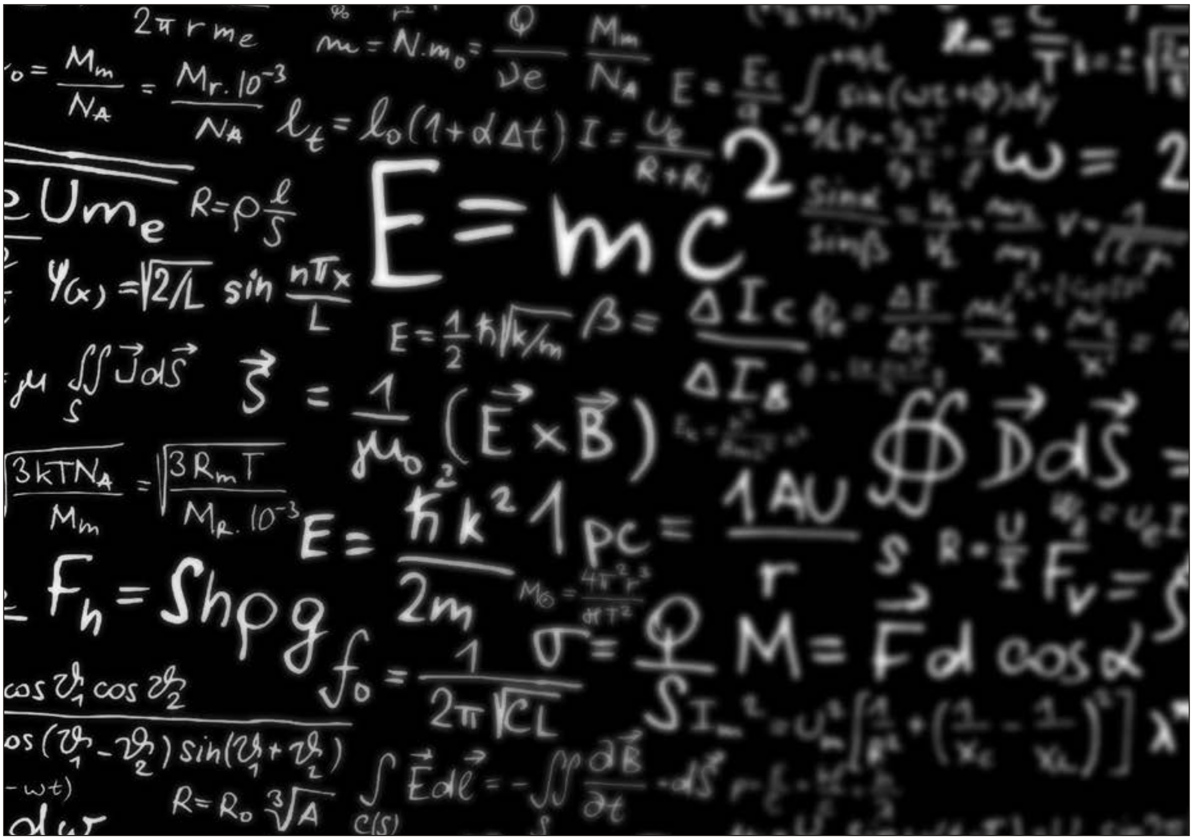

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Upper Bounds for Ruin Probability in a Generalized Risk Process under Rates of Interest with Homogenous Markov Chain Claims

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ABSTRACT

The aim of this study is to give upper bounds for ruin probabilities of generalized risk processes under interest force with homogenous Markov chain claims. Generalized Lundberg inequalities for ruin probabilities of these processes are derived by the martingale approach.

Key words: Supermartingale, optional stopping theorem, ruin probability, homogeneous markov chain

INTRODUCTION

Modern insurance businesses allow insurers to invest their wealth into financial assets. Since a large part of the surplus of insurance businesses comes from investment income, actuaries have been studying ruin problems under risk models with interest force. For example, Sundt and Teugels (1995, 1997) studied the effects of constant rate on the ruin probability under the compound Poisson risk model. Yang (1999) established both exponential and non-exponential upper bounds for ruin probabilities in a risk model with constant interest force and independent premiums and claims. Cai (2002a, b) investigated the ruin probabilities in two risk models, with independent premiums and claims and used a first-order autoregressive process to model the rates of in interest. Cai and Dickson (2004) obtained Lundberg inequalities for ruin probabilities in two discrete-time risk process with a Markov chain interest model and independent premiums and claims.

