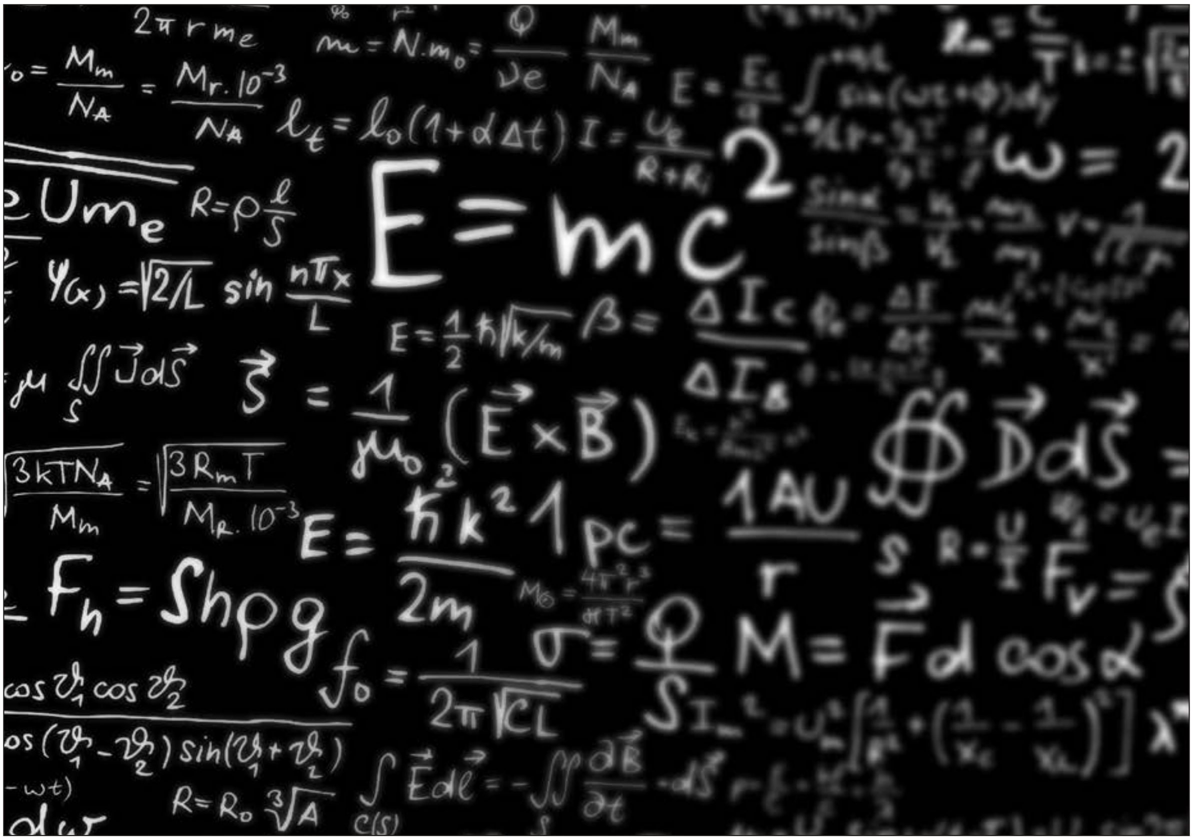

Asian Journal of Mathematics & Statistics

ISSN 1994-5418

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OPEN ACCESS Asian Journal of Mathematics and Statistics

ISSN 1994-5418
DOI: 10.3923/ajms.2016.1.5

Research Article
Solution of Differential Equations Using Differential Transform Method

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Abstract
Objective: The objective of the study was to solve differential equations. **Methodology:** It was evaluated by using Differential Transform Method (DTM). The solution obtained by DTM and Laplace transform are compared. **Results:** The results obtained show that the DTM technique is accurate and efficient and require less computational effort in comparison to the other methods. **Conclusion:** Results revealed that DTM save time and space and it is easy to implement.

Key words: Differential equation, differential transform method, Laplace transform method

Received: April 21, 2016 Accepted: May 20, 2016 Published: June 15, 2016

Citation: Giriraj Methi, 2016. Solution of differential equations using differential transform method. Asian J. Math. Stat., 9: 1-5.

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Competing Interest: The author has declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Aim of the study is to solve the differential equations using differential transform method which are often encounter in applied sciences and engineering. There are different methods available exact, approximate and numerical for the solution of differential equations. Most of these methods are computationally exhaustive because they require lot of time and space.

The concept of differential transform was first proposed by Zhou[1] and it was applied to solve linear and non-linear initial value problems in electric circuit analysis. Jang *et al.*[2] stated that the differential transform is an iterative method for obtaining Taylor series solutions of differential equations. Although, the Taylor series method requires more computational work for large orders, the present method reduces the size of computational domain and is applicable to many problems easily[3-6]. A method of differential transform was used to obtain approximate solutions of the linear and non-linear equations related to engineering problems and observed that the numerical results are in good agreement with the analytical solutions. In this study, the differential transformation technique is applied to solve differential equations. The differential transformation technique can be used to obtain both numerical and analytical solutions of both linear and non-linear differential equations.

