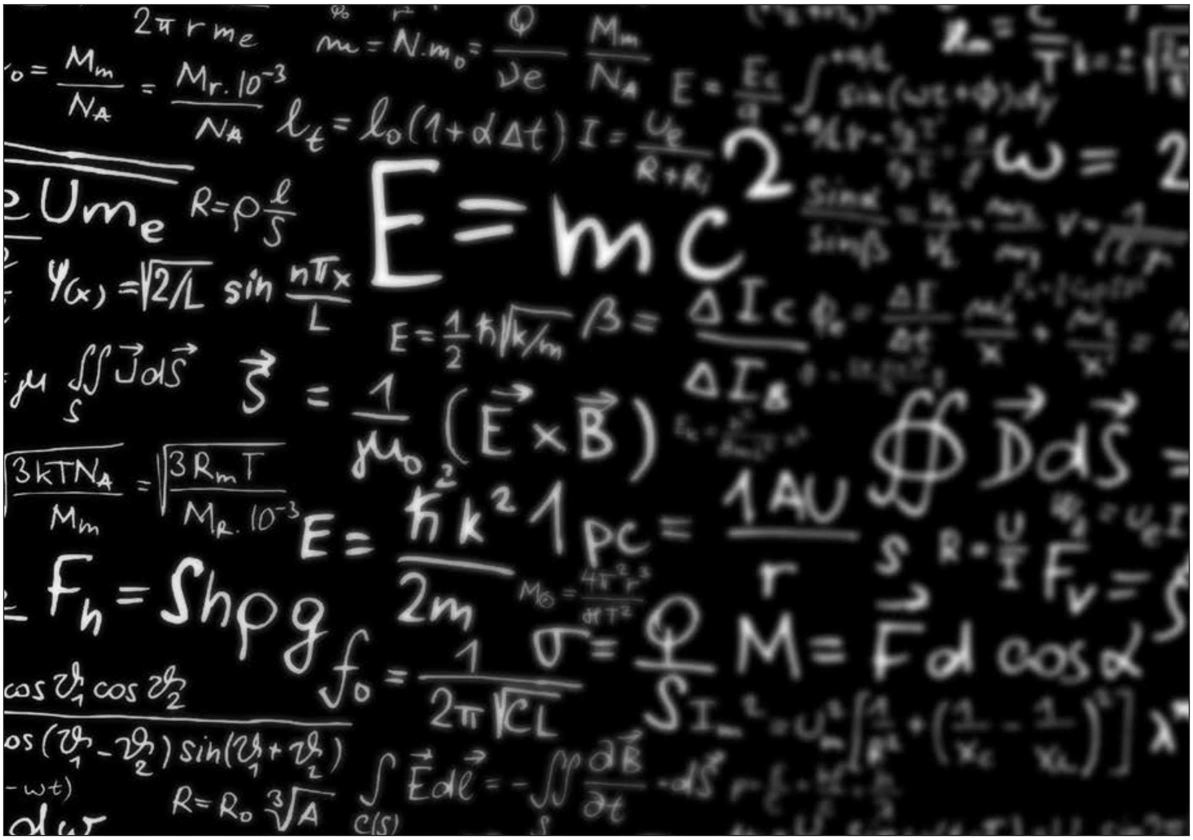

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Bayesian Generalized Least Squares with Parametric Heteroscedasticity Linear Model

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ABSTRACT

We investigate the asymptotic finite properties of estimator to ascertain its behaviour from small to large sample when there is presence of heteroscedasticity. We explore full Bayesian experiments with Generalized Least Squares estimator incorporating heteroscedastic error structure. Estimates were obtained through Markov Chain Monte Carlo approach that draws simulated sample of parameters from joint posterior distribution. Burnin and thinning were chosen as 1000 and 5, respectively. Bias and Mean Squares Error criteria were used to evaluate finite properties of the estimator. We choose the following sample sizes: 25, 50, 100, 200, 500 and 1000. Thus, 10,000 simulations with varying degree of heteroscedasticity were carried out. This is subjected to the level of convergence. Bias and Minimum Mean Squares Error criteria revealed improving performance asymptotically regardless of the degree of heteroscedasticity. Considering heteroscedasticity at scale 0.3, from the results, we observed an increase in sample sizes: 25, 50, 100, 200, 500 and 1000 led to decrease in mean squares error: 0.068436, 0.033896, 0.015071, 0.006772, 0.001935 and 0.00101, respectively. This implies efficiency of the estimator asymptotically, ditto for all other scales.

