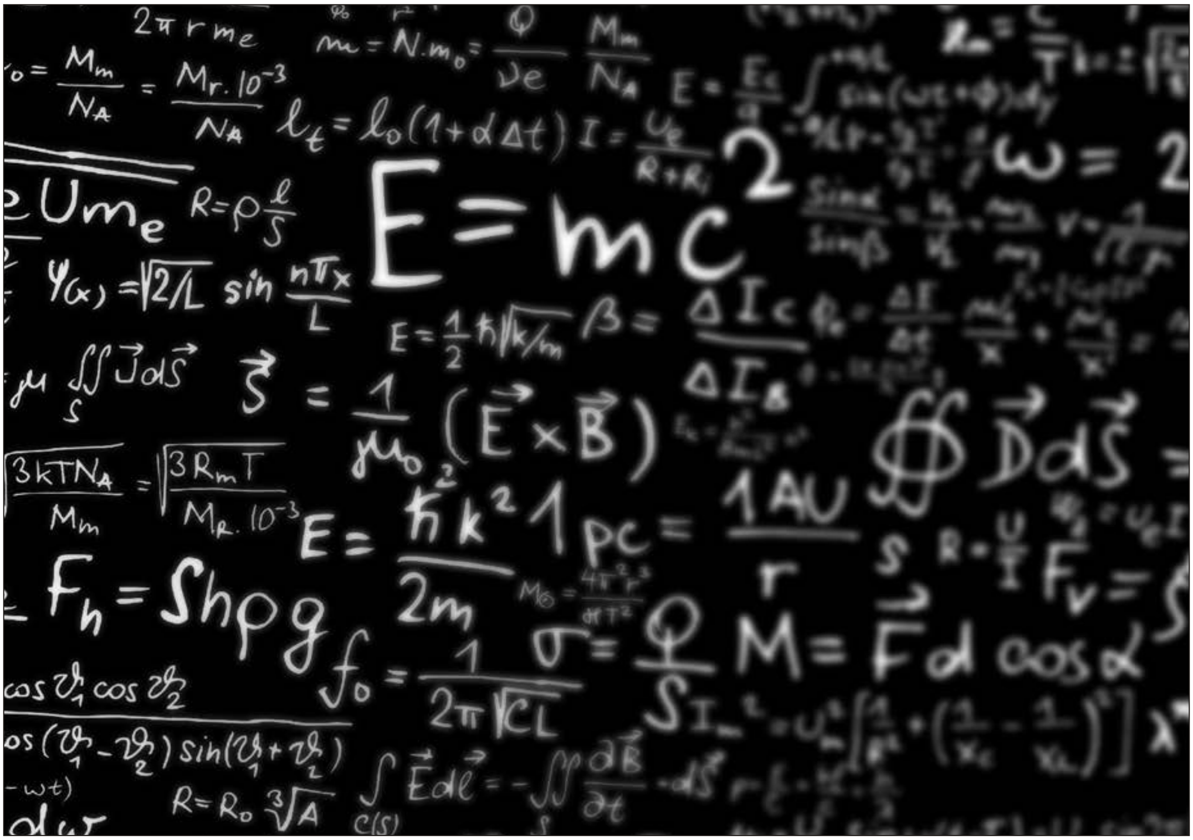

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# Joint Probability Distribution Function of the Repairing Time for the Defective Machines in the Multiple Queues System

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## ABSTRACT

This study dealt with the queues system in one of the industrial workshops which it was composed of four queues, the first and the fourth queues were connected in series, while the second and the third queues were connected in parallel to repair defective machines such that the repairing process included two parts, it were electric and mechanic parts. The first stage was the dismantling of the machine into two parts with the time represented by a continuous random variable $T_1$. The continuous random variables $T_2$ and $T_3$ represented the time of the repairing stages which it were electrical and mechanical repairing respectively and the continuous random variable $T_4$ represented a compile-time of the machine after repairing it. All the random variables had a normal distribution with different means and variances. The joint probability distribution function $F_T(t)$ was obtained as multiple linear regression form by calculating the probability density function $f(T_i)$, i = 1, 2, 3, 4 where T is a continuous random variable represented the whole time repairs in the system and t represented a maximum value of T. On the other hand the probability that the system will be failure or defective at $T>t$ is equal to $(1-F_T(t))$ can be computed and predicted its value easily by using the multiple linear regression form of $F_T(t)$ without performed many mathematical processes.