In this study, we study the models considered by Cai and Dickson (2004) to the case homogenous markov chain claims, independent rates of interest and independent premiums. The main difference between the model in our study and the one in Cai and Dickson (2004) is that claims in our model are assumed to follow homogeneous Markov chains. Generalized Lundberg inequalities for ruin probabilities of these processes are derived by the martingale approach.

In this study, we study two style of premium collections. On one hand of the premiums are collected at the beging of each period then the surplus process $\{U_n^{(1)}\}_{n\geq 1}$ with initial surplus u can be written as:

$$U_n^{(1)} = U_{n-1}^{(1)}(1+I_n) + X_n - Y_n \quad (1)$$

which can be rearranged as:

$$U_n^{(1)} = u.\prod_{k=1}^{n}(1+I_k) + \sum_{k=1}^{n}\left(X_k - Y_k\right)\prod_{j=k+1}^{n}(1+I_j) \quad (2)$$

On the other hand, if the premiums are collected at the end of each period, then the surplus process $\{U_n^{(2)}\}_{n\geq 1}$ with initial surplus u can be written as:

$$U_n^{(2)} = (U_{n-1}^{(2)} + X_n)(1 + I_n) - Y_n \tag{3}$$

which is equivalent to:

$$U_n^{(2)} = u.\prod_{k=1}^{n}(1 + I_k) + \sum_{k=1}^{n}\left[X_k(1 + I_k) - Y_k\right]\prod_{j=k+1}^{n}(1 + I_j) \tag{4}$$

where, throughout this study, we denote:

$$\prod_{t=a}^{b} x_t = 1$$

and:

$$\sum_{t=a}^{b} x_t = 0$$

if a>b.

We assume that:

Assumption 1: $U_o^{(1)} = U_o^{(2)} = u>0$

Assumption 2: $X = \{X_n\}_{n\geq 0}$ is sequence of independent and identically distributed non-negative continuous random variables with the same distribution function $F(x) = P(X_0 \leq x)$

Assumption 3: $\{I_n\}_{n\geq 0}$ is sequence of independent and identically distributed non-negative continuous random variables with the same distribution function $G(t) = P(I_0 \leq t)$

Assumption 4: $\{Y_n\}_{n\geq 0}$ is a homogeneous Markov chain such that for any n, Y_n takes values in a countable set of non-negative numbers $E = \{y_1, y_2, \ldots, y_n, \ldots\}$ with $Y_0 = y_i \in E$ and:

$$p_{ij} = P\left[Y_{m+1} = y_j \middle| Y_m = y_i\right], (m \in N); y_i, y_j \in E$$

Where:

$$0 \leq p_{ij} \leq 1, \sum_{j=1}^{+\infty} p_{ij} = 1$$

Assumption 5: X, Y and I are assumed to be independent

We define the finite time and ultimate ruin probabilities in model (1) with assumption 1 to assumption 5, respectively, by:

$$\psi_n^{(1)}(u,y_i) = P\left(\bigcup_{k=1}^{n}(U_k^{(1)} < 0)\middle| U_o^{(1)} = u, Y_o = y_i\right) \tag{5}$$

$$\psi^1(u,y_i) = \lim_{n\to\infty}\psi_n^{(1)}(u,y_i) = P\left(\bigcup_{k=1}^{\infty}(U_k^{(1)} < 0)\middle| U_o^{(1)} = u, Y_o = y_i\right) \tag{6}$$

Similarly, we define the finite time and ultimate ruin probabilities in model (3) with assumption 1 to assumption 5, respectively, by:

$$\psi_n^{(2)}(u,y_i) = P\left(\bigcup_{k=1}^{n}(U_k^{(2)} < 0)\middle| U_o^{(2)} = u, Y_o = y_i\right) \tag{7}$$

$$\psi^{(2)}(u,y_i) = \lim_{n\to\infty}\psi_n^{(2)}(u,y_i) = P\left(\bigcup_{k=1}^{\infty}(U_k^{(2)} < 0)\middle| U_o^{(2)} = u, Y_o = y_i\right) \tag{8}$$

In this study, we derive probability inequalities for $\psi^{(1)}(u, y_i)$ and $\psi^{(2)}(u, y_i)$ by the martingale approach.