Key words: Markov Chain Monte Carlo method, heteroscedasticity, bayesian generalized least squares, metropolis-hasting algorithm

INTRODUCTION

Bayesian inference is an alternative to classical statistical point of view. In a Bayesian framework, the knowledge about the parameters of the model is described by a probability distribution.

The generalized least squares estimation does not bring about uncertainty of model estimates for both error variance and variance of parameters β (Reis *et al*., 2005). It has been observed in the previous studies that the consequences of heteroscedasticity when it is present in the data and/or model lead to poor inferences of parameter estimates. Though, the ordinary least squares estimation may be unbiased but it is no longer efficient. Thus, standard error that uses ordinary least squares estimates becomes invalid while confidence interval and hypothesis test that make use of the computed standard error are equally invalid, therefore, estimator lost its properties of efficiency and consistency (Guermat and Hadri, 1999; Robinson, 1987; White, 1980).

The joint posterior distribution is the product of likelihood and prior which is divided by normalizing constant, thus, normalizing constant often portend computational intensive. It is usually assumed equal to unity. Meanwhile, Markov Chain Monte Carlo simulation technique that draw correlated sample of parameters from the joint posterior distribution with normalizing constant set to unity would proffer solution to the problem of intensive computation (Gilks *et al*., 1996).

The consequences of heteroscedasticity when it is present in the model and/or data, if the researcher fails to correct for it will lead to substantial bias, inefficiency of the estimator and poor inferences of parameters estimate. Thus wrong conclusion will be made.

To avoid those problems, we use a fully Bayesian approach, which automatically averages over our uncertainty in the model parameters. Ohtani (1982) opined that the Bayesian estimator for heteroscedasticity linear model produced best properties using the mean squares error criterion compare with traditional estimator.

In this study, multiplicative double sided error structure with one component was incorporated. We extend our study to the multidimensional and more complicated cases and carry out simulation using MCMC; this study also examined the finite sample properties of the estimator. These are the gaps, this study decides to fill.

MODEL DESIGNS

Let $y = X\boldsymbol{\beta}+u$ with $u \sim N(0_t \sigma_i^2 \Omega)$, where, Ω is a positive definite matrix of order $n \times n$. A case where $u \sim N(0_t \sigma^2 \Omega)$ is a homoscedastic model with constant variance, but when $u \sim N(0_t \sigma_i^2 \Omega)$ indicates unequal variances of the diagonal element of $n \times n$ matrix of $E(uu')$ which is regarded as heteroscedastic error structure:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u \tag{1}$$

Let X denote X_1 and X_2 with multiplicative heteroscedasticity using Harvey (1976) which can be expressed as $\sigma_i^2 = \sigma_2(\boldsymbol{\beta}_0+\boldsymbol{\beta}_1 X_1+\boldsymbol{\beta}_2 X_2)\lambda$, where, λ is an unknown parameter which determine the degree of heteroscedasticity. Adopting a full Bayesian inference, we examine the likelihood function, prior distribution for the parameters, hyper-parameters in the model, with MCMC algorithm.