**Key words:** Queues system, normal distribution, joint probability distribution function, joint probability density function, multiple linear regression

## INTRODUCTION

There was always a difficulty in handling the mathematical formulas obtained in some of the research such as the obtaining some important information of the repairing time for the defective machines in the multiple queues system of the industrial workshops. In this study, the methods and ideas of the previous studies were updated for providing the important relevant statistics in order to get and predict some information about the repairing time of the defective machines in the industrial workshops. Research has done on transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow in presence of dust particles by Azad *et al.* (2012). Contextual mobile learning for repairing industrial machines: System architecture and development process was in working by David *et al.* (2008) but the work of Ekpenyong *et al.* (2008) was in polynomial (Non Linear) regression method for improved estimation based on sampling. El Genidy (2011) carried out review on maximal queue size with standard normal distribution for arrival times. Ismail *et al.* (2009) performed forecasting gold prices using multiple linear regression method. Jin *et al.* (2006) deal with an algorithm to estimate continuous-time traffic speed using multiple regression model. A linear regression model to study the relationship of pesticide imports with agricultural productivity growth in Pakistan was in working by Khooharo *et al*. (2006). Okereke (2011) has done work in effect of transformation on the parameter estimates of a simple linear regression model: A case study of division of variables by constants. Hofer (2007) deals with the joint distribution of the weighted sum-of-digits function modulo one in case of pairwise coprime bases. The results of this study will enable the researchers to deal with the different systems of repairing the defective machines with an economical manner and minimal costs.

## DESCRIPTION OF THE PROBLEM AND ITS SOLVING

The system of the machines repairing is a multiple queues system which it consists of four queues. The first and the fourth queues were connected in series, while the second and the third queues were connected in parallel to repair defective machines such that the repairing process included two parts, it were electrical part and mechanic part. The first service was dismantling process which the time of the first service represented by a continuous random variable $T_1$ on the interval $[0,t_1]$ while the continuous random variables $T_2$ on the interval $[0,t_2]$ and $T_3$ on the interval $[0,t_3]$ represented the time of the repairing stages which it were electrical repairing and mechanical repairing, respectively, lastly the continuous random variable $T_4$ on the interval $[0,t_4]$ represented a compile-time of the machine after repairing it. The continuous random variables $T_1$, $T_2$, $T_3$ and $T_4$ were mutually independent and had a normal distribution with different means and variances $(\mu_1, \sigma_1^2)$, $(\mu_2, \sigma_2^2)$, $(\mu_3, \sigma_3^2)$ and $(\mu_4, \sigma_4^2)$ , respectively. Therefore the whole time T of the time repairing in the system was written as follows:

$$T = T_1+\max\{T_2, T_3\}+T_4 = T_1+M+T_4$$

where, M is a random variable such that, $M = \max\{T_2, T_3\}$.

On the other hand, the probability density functions of the continuous random variables $T_1$, $T_2$, $T_3$ and $T_4$ were written as follows:

$$f(T_1) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{T_1-\mu_1}{\sigma_1}\right)^2}, \quad 0 < T_1 < t_1$$

$$f(T_2) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{T_2-\mu_2}{\sigma_2}\right)^2}, \quad 0 < T_2 < t_2$$

$$f(T_3) = \frac{1}{\sigma_3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{T_3-\mu_3}{\sigma_3}\right)^2}, \quad 0 < T_3 < t_3$$

and:

$$f(T_4) = \frac{1}{\sigma_4\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{T_4-\mu_4}{\sigma_4}\right)^2}, \quad 0 < T_4 < t_4$$

Such that: $t_1$, $t_2$, $t_3$ and $t_4$ were the values of $T_1$, $T_2$, $T_3$ and $T_4$, respectively.

Therefore, the joint probability distribution function $F_T(t)$ was obtained as follows:

$$F_T(t) = p(T \le t) = p((T_1+M+T_4) \le t)$$

where, $t = t_1+m+t_4$ and $m = \max\{t_2, t_3\}$.