UPPER BOUNDS FOR PROBABILITY BY THE MARTINGALE APPROACH

To establish probability inequalities for ruin probabilities of model (1), we first proof the following Lemma.

Lemma 1: Let model (1) satisfy assumptions 1 to 5.

Any $y_i \in E$, if:

$$M = \max\{y_i; y_i \in E\} < +\propto$$

$$E(Y_1 | Y_o = y_i) < E(X_1) \text{ and } P\left((Y_1 - X_1)(1+I_1)^{-1} > 0 \middle| Y_o = y_i\right) > 0 \tag{9}$$

then, there exists a unique positive constant R_i satisfying:

$$E\left(e^{R_i(Y_1-X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right) = 1 \tag{10}$$

Proof: Define:

$$f_i(t) = E\left\{e^{t(Y_1-X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right\} - 1; t \in (0, +\infty)$$

We have:

$$f_i(t) = h_i(t) - 1$$

Where:

$$h_i(t) = E\{e^{t(y_1-x_1)(1+I_1)^{-1}} | Y_0 = y_i\} = \sum_{j=1}^{+\infty} p_{ij} \int_0^{+\infty}\int_0^{+\infty} e^{\frac{t(y_1-x)}{1+y}}.f(x).g(y)dxdy$$

With:

$$f(x) = F'(x),\ g(y) = G'(y)$$

With:

$$n\in\mathbb{N}^* = \mathbb{N}\setminus\{0\}$$

Then:

$$\sum_{j=1}^{+\infty} p_{ij}\int_0^{+\infty}\int_0^{+\infty}\left[\frac{y_j-x}{1+y}\right]^n \cdot e^{\frac{t(y_j-x)}{1+y}} f(x)g(y)dxdy \le \sum_{j=1}^{+\infty} P_{ij}\int_0^{+\infty}\int_0^{+\infty} y_j^n e^{ty_j} f(x)g(y)dxdy$$
$$= \sum_{j=1}^{+\infty} p_{ij} y_j^n e^{ty_j} \le M^n \cdot e^{tM} \forall t \in (o, +\infty)$$

This implies that $h_i(t)$ has n-th derivative function on $(0, +\infty)$ with $n\in\mathbb{N}^*$. Thus, $f_i(t)$ has n-th derivative function on $(0, +\infty)$ with $n\in\mathbb{N}^*$ and:

$$f_i^{'}(t) = E\left\{(Y_1 - X_1)(1+I_1)^{-1} e^{t(Y_1-X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right\}$$

$$f_i^{''}(t) = E\left\{\left[(Y_1 - X_1)(1+I_1)^{-1}\right]^2 e^{t(Y_1-X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right\} \ge 0$$

Which implies that:

$$f_i(t) \text{ is a convex function with } f_i(0) = 0 \tag{11}$$

and:

$$f_i^{'}(0) = E\left\{(Y_1 - X_1)(1+I_1)^{-1} \middle| Y_o = y_i\right\} \le E\left(Y_1 \middle| Y_o = y_i\right) - E(X_1) < 0 \tag{12}$$

By $P((Y_1\text{-}X_1)(1+I_1)^{-1}>0|Y_0 = y_i)>0$, we can find some constant $\delta>0$ such that:

$$P((Y_1\text{-}X_1)(1+I_1)^{-1}>\delta>0|Y_0 = y_i)>0$$

Then, we can get that:

$$f_i(t) = E\left\{e^{t(Y_1-X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right\} - 1 \ge E\left\{e^{t(Y_1-X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right\}.1_{\left\{(Y_1-X_1)(1+I_1)^{-1}>\delta \middle| Y_o = y_i\right\}} - 1 \ge e^{t\delta}.P(\left\{(Y_1 - X_1)(1+I_1)^{-1} > \delta \middle| Y_o = y_i\right\} - 1$$

This implies that:

$$\lim_{t \to +\infty} f_i(t) = +\infty \tag{13}$$

From (11-13) there exists a unique positive constant R_i satisfying (10).
This completes the proof .
Let:

$$R_o = \min\left\{R_i > 0 : E\left(e^{R_i (Y_1 - X_1)(1+I_1)^{-1}} \middle| Y_o = y_i\right) = 1\,(y_i \in E)\right\}$$

Use Lemma 1, we now obtain a probability inequality for $\psi^{(1)}(u, y_i)$ by the martingale approach.