The likelihood function of $\boldsymbol{\theta}$, where $\boldsymbol{\theta} = (\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \lambda)$ give the sample vector X_1, $X_2 = (1, 2, ..., n)$ and $y = (y_1, y_2, ..., y_n)$ is expressed as:

$$L(\theta, \sigma \mid X, y) = (2\pi\sigma^2)^{-n/2} \Pi_{i=1}^{n} \exp\left\{-\frac{1}{2\sigma^2} \sum\nolimits_{i=1}^{n} [y_i - x\beta]^2\right\} \tag{2}$$

Incorporating multiplicative heteroscedastic into our likelihood estimator we derived from the product of the error density function. Thus our error u is changed to w:

$$L(\theta, \sigma \mid X, y) = (2\pi\sigma^2)^{-n/2} \Pi_{i=1}^{n} \mid w^{-\lambda/2} \exp\left\{-\frac{1}{2\sigma^2} \sum\nolimits_{i=1}^{n} (y_i - x\beta) w^{-\lambda} (y_i - x\beta)\right\} \tag{3}$$

To derive the full Bayesian density, we truncate the error density function Eq. 3 with multinomial distribution density and inverse-gamma distribution.

Marginal posterior density is obtained by integrating the joint posterior density with respect to each parameter, thus, expert opinion can be adopted by assuming the set of parameters $\boldsymbol{\beta}_0$, $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, λ and σ as independent marginal distribution.

We assumed a prior density $\pi(\beta_0, \beta_1, \beta_2, \lambda, \sigma) = \pi(\beta_0)\pi(\beta_1)\pi(\beta_2)\pi(\lambda)\pi(\sigma)$. Thus, multivariate normal distribution is considered for β, while inverse gamma is considered for σ and a uniform distribution is considered for λ such that:

$$\pi(\beta) \propto (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2\sigma^2}(\beta - mn)^2\right\}, \beta > 0 \tag{4}$$

$$\pi(\sigma^2) \propto (\sigma^2)^{-a1+1} \exp(-b_1 / \sigma^2), \sigma^2 > 0 \tag{5}$$

$$\pi(\lambda) \propto c \quad c \text{ is constant} \tag{6}$$

The posterior distribution of $\theta = (\beta_0, \beta_1, \beta_2, \lambda, \sigma)$. Considering independence among the parameters is given by:

$$\begin{gathered}\pi(\beta_0, \beta_1, \beta_2, \lambda, \sigma \mid X, y) \propto (2\pi\sigma^2)^{-\frac{n}{2}} (\sigma^2)^{-(a_1 - 1 - n/2)} \\ \exp\left\{-\frac{1}{2\sigma^2}(\beta - mu)^2\right\} \Pi_{i=1}^{n} \quad | w^{-\lambda/2} | \exp\{-\frac{1}{2}\sum_{i=1}^{n} \ (y_i - x\beta) w^{-\lambda} (y_i - x\beta)\}\end{gathered} \tag{7}$$

where, a_1, b_1 are the hyper-parameters for the inverse-gamma distribution. Hyper-parameters are excluded for β-parameters since they would be estimated from the data and may be arbitrarily small leading to problems which may eventually affect the inferences. Integrating the posterior $\pi(\beta, \lambda, \sigma \mid X, y)$ with respect to σ, thus, we have joint a posterior distribution for (β, λ):

$$\begin{gathered}\pi(\beta_0, \beta_1, \beta_2, \lambda, \sigma \mid X, y) \propto (2\pi)^{-\frac{n}{2}} \\ \exp\left\{-\frac{1}{2}(\beta - mu)^2\right\} \Pi_{i=1}^{n} \quad | w^{-\lambda/2} | \exp\{-b_1 - \frac{1}{2}\sum_{i=1}^{n} \ (y_i - x\beta) w^{-\lambda} (y_i - x\beta)\}^{-(a_1 - n/2)}\end{gathered} \tag{8}$$

Metropolis Hasting Algorithm update is performed on the full conditional distribution of:

$$\sigma^2 \propto IG(a_1 + \frac{n}{2}, b_1 + \frac{1}{2}\sum_{i=1}^{n} \ (y_i - x\beta) w^{-\lambda})(y_i - x\beta))$$

