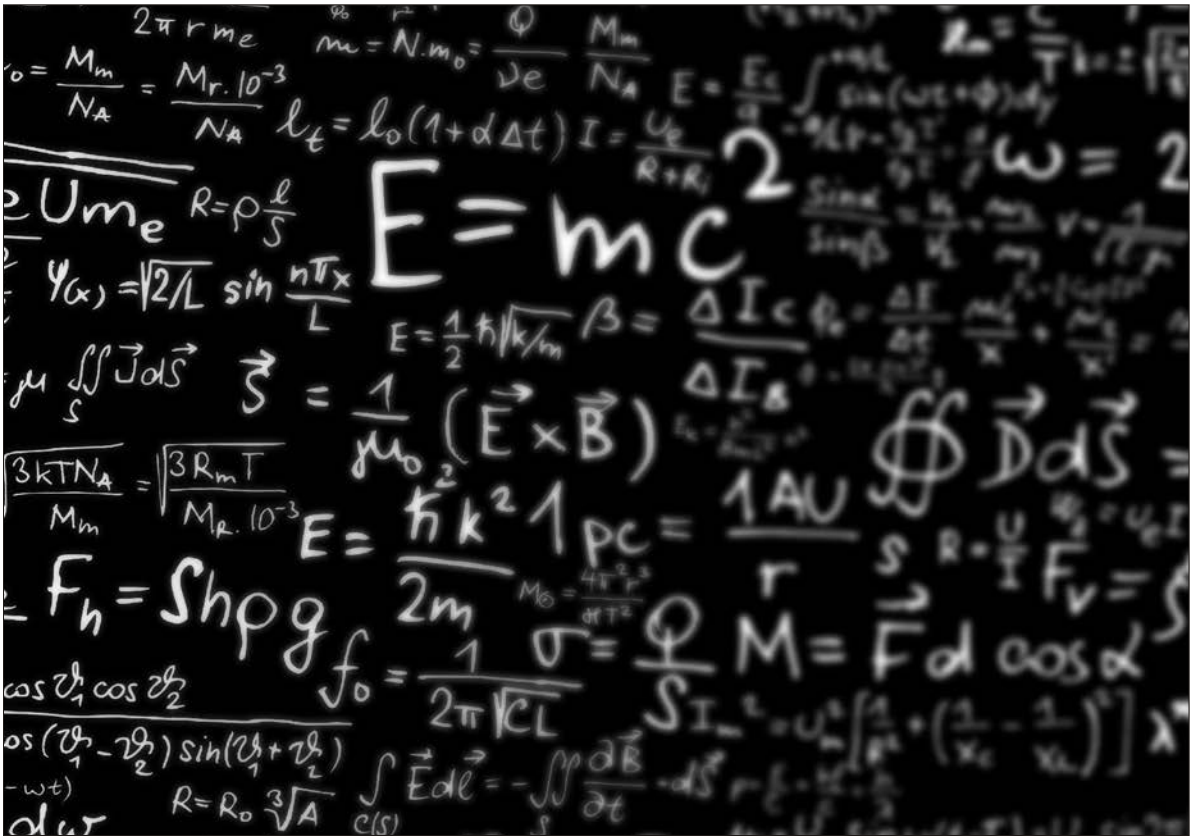

Asian Journal of Mathematics & Statistics

ISSN 1994-5418

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Asian Journal of Mathematics and Statistics 7 (2): 68-69, 2014
ISSN 1994-5418 / DOI: 10.3923/ajms.2014.68.69

A Generalization of a Best Approximation Problem in Real Line

Asiyeh Erfanmanesh
Faculty of Iranian Academic Center for Education Culture and Research, Ahvaz, Iran

ABSTRACT

In the present study, we generalize a problem dealing with the minimum distance between two disjoint, closed and compact subsets of real line. This result asserts that the disjointness of a closed set and a compact set in a metric space implies that the distance between these sets is always nonzero.

Key words: Metric space, compact set, closed set

INTRODUCTION

Maurice Fréchet introduced metric spaces in (Frechet, 1906). A metric space is a set where a notion of distance (called a metric) between elements of the set is defined.

The metric space which most closely corresponds to our intuitive understanding of space is the 3-dimensional Euclidean space. In fact, the notion of "metric" is a generalization of the Euclidean metric arising from the four long-known properties of the Euclidean distance. The Euclidean metric defines the distance between two points as the length of the straight line segment connecting them. Other metric spaces occur for example in elliptic geometry and hyperbolic geometry, where distance on a sphere measured by angle is a metric and the hyperboloid model of hyperbolic geometry is used by special relativity as a metric space of velocities. For more detail see (Rudin, 1976; Bryant, 1985; Burago *et al*., 2001; Papadopoulos, 2004; O'Searcoid, 2006).

A metric space also induces topological properties like open and closed sets which leads to the study of even more abstract topological spaces.

In this study, we have discussed on the minimum distance between a closed set and a compact set in a metric space which are distinct.

GENERALIZATION

In the section, first, we have a problem which is in (Rudin, 1986). Next, a generalization of it, is proved.

Let us to present the problem in special case.

Problem 1: Let, A and B be compact and closed subsets of $\mathbb{R}$, respectively. Further, suppose that A and B be distinct. Then there is a positive real number δ such that $|a-b|\geq\delta$ for all $a\in A$ and $b\in B$.

Remark 1: Notice that the condition "distinctness" in the above problem is necessary. Now, we are ready to prove the generalization of the previous problem.

Generalization 1: Let, A and B be compact and closed subsets of a metric space (X, d), respectively. Further, suppose that A and B be distinct. Then there is a positive real number δ such that $d(a, b)\geq\delta$ for all $a\in A$ and $b\in B$.

Proof: By contrary, let for each $\delta>0$, there are $a\in A$ and $b\in B$ so that:

$$d(a, b)\geq\delta$$

Let, n be an arbitrary positive integer. Set $\delta = 1/n$. Then we can obtain two sequences (a_n) and (b_n) of elements in A and B, respectively so that:

$$d(a_n, b_n)<1/n$$

For sufficiently large n, we have:

$$d(a_n, b_n)\to 0$$

Since A is compact, so (a_n) has a convergent subsequence. Without loss of generality one may consider (a_n) as this subsequence. Hence:

$$a_n\to a^*$$

For some $a^*\in A$. On the other hand:

$$d(b_n, a^*)\leq d(b_n, a_n)+d(a_n, a^*)$$

If $n\to\infty$, then the right side of the last inequality approaches to zero. Therefore:

$$b_n\to a^*$$

As $n\to\infty$. Since B is closed, so $a^*\in A$ which is impossible; since A and B are distinct.

Remark 2: According to the generalization 1, $D(A, B) = \delta$, where:

$$D(A, B) = \inf\{d(a, b):a\in A, b\in B\}$$

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