Theorem 1: If model (1) satisfies assumptions 1 to 5, $M = \max\{y_i; y_i \in E\} < +\infty$ and (9) then for any $u>0$ and $y_i \in E$:

$$\psi^{(1)}(u, y_i) \le e^{-R_o u} \tag{14}$$

Proof: Consider the process $\{U_n^{(1)}\}$ given by (2), we let:

$$V_n^{(1)} = U_n^{(1)} \prod_{j=1}^{n} (1 + I_j)^{-1} = u + \sum_{j=1}^{n} \left(X_j - Y_j\right) \prod_{t=1}^{j} (1 + I_t)^{-1} \tag{15}$$

and $S_n^{(1)} = e^{-R_o V_n^{(1)}}$. Thus, we have:

$$S_{n+1}^{(1)} = S_n^{(1)} e^{-R_o (X_{n+1} - Y_{n+1}) \prod_{t=1}^{n+1} (1+I_t)^{-1}}$$

With any $n \ge 1$:

$$\begin{aligned}
&E\left(S_{n+1}^{(1)} \middle| X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n, I_1, I_2, \ldots, I_n\right) \\
&= S_n^{(1)} E\left(e^{-R_o (X_{n+1} - Y_{n+1}) \prod_{t=1}^{n+1} (1+I_t)^{-1}} \middle| X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n, I_1, I_2, \ldots, I_n\right) \\
&= S_n^{(1)} E\left(e^{-R_o (X_{n+1} - Y_{n+1})(1+I_{n+1})^{-1} \prod_{t=1}^{n} (1+I_t)^{-1}} \middle| Y_1, Y_2, \ldots, Y_n, I_1, I_2, \ldots, I_n\right)
\end{aligned}$$

From:

$$0 \le \prod_{t=1}^{n} (1 + I_t)^{-1} \le 1$$

and Jensen's inequality implies:

$$E\left(S_{n+1}^{(1)}\middle|X_1,X_2,\dots,X_n,Y_1,Y_2,\dots,Y_n,I_1,I_2,\dots,I_n\right)$$
$$\le S_n^{(1)}E\left(e^{-R_o(X_{n+1}-Y_{n+1})(1+I_{n+1})^{-1}}\middle|Y_1,Y_2,\dots,Y_n,I_1,I_2,\dots,I_n\right)^{\prod_{t=1}^{n}(1+I_t)^{-1}}$$

In addition:

$$E\left(e^{-R_o(X_{n+1}-Y_{n+1})(1+I_{n+1})^{-1}}\middle|Y_1,Y_2,\dots,Y_n,I_1,I_2,\dots,I_n\right)=E\left(e^{-R_o(X_{n+1}-Y_{n+1})(1+I_{n+1})^{-1}}\middle|Y_n\right)=E\left(e^{-R_o(X_1-Y_1)(1+I_1)^{-1}}\middle|Y_o\right)=1$$

Thus, we have:

$$E\left(S_{n+1}^{(1)}\middle|X_1,X_2,\dots,X_n,Y_1,Y_2,\dots,Y_n,I_1,I_2,\dots,I_n\right)\le S_n^{(1)}$$

Hence, $\{S_n^{(1)}, n = 1, 2\dots\}$ is a supermartingale with respect to the σ-filtration:

$$\Im_n^{(1)} = \sigma\{X_1,\dots, X_n, Y_1,\dots Y_n, I_1,\dots, I_n\}$$

Define $T_i^{(1)} = \min\{n: V_n^{(1)}<0 | U_O^{(1)} = u, Y_0 = y_i\}$, with $V_n^{(1)}$ is given by (15). Hence, $T_i^{(1)}$ is a stopping time and $n\wedge T_i^{(1)} = \min(n, T_i^{(1)})$ is a finite stopping time.