This yields the following full conditional density of the parameters β and σ:

$$\pi(\beta_0 \mid \lambda, X, y) \propto \exp\left\{-\frac{1}{2}(\beta_0 - mu)^2\right\} \prod_{i=1}^{n} \ | w^{-\lambda/2} | \exp\{-\frac{1}{2}\sum_{i=1}^{n} \ (y_i - x\beta) w^{-\lambda} (y_i - x\beta)\}^{-(a_1 - n/2)} \tag{9}$$

$$\pi(\beta_1 \mid \lambda, X, y) \propto \exp\left\{-\frac{1}{2}(\beta_1 - mu)^2\right\} \prod_{i=1}^{n} \ | w^{-\lambda/2} | \exp\{-\frac{1}{2}\sum_{i=1}^{n} \ (y_i - x\beta) w^{-\lambda} (y_i - x\beta)\}^{-(a_1 - n/2)} \tag{10}$$

$$\pi(\beta_2 \mid \lambda, X, y) \propto \exp\left\{-\frac{1}{2}(\beta_2 - mu)^2\right\} \prod_{i=1}^{n} \ | w^{-\lambda/2} | \exp\{-\frac{1}{2}\sum_{i=1}^{n} \ (y_i - x\beta) w^{-\lambda} (y_i - x\beta)\}^{-(a_1 - n/2)} \tag{11}$$

$$\pi(\sigma \mid \theta, X, y) \propto (\sigma^2)^{-(a_1-1-n/2)} \exp\{-b_1/\sigma^2\}\prod_{i=1}^{n} \mid w^{-\lambda/2} \mid \exp\{-\frac{1}{\sigma^2}(b_4 + \frac{1}{2}\sum_{i=1}^{n} (y_i - x\beta)w^{-\lambda}(y_i - x\beta))\}^{-(a_1-n/2)} \quad (12)$$

$$\pi(\lambda \mid \beta, X, y) \propto \prod_{i=1}^{n} \mid w^{-\lambda/2} \mid (b_1 + \frac{1}{2}\sum_{i=1}^{n} (y_i - x\beta)w^{-\lambda}(y_i - x\beta))\}^{-(a_4-n/2)} \quad (13)$$

POSTERIOR SIMULATION:MCMC

- Simulating $\boldsymbol{\beta}$:

Let $\theta_{-\beta}$ denote a vector θ without $\boldsymbol{\beta}$ parameters, then the MH sampler for $\boldsymbol{\beta}$ starts out with:

$$\beta^{(1)} \mid \theta_{-\beta}^{(0)} \sim p\left(\beta \mid \theta_{-\beta}^{(0)}, y, X\right)$$

where, the superscript r in $\boldsymbol{\beta}^{(r)}$ denotes the rth simulated $\boldsymbol{\beta}$, thus r = 0 is the initial value. For =1,...,R and R is the number of replications in the MCMC. The conditional probability in the right hand side denotes a fully conditional posterior pdf of $\boldsymbol{\beta}$. We scale λ as 0.0, 0.3, 0.5,0.6, 0.9,1 and 2. We chose 0.2 as our initial value of sigma squares. The full conditional distribution of $\boldsymbol{\beta}$ given $\theta_{-\beta}$ is $\beta \mid (\theta - \beta, y, X) \sim N(\hat{\beta}), \sigma^{-2}(X\Omega^{-1}X)^{-1}$.

- Simulating $\boldsymbol{\sigma}^2$:

The kernel of an inverse gamma pdf:

$$IG\left(\frac{n}{2}, \frac{\in \Omega^{-1} \in}{2}\right)$$

The initial value of $\theta\text{-}\sigma^2$ becomes $\theta^{(0)}{}_{-\sigma}{}^2 = (\boldsymbol{\beta}^{(1)}\text{ in})$, hence $\boldsymbol{\sigma}^{(1)}$ is generated as follows:

$$\sigma^{(1)} \mid \theta^{(0)}{}_{-\sigma^2} \sim IG\left(\frac{n}{2}, \frac{\in^{(1)} \Omega^{-1} \in^{(1)}}{2}\right)$$

where, $\epsilon^{(1)} = y\text{-}X\boldsymbol{\beta}^{(1)}$. For succeeding simulations we have:

$$\sigma^{(r)} \mid \theta^{(r-1)}{}_{-\sigma^2} \sim IG\left(\frac{n}{2}, \frac{\in^{(r)} \Omega^{-1} \in^{(r)}}{2}\right)$$

DATA GENERATION PROCESSES

In an attempt to investigate the asymptotic finite property of estimator of econometric model in the presence of heteroscedastic error structure, we adopted Markov Chain Monte Carlo Experiments. The sample sizes are specified with 6 sets as follow: 25, 50, 100, 200, 500 and 1000. Harvey (1976) multiplicative heteroscedastic error structure was adopted to truncate linear econometric model.