While, the continuous random variables $T_1$, $T_2$, $T_3$ and $T_4$ were mutually independent and had a normal distribution, then:

$$F_T(t) = p(T_1 \le t_1) + p(M \le m) + p(T_4 \le t_4) = \int_0^{t_1} f(T_1)dT_1 + \int_0^{m} f(M)dM + \int_0^{t_4} f(T_4)dT_4$$

Let $M = \max\{T_2, T_3\} = T_2$, thus:

$$F_T(t) = \frac{1}{\sigma_1\sqrt{2\pi}}\int_0^{t_1} e^{-\frac{1}{2}\left(\frac{T_1-\mu_1}{\sigma_1}\right)^2} dT_1 +$$
$$= \frac{1}{\sigma_2\sqrt{2\pi}}\int_0^{t_2} e^{-\frac{1}{2}\left(\frac{T_2-\mu_2}{\sigma_2}\right)^2} dT_2 +$$
$$= \frac{1}{\sigma_4\sqrt{2\pi}}\int_0^{t_4} e^{-\frac{1}{2}\left(\frac{T_4-\mu_4}{\sigma_4}\right)^2} dT_4$$

## APPLICATION WITH NUMERICAL RESULTS

Suppose, samples were taken from the industrial workshops and were found that:

$\mu_1 = 0.2$, $\mu_2 = 0.3$, $\mu_4 = 0.4$, $\sigma_1 = \sigma_2 = \sigma_4 = 0.5$

such that, the values of $t_1$, $t_2$ and $t_4$ were supposed as follows:

$$t_1 = 0.21,\dots, 0.3$$

$$t_2 = 0.31,\dots, 0.4$$

$$t_4 = 0.41,\dots, 0.5$$

Then, the values of $F_T(t)$ were computed as follows:

$$F_T(0.21,0.31,0.41) = \frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.21} e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2} dT_1 + \int_0^{0.31} e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2} dT_2 + \int_0^{0.41} e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2} dT_4\right) = 0.693248$$

$$F_T(0.22,0.32,0.42) = \frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.22} e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2} dT_1 + \int_0^{0.32} e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2} dT_2 + \int_0^{0.42} e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2} dT_4\right) = 0.717174$$

$$F_T(0.23,0.33,0.43)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.23}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.33}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.43}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.74108$$

$$F_T(0.24,0.34,0.44)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.24}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.34}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.44}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.764957$$

$$F_T(0.25,0.35,0.45)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.25}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.35}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.45}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.788797$$

$$F_T(0.26,0.36,0.46)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.26}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.36}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.46}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.812589$$

$$F_T(0.27,0.37,0.47)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.27}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.37}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.47}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.836323$$

$$F_T(0.28,0.38,0.48)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.28}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.38}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.49}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.867856$$

$$F_T(0.29,0.39,0.49)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.29}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.39}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.49}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.883584$$

$$F_T(0.3,0.4,0.5)=\frac{1}{0.5\sqrt{2\pi}}\left(\int_0^{0.3}e^{-\frac{1}{2}\left(\frac{T_1-0.2}{0.5}\right)^2}dT_1+\int_0^{0.4}e^{-\frac{1}{2}\left(\frac{T_2-0..3}{0.5}\right)^2}dT_2+\int_0^{0.5}e^{-\frac{1}{2}\left(\frac{T_4-0.4}{0.5}\right)^2}dT_4\right)=0.907092$$

Apply the FIT command on the obvious results to get the multiple linear regression. $Z = z(T_1, T_2, T_4)$ for the joint probability distribution function $F_T(t)$ as follows:

Fit[{{0.21,0.31,0.41,0.693248},{0.22,0.32,0.42,0.717174},{0.23,0.33,0.43,0.74108},
{0.24,0.34,0.44,0.764957},{0.25,0.35,0.45,0.788797},{0.26,0.36,0.46,0.812589},
{0.27,0.37,0.47,0.836323},{0.28,0.38,0.48,0.867856},{0.29,0.39,0.49,0.883584},
{0.3,0.4,0.5,0.907092}},{1, $T_1$, $T_2$, $T_4$},{ $T_1$, $T_2$, $T_4$}]

Then $Z = z(T_1, T_2, T_4)$ was obtained as follows:

$$Z = z(T_1, T_2, T_4) = -0.049996+0.805249\ T_1+0.80025\ T_2+0.79525\ T_4$$

## CONCLUSION

If $T_1 = 0.3$, $T_2 = 0.4$ and $T_4 = 0.5$ then, $Z = 0.909304$ but also $z \cong F_T(t)$ for all values of t therefore, the function Z is good estimator for the function $F_T(t)$. On the other hand the probability that the system will be failure or defective at T>1.2 is equal to $(1-F_T(t)) \sim 0.09$. This method can apply on other different distributions as Exponential and Pareto distributions. The results of this study will enable the researchers to deal with this system in an economical manner and with minimal costs. Lastly, the idea of this study allows the researchers to apply it in various scientific fields.

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