Therefore, from the optional stopping theorem for supermartingales, we have:

$$E\left(S_{n\wedge T_i^{(1)}}^{(1)}\right)\le E(S_o^{(1)})=e^{-R_o u}$$

This implies that:

$$e^{-R_o u}\ge E\left(S_{n\wedge T_i^{(1)}}\right)\ge E\left(S_{n\wedge T_i^{(1)}}^{(1)}.1_{(T_i^{(1)}\le n)}\right)=E\left(S_{T_i^{(1)}}^{(1)}.1_{(T_i^{(1)}\le n)}\right)=E\left(e^{-R_o V_{T_i^{(1)}}^{(1)}}.1_{(T_i^{(1)}\le n)}\right)\tag{16}$$

From $V_{T_i^{(1)}}^{(1)}<0$ then (16) becomes:

$$e^{-R_o u}\ge E\left(1_{(T_i^{(1)}\le n)}\right)=P(T_i^{(1)}\le n)\tag{17}$$

In addition:

$$\begin{aligned}\psi_n^{(1)}(u,y_i)&=P\left(\bigcup_{k=1}^{n}(U_k^{(1)}<0)\middle|U_o^{(1)}=u,Y_o=y_i\right)\\&=P\left(\bigcup_{k=1}^{n}(V_k^{(1)}<0)\middle|U_o^{(1)}=u,Y_o=y_i\right)=P(T_i^{(1)}\le n)\end{aligned}\tag{18}$$

Combining (17) and (18) imply that:

$$\psi_n^{(1)}(u,y_i)\le e^{-R_o u}\tag{19}$$

Thus, (14) follows by letting $n\to\infty$ in (19).

Similarly, we have Lemma 2.

Lemma 2: Assume that model (3) satisfies assumptions 1 to 5 and $E(X_1^k)<+\infty (k = 1, 2)$.

Any $y_i \in E$, if:

$$E\left[Y_1 \middle| Y_o = y_i\right] < E(X_1)$$

and:

$$P\left(Y_1(1+I_1)^{-1} - X_1 > 0 \middle| Y_o = y_i\right) > 0 \tag{20}$$

Then, there exists a unique positive constant R_i satisfying:

$$E\left(e^{R_i\left[Y_1(1+I_1)^{-1}-X_1\right]} \middle| Y_o = y_i\right) = 1 \tag{21}$$

Proof: Define:

$$f_i(t) = E\left\{e^{t\left(Y_1(1+I_1)^{-1}-X_1\right)} \middle| Y_o = y_i\right\} - 1; t \in (0, +\infty)$$

We have:

$$f_i(t) = E\left\{e^{tY_1(1+I_1)^{-1}} \middle| Y_o = y_i\right\}.E\left(e^{-tX_1}\right) - 1 = g_i(t).h(t) - 1$$

From Y_1 is discrete random variables and it takes values in $E = \{y_1, y_2,\dots, y_n,\dots\}$ then:

$$g_i(t) = E\left\{e^{tY_1(1+I_1)^{-1}} \middle| Y_o = y_i\right\} = \sum_{j=1}^{+\infty} p_{ij} \int_0^{+\infty} e^{\frac{ty_j}{1+y}} g(y)dy$$

with $g(y) = G'(y)$.

We have:

$$\int_0^{+\infty} e^{\frac{ty_j}{1+y}} g(y)dy \le \int_0^{+\infty} e^{ty_j} g(y)dy = e^{ty_j}$$

and:

$$\int_0^{+\infty} \left(\frac{y_j}{1+y}\right)^n e^{\frac{ty_j}{1+y}} g(y)dy \le \int_0^{+\infty} \left(y_j\right)^n e^{ty_j} g(y)dy = \left(y_j\right)^n e^{ty_j}$$

This implies that $g_i(t)$ has n-th derivative function on $(0, +\infty)$ (any $n\in N^* = N\setminus\{0\}$).
In addition:

$$h(t) = E\left(e^{-tX_1}\right) = \int_0^{+\infty} e^{-tx} f(x)dx$$

with $f(x) = F'(x)$ satisfying:

$$h(t) = \int_0^{+\infty} e^{-tx} f(x)dx \le \int_0^{+\infty} f(x)dx = 1$$

and:

$$\int_0^{+\infty} x^k e^{-tx} f(x)dx \le \int_0^{+\infty} x^k f(x)dx = E\left(X_1^k\right) < +\infty\,(k = 1,2)$$