The scale of λ-heteroscedastic error structure is selected as 0.0 homoscedastic, 0.3 less heteroscedastic, 0.5 and 0.6-moderately heteroscedastic, 0.9-mildly heteroscedastic and 1 and 2-severely heteroscedastic.

According to Germa and Saez (2000), the distribution of the main regression is assumed to be moderately heteroscedastic when variance is proportional to x_1 where, x_1 ranged from 11 to 15-mesokurtos) and strongly heteroscedastic where x_2 ranged from 4 to 8-platokurtos.

The error term U is generated based on E(U) = 0 and $E(UU') = (\delta_0+\delta_1+x_1+\delta_2x_2)^{\lambda i}$, λi = 0.0, 0.3, 0.5, 0.6, 0.9, 1 and 2 δ_0, δ_1 and δ_2 are set at -2, 0.25 and 1, respectively. Thereafter, we incorporated U into the model and the parameters β_0, β_1 and β_2 are set at 10, 1 and 1, respectively to generate variable y. The number of replications of our experiment is set at 10,000 with burn-in of 1000 which specified the draws that were discarded to remove the effect of the initial values. The thinning is set at 5 to ensure the removal of the effect of autocorrelation in our MCMC simulation. For the Bayesian experiment, a Metropolis Hasting Algorithm was developed to simulate our heteroscedastic based models. This was invoked in [R 2.3.0.0] Statistical software.

RESULTS

In this study, we presented heteroscedastic truncated linear model, considering multiplicative heteroscedasticity structure. For the parameters obtained through the posterior point estimate of Metropolis-Hasting Algorithm simulation, we computed bias-measure of consistency-and mean squared error criterion-measure of efficiency. Hyper-parameter were arbitrarily chosen for σ^2. The levels of convergence of the chains were monitored using the method proposed by Gelman and Rubin (1992) and graphic analysis was carried out using coda package in R package. Multivariate normal and inverse gamma distributions were chosen as priors for parameter estimates and σ^2, respectively.

Performances of the GLS heteroscedastic linear model on the basis of bias criterion: Table 1 revealed the outcome of our estimation of GLS heteroscedastic linear model. The Bias for $\hat{\beta}_0$ at $\lambda = 0$ degree of heteroscedasticity decreases algebraically, at sample size 25 the bias is 0.02929, it decreases to 0.02457 as the sample size increases to 50, it equally decreases to 0.001261 when we increased the sample size to 100, the sample size 200 appeared to be turning point where the bias increases to 0.031881, as we increased the sample size to 500, the bias decreases algebraically to 0.010511, it was followed by a decrease in bias of -0.0293 as we increased the sample size to 1000. For all the other scales of heteroscedasticity, the bias for $\hat{\beta}_0$ where, λ equals 0.3, 0.5, 0.6, 0.9, 1 and 2 have the same characteristics as $\lambda = 0$. Thus, there is consistency for $\hat{\beta}_0$. The bias for β_1 is negative and absolutely decreases algebraically as sample size increases. The Bias for $\hat{\beta}_0$ at $\lambda = 0.3$ degree of heteroscedasticity absolutely decreases algebraically, at sample size 25 the bias is 0.00909, it decreases to 0.0039 as the sample size increases to 50, it decreases to 0.00075 when we increased the sample size to 100, the sample size 200 appeared to be turning point where the bias increases absolutely to 0.00187, as we increased the sample size to 500, the bias decreases algebraically to 0.00111, it was followed by an increase in bias of 0.001716 as we increased the sample size to 1000. For all the other scales of heteroscedasticity, the bias for $\hat{\beta}_1$ where λ equals 0.0, 0.5, 0.6, 0.9, 1 and 2 have the same characteristics as $\lambda = 0.3$. Thus, there is consistency for $\hat{\beta}_1$. The bias for $\hat{\beta}_2$ is interchangeable, it increases and decreases algebraically as sample size increases. The Bias for $\hat{\beta}_2$ at $\lambda = 0.9$ degree of heteroscedasticity absolutely decreases algebraically, at sample size 25 the bias is 0.00517, it decreases to 0.00014 as the sample size increases to 50, the sample size 100 appeared to be turning point where the bias increases to 0.002178, as we increased the sample size to 200, the bias decreases algebraically to 0.00309, the bias decreases to 0.00161 as the sample size increases to 500, it was followed by absolute increase, the bias is 0.00998 as we increased the sample size to 1000. For all the other scales of heteroscedasticity, the bias for $\hat{\beta}_2$ where λ equals 0.0,0.3, 0.5, 0.6, 1 and 2 have the same characteristics as $\lambda = 0.9$. Thus, there is consistency for $\hat{\beta}_2$.