This implies that $h(t)$ has n-th derivative function on $(0, +\infty)$ with $n = 1, 2$. Thus, $f_i(t)$ has n-th derivative function on $(0, +\infty)$ with $n = 1, 2$ and:

$$f_i^{'}(t) = E\left\{\left(Y_1(1+I_1)^{-1} - X_1\right)e^{t(Y_1(1+I_1)^{-1}-X_1)} \middle| Y_0 = y_i\right\}$$

$$f_i^{''}(t) = E\left\{\left(Y_1(1+I_1)^{-1} - X_1\right)^2 e^{t(Y_1(1+I_1)^{-1}-X_1)} \middle| Y_0 = y_i\right\} \ge 0$$

Which implies that:

$$f_i(t) \text{ is a convex function with } f_i(0) = 0 \tag{22}$$

and:

$$f_i^{'}(0) = E\left\{(Y_1(1+I_1)^{-1} - X_1) \middle| Y_0 = y_i\right\} \le E\left(Y_1 \middle| Y_0 = y_i\right) - E(X_1) < 0 \tag{23}$$

By $P((Y_1(1+I_1)^{-1}-X_1)>0|Y_0 = y_i)>0$, we can find some constant $\delta>0$ such that:

$$P((Y_1(1+I_1)^{-1}-X_1)>\delta>0|Y_0 = y_i)>0$$

Then, we can get that:

$$f_i(t) = E\left\{e^{t(Y_1(1+I_1)^{-1}-X_1)} \middle| Y_0 = y_i\right\} - 1 \ge E\left\{e^{t(Y_1(1+I_1)^{-1}-X_1)} \middle| Y_0 = y_i\right\}.1_{\left\{\left(Y_1(1+I_1)^{-1}-X_1\right)>\delta \middle| Y_0 = y_i\right\}} - 1$$
$$\ge e^{t\delta}.P(\left\{\left(Y_1(1+I_1)^{-1} - X_1\right) > \delta \middle| Y_0 = y_i\right\} - 1$$

Imply:

$$\lim_{t\to+\infty} f_i(t) = +\infty \tag{24}$$

From (22-24) there exists a unique positive constant R_i satisfying (21).
This completes the proof.
Let:

$$\overline{R}_o = \min\left\{R_i > 0 : E\left(e^{R_i(Y_1(1+I_1)^{-1}-X_1)}\middle| Y_o = y_i\right) = 1(y_i \in E)\right\}$$

Use Lemma 2 we now obtain a probability inequality for $\psi^{(2)}(u, y_i)$ by the martingale approach.

Theorem 2: If model (3) satisfies assumptions 1 to 5, $E(X_1^k)<+\infty(k = 1, 2)$ and (20) then for any $u>0$ and $y_i \in E$:

$$\psi^{(2)}(u,y_i) \le e^{-\overline{R}_o u} \tag{25}$$

Proof: Consider the process $\{U_n^{(2)}\}$ given by (4), we let:

$$V_n^{(2)} = U_n^{(2)}\prod_{j=1}^{n}(1+I_j)^{-1} = u + \sum_{j=1}^{n}\left(X_j(1+I_j) - Y_j\right)\prod_{t=1}^{j}(1+I_t)^{-1} \tag{26}$$

and $S_n^{(2)} = e^{-\overline{R}_o V_n^{(2)}}$. Thus, we have:

$$S_{n+1}^{(2)} = S_n^{(2)} e^{-\overline{R}_o\left(X_{n+1}-Y_{n+1}(1+I_{n+1})^{-1}\right)\prod_{t=1}^{n}(1+I_t)^{-1}}$$

with any $n \ge 1$:

$$\begin{aligned}
&E\left(S_{n+1}^{(2)}\middle| X_1, X_2,\dots,X_n, Y_1, Y_2,\dots,Y_n, I_1, I_2,\dots,I_n\right)\\
&= S_n^{(2)} E\left(e^{-\overline{R}_o(X_{n+1}-Y_{n+1}(1+I_{n+1})^{-1})\prod_{t=1}^{n}(1+I_t)^{-1}}\middle| X_1, X_2,\dots,X_n, Y_1, Y_2,\dots,Y_n, I_1, I_2,\dots,I_n\right)\\
&= S_n^{(2)} E\left(e^{-\overline{R}_o(X_{n+1}-Y_{n+1}(1+I_{n+1})^{-1})\prod_{t=1}^{n}(1+I_t)^{-1}}\middle| Y_1, Y_2,\dots,Y_n, I_1, I_2,\dots,I_n\right)
\end{aligned}$$