Table 1: Bias of posterior estimation of heteroscedastic linear model

		Bias criterion		
Scales λ	Samples	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
0.0	25	0.02929	-0.00286	0.000844
	50	0.024579	-0.00176	-0.00013
	100	0.001261	-0.00056	0.00093
	200	0.031889	-0.00187	-0.00172
	500	0.010511	-0.00109	0.000511
	1000	-0.02953	0.003151	-0.00147
0.3	25	0.089236	-0.00909	0.003074
	50	0.052681	-0.0039	-5.87E-05
	100	0.001848	-0.00075	0.001201
	200	0.031889	-0.00187	-0.00172
	500	0.009018	-0.00111	0.000788
	1000	0.00877	0.001716	-0.00493
0.5	25	0.108772	-0.01104	0.003661
	50	0.063666	-0.00471	-7.85E-05
	100	0.002254	-0.00092	0.001465
	200	0.038697	-0.00227	-0.00209
	500	0.010536	-0.00129	0.000924
	1000	0.010221	0.001999	-0.00577
0.6	25	0.120072	-0.01216	0.003993
	50	0.06998	-0.00518	-9.08E-05
	100	0.002488	-0.00101	0.001618
	200	0.042621	-0.00251	-0.0023
	500	0.011723	-0.00144	0.001031
	1000	0.011355	0.002221	-0.00642
0.9	25	0.161422	-0.01626	0.00517
	50	0.092877	-0.00687	-0.00014
	100	0.003344	-0.00135	0.002178
	200	0.056914	-0.00336	-0.00309
	500	0.018118	-0.00223	0.00161
	1000	0.017452	0.003415	-0.00998
1	25	0.178124	-0.0179	0.005632
	50	0.102047	-0.00755	-0.00016
	100	0.00369	-0.00149	0.002405
	200	0.062661	-0.00371	-0.00341
	500	0.021757	-0.00268	0.001944
	1000	0.020919	0.004094	-0.01203
2	25	0.474372	-0.04673	0.012949
	50	0.2603	-0.01925	-0.00067
	100	0.009761	-0.00391	0.006475
	200	0.163077	-0.00971	-0.00912
	500	0.370433	-0.04635	0.036835
	1000	0.358685	0.071985	-0.22717