From:

$$0 \le \prod_{t=1}^{n}(1+I_t)^{-1} \le 1$$

and Jensen's inequality implies:

$$E\left(S_{n+1}^{(2)}\middle|X_1,X_2,...,X_n,Y_1,Y_2,...,Y_n,I_1,I_2,...,I_n\right)$$
$$\leq S_n^{(2)}E\left(e^{-\bar{R}_o(X_{n+1}-Y_{n+1}(1+I_{n+1})^{-1})}\middle|Y_1,Y_2,...,Y_n,I_1,I_2,...,I_n\right)^{\prod_{t=1}^{n}(1+I_t)^{-1}}$$

In addition:

$$E\left(e^{-\bar{R}_o(X_{n+1}-Y_{n+1}(1+I_{n+1})^{-1})}\middle|Y_1,Y_2,...,Y_n,I_1,I_2,...,I_n\right)=E\left(e^{-\bar{R}_o(X_{n+1}-Y_{n+1}(1+I_{n+1})^{-1})}\middle|Y_n\right)=E\left(e^{-\bar{R}_o(X_1-Y_1(1+I_1)^{-1})}\middle|Y_o\right)=1$$

Thus, we have:

$$E\left(S_{n+1}^{(2)}\middle|X_1,X_2,...,X_n,Y_1,Y_2,...,Y_n,I_1,I_2,...,I_n\right)\leq S_n^{(2)}$$

Hence, $\{S_n^{(2)}, n = 1, 2,...\}$ is a supermartingale with respect to the σ-filtration:

$$\Im_n^{(2)} = \sigma\{X_1,... X_n, Y_1,..., Y_n, I_1,..., I_n\}$$

Define $T_i^{(2)} = \min\{n: V_n^{(2)}<0|U_O^{(2)} = u, Y_0 = y\}$, with $V_n^{(2)}$ is given by (15). Hence, $T_i^{(2)}$ is a stopping time and $n\wedge T_i^{(2)} = \min(n, T_i^{(2)})$ is a finite stopping time.

Therefore, from the optional stopping theorem for supermartingales, we have:

$$E\left(S_{n\wedge T_i^{(2)}}^{(2)}\right)\leq E(S_o^{(2)})=e^{-\bar{R}_o u}$$

This implies that:

$$e^{-\bar{R}_o u}\geq E\left(S_{n\wedge T_i^{(2)}}^{(2)}\right)\geq E\left(S_{n\wedge T_i^{(2)}}^{(2)}.1_{(T_i^{(2)}\leq n)}\right)=E\left(S_{T_i^{(2)}}^{(2)}.1_{(T_i^{(2)}\leq n)}\right)=E\left(e^{-R_o V_{T_i^{(2)}}^{(2)}}.1_{(T_i^{(2)}\leq n)}\right)\tag{27}$$

From $V_{T_i^{(2)}}^{(2)}<0$ then (27) becomes:

$$e^{-\bar{R}_o u}\geq E\left(1_{(T_i^{(2)}\leq n)}\right)=P(T_i^{(2)}\leq n)\tag{28}$$

In addition:

$$\begin{aligned}\psi_n^{(2)}(u,y_i)&=P\left(\bigcup_{k=1}^{n}(U_k^{(2)}<0)\middle|U_o^{(2)}=u,Y_o=y_i\right)\\&=P\left(\bigcup_{k=1}^{n}(V_k^{(2)}<0)\middle|U_o^{(2)}=u,Y_o=y_i\right)=P(T_i^{(2)}\leq n)\end{aligned}\tag{29}$$

Combining (28) and (29) imply that:

$$\psi_n^{(2)}(u, y_i) \leq e^{-\bar{R}_0 u} \tag{30}$$

Thus, (30) follows by letting $n \to \infty$ in (25).

This completes the proof.

CONCLUSION

Our main results in this study, Theorem 1 and Theorem 2 give upper bounds for $\psi_n^{(1)}(u, y_i)$ and $\psi_n^{(2)}(u, y_i)$ by the martingale approach.

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