Considering the degree of heteroscedasticity, we observed that the bias for $\hat{\beta}_0$ increases algebraically as the scale of heteroscedasticity increases, with sample size 25 at $\lambda = 0$ the bias is 0.02929 which increases to 0.089236 as λ increases to 0.3, as we increased λ to 0.5 so also the bias increases to 0.108772, the bias increases to 0.120072 when λ increases to 0.6, thus the bias increases to 0.161422 as λ increases to 0.9, we observed that the bias increases to 0.178124 as we increased λ to 1, we recorded an increased bias of 0.474372 when we increased λ to 2. Thus, we have the same pattern for all other sample sizes. The bias for $\hat{\beta}_1$ is negative and absolutely increases algebraically as the scale of heteroscedasticity increases, with sample size 50 at $\lambda = 0$ the bias is 0.00176 which increases to 0.0039 as the λ increases to 0.3, as we increased λ to 0.5 so also the bias increases to 0.00471, the bias increases to 0.00518 when λ increases to 0.6, thus, the bias increases to 0.00687 as λ increases to 0.9, we observed that the bias increases to 0.00755 as we increased λ to 1, we recorded an increased bias of 0.01925 when we increased λ to 2. Thus, we have the same pattern for all other sample sizes. The bias for $\hat{\beta}_2$ absolutely increases algebraically as the scale of heteroscedasticity increases, with sample size 100 at $\lambda = 0$ the bias is 0.00056 which increases to 0.0012101 as λ increases to 0.3, as we increased λ to 0.5 so also the bias increases to 0.001465, the bias increases to 0.001618 when λ increases to 0.6, thus the bias increases to 0.002178 as λ increases to 0.9, we observed that the bias increases to 0.002405 as we increased λ to 1, we recorded an increased bias of 0.006475 when we increased λ to 2. Thus, we have the same pattern for all other sample sizes.

Performances of the BGLS heteroscedastic linear model on the basis of mean squares error criterion: Table 2 revealed the mean squared error (mse) criterion, the mean squares error for $\hat{\beta}_0$ decreases algebraically as the sample size increases irrespective of the scale of heteroscedasticity. The mse for $\hat{\beta}_0$ at $\lambda = 0$ degree of heteroscedasticity decreases as the sample size increases, at sample size 25 the mse is 1.272, it decreases to 1.3202 as the sample size increases to 50, it decreases to 1.2979 when we increased the sample size to 100, it decreases to 1.2947 as the sample size increases to 200, as we increased the sample size to 500, the mse decreases to 0.3289, likewise we observed decrease of 0.15224 in mse as the sample size increases to 1000. For all the other scales of heteroscedasticity, the mse for $\hat{\beta}_0$ where, λ equals 0.3, 0.5, 0.6, 0.9, 1 and 2 have the same characteristics as $\lambda = 0$. Thus, there is efficiency for $\hat{\beta}_0$. The mse for $\hat{\beta}_1$ at $\lambda = 0.3$ degree of heteroscedasticity decreases as the sample size increases, at sample size 25 the mse is 0.0684, it decreases to 0.03389 as the sample size increases to 50, it decreases to 0.01507 when we increased the sample size to 100, it decreases to 0.006772 as the sample size increases to 200, as we increased the sample size to 500, the mse decreases to 0.001935, likewise we observed decrease of 0.00101 in mse as the sample size increases to 1000. For all the other scales of heteroscedasticity, the mse for $\hat{\beta}_1$ where λ equals 0.0, 0.5, 0.6, 0.9, 1 and 2 have the same characteristics as $\lambda = 0.3$. Thus, there is efficiency for $\hat{\beta}_1$. The mse for $\hat{\beta}_2$ at $\lambda = 0.6$ degree of heteroscedasticity decreases as the sample size increases, at sample size 25 the mse is 0.128, it decreases to 0.04727 as the sample size increases to 50, it decreases to 0.02337 when we increased the sample size to 100, it decreases to 0.01274 as the sample size increases to 200, as we increased the sample size to 500, the mse decreases to 0.002859, thus we observed decrease of 0.00133 in mse as the sample size increases to 1000. For all the other scales of heteroscedaticity, the mse for $\hat{\beta}_2$ where λ equals 0.0, 0.3, 0.5, 0.9, 1 and 2 have the same characteristics as $\lambda = 0.6$. Thus, there is efficiency for $\hat{\beta}_2$. Moreover, the mean squares error for all parameters have asymptotic efficiency since their mse decreases as the sample size increases.

Considering the degree of heteroscedasticity, we observed that the mean squares error (mse) for $\hat{\beta}_0$ increases algebraically as the scale of heteroscedasticity increases, with sample size 25 at $\lambda = 0$ the mse is 1.272 which increases to 11.7418 as λ increases to 0.3, as we increased λ to 0.5 so

Table 2: Mean squared error criterion of posterior estimation

		Mean squared errors		
Scale λ	Samples	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
0.0	25	1.272007	0.006828	0.006972
	50	1.320185	0.00715	0.006918
	100	1.297923	0.007015	0.007025
	200	1.294664	0.006772	0.007068
	500	0.328895	0.001629	0.001401
	1000	0.152244	0.000859	0.000662
0.3	25	11.74183	0.068436	0.070956
	50	6.052991	0.033896	0.026202
	100	2.992254	0.015071	0.012969
	200	1.294664	0.006772	0.007068
	500	0.389045	0.001935	0.001685
	1000	0.173	0.00101	0.000782
0.5	25	17.4542	0.101115	0.105142
	50	8.842747	0.049496	0.038832
	100	4.413156	0.02236	0.019202
	200	1.906254	0.010019	0.010467
	500	0.530549	0.00265	0.002305
	1000	0.235836	0.001381	0.001071
0.6	25	21.27469	0.122864	0.128
	50	10.68487	0.059791	0.047277
	100	5.358008	0.027226	0.023366
	200	2.312379	0.012181	0.012738
	500	0.656467	0.003287	0.002859
	1000	0.29174	0.001712	0.001329
0.9	25	38.48439	0.220104	0.231033
	50	18.82841	0.105264	0.085348
	100	9.578117	0.049077	0.042114
	200	4.122471	0.021858	0.022958
	500	1.564625	0.00791	0.006904
	1000	0.694781	0.004114	0.003213
1	25	46.87465	0.267191	0.281332
	50	22.73329	0.127049	0.103933
	100	11.62012	0.059698	0.051256
	200	4.996642	0.026547	0.027938
	500	2.254817	0.011442	0.010022
	1000	1.001007	0.005948	0.004664
2	25	333.7871	1.833107	2.024595
	50	148.1523	0.823949	0.746992
	100	79.44869	0.417739	0.366433
	200	33.80501	0.182878	0.19899
	500	656.4161	3.424931	3.701633
	1000	291.0903	1.80265	1.681897

also the mse increases to 17.454, the mse increases to 21.2747 when λ increases to 0.6, thus the mse increases to 38.4843 as λ increases to 0.9, we observed that the mse increases to 46.8746 as we increased λ to 1, we recorded an increase mse of 333.787 when we increased λ to 2. Thus, we have the same pattern for all other sample sizes. The mse for $\hat{\beta}_1$ increases algebraically as the scale of heteroscedasticity increases, with sample size 50 at $\lambda = 0$ the mse is 0.00715 which increases to 0.03389 as λ increases to 0.3, as we increased λ to 0.5 so also the mse increases to 0.04949, the mse increases to 0.05979 when λ increases to 0.6, thus, the mse increases to 0.10526 as λ increases to 0.9, we observed that the mse increases to 0.12705 as we increased λ to 1, we recorded an increase mse of 0.82395 when we increased λ to 2. Thus, we have the same pattern for all other sample sizes. The mse for $\hat{\beta}_2$ increases algebraically as the scale of heteroscedasticity increases, with sample size 100 at λ the mse is 0.00703 which increases to 0.01297 as λ increases to 0.3, as we increased λ to 0.5 so also the mse increases to 0.0192, the mse increases to 0.02337 when λ increases to 0.6, thus, the mse increases to 0.04211 as λ increases to 0.9, we observed that the mse increases to 0.05125 as we increased λ to 1, we recorded an increase mse of 0.3664 when we increased λ to 2. Thus, we have the same pattern for all other sample sizes.

CONCLUSION

In this study, we have presented a simple way of modeling and estimating heteroscedastic linear model under simulation approach (MCMC). We observed that modeling heteroscedasticity in a full Bayesian improve the precision of the inferences of the parameter estimates. We conclude that asymptotically there exist consistency and efficiency in the estimation. Our approach can be applied to further studies in the area of simultaneous equation and other econometric models.

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