The differential transformation technique uses the polynomials as the approximation to the exact solution. The high-order Taylor series method can also be applied to differential equations. However, the Taylor method requires the calculation of high-order derivatives, a difficult symbolic and complex problem[7-10].

MATERIALS AND METHODS

Basic definitions: The basic idea of differential transform method is illustrated.

Definition 1: The one dimensional differential transform of function v(t) is defined as follows:

$$V(k)=\left[\frac{d^k v(t)}{dt^k}\right] \tag{1}$$

Definition 2: If v(t) can be expressed by Taylor series, then v(t) is given by:

$$v(t)=\sum_{k=0}^{\infty}\left[\frac{(t-t_i)^k}{k!}\right]V(k) \tag{2}$$

Equation 2 is called the inverse of V(k). Now combining Eq. 1 and 2:

$$v(t)=\sum_{k=0}^{\infty}\left[\frac{(t-t_i)^k}{k!}\right]V(k)=D^{-1}V(k) \tag{3}$$

Properties of DTM: If U(k) and V(k) are corresponding transformed functions of u(t) and v(t). Then we have following DTM properties:

- If $z(t) = u(t)\pm v(t)$ then:

  $Z(k) = U(k)\pm V(k)$

- If $z(t) = \alpha u(t)$ then:

  $Z(k) = \alpha U(k)$

- If $z(t)=\frac{du(t)}{dt}$ then:

  $Z(k) = (k+1)U(k+1)$

- If $z(t)=\frac{d^2u(t)}{dt^2}$ then:

  $Z(k) = (k+1)(k+2)U(k+2)$

- If $z(t) = u(t)v(t)$ then:

  $$Z(k)=\sum_{l=0}^{k}V(l)U(k-l)$$

- If $z(t) = e^{\lambda t}$ then:

  $$Z(k)=\frac{\lambda^k}{k!}$$

- If $z(t) = \sin(wt+\alpha)$ then:

  $$Z(k)=\frac{w^k}{k!}\sin\left(\frac{\pi k}{2}+\alpha\right)$$

RESULTS ANS DISCUSSION

Numerical example 1: Consider the differential equation:

$$\frac{d^2v}{dt^2}-2\frac{dv}{dt}-8v=0 \tag{4}$$

with the initial condition:

$$v(0) = 3, v(0) = 6 \quad (5)$$

Now, taking the DTM to Eq. 4 and 5:

$$V(k+2) = \frac{2(k+1)\, V(k+1) + 8V(k)}{(k+1)\,(k+2)} \quad (6)$$

and

$$V(0) = 3, V(1) = 6 \quad (7)$$

The numerical results of example 1 by DTM are presented in Table 1 and the approximate solution is:

$$v(t) = \sum_{k=0}^{k=5} V(k)t^k \quad (8)$$

$$= V(0)+V(1)t+V(2)t^2+V(3)t^3+V(4)t^4+V(5)t^5+... \quad (9)$$

$$= 3+6t+18t^2+20t^3+22t^4+\frac{84}{5}t^5+... \quad (10)$$

Using the Laplace transform method, the exact solution is given by:

$$v(t) = 2e^{4t}+e^{-2t} \quad (11)$$

Example 2: Consider the differential equation:

$$\frac{d^2v}{dt^2}+7\frac{dv}{dt}+10v = 4e^{-3t} \quad (12)$$

with initial conditions:

$$v(0) = 0, \ v(0) = -1 \quad (13)$$

Taking the DTM of Eq. 12 and 13:

$$(k+1)(k+2)V(k+2)+7(k+1)V(k+1)+10V(k) = 4\frac{(-3)^k}{k!} \quad (14)$$

The numerical results for example 2 by DTM are presented in Table 2 and the approximate solution is given by:

$$v(t) = \sum_{k=0}^{k=2} V(k)t^k \quad (15)$$

$$= V(0)+V(1)t+V(2)t^2+V(3)t^3+V(4)t^4+... \quad (16)$$

$$= t+11/2t^2-79/6t^3+479/2t^4+... \quad (17)$$

Using the Laplace transform, the exact solution is given by:

$$v(t) = -2e^{-3t}+e^{-2t}+e^{-5t}$$

Example 3: Consider the simultaneous differential equations:

$$2\frac{dx}{dt}+\frac{dy}{dt}-x-y = e^{-t} \quad (18)$$

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y = e^{-t} \quad (19)$$

with initial conditions:

$$x(0) = 2, y(0) = 1 \quad (20)$$

Taking the DTM of Eq. 18 and 19:

$$2(k+1)X(k+1)+(k+1)Y(k+1)-X(k)-Y(k) = \frac{-1}{k!} \quad (21)$$

$$(k+1)X(k+1)+(k+1)Y(k+1)+2X(k)+Y(k) = \frac{-1}{k!} \quad (22)$$

The numerical results for example 3 by DTM are presented in Table 3 and the approximate solution for x(t) is given by:

Table 1: Numerical results of example 1 by DTM

k	k+2	V (k+2)
0	2	18
1	3	20
2	4	22
3	5	84/5

Table 2: Numerical results for example 2 by DTM

k	k+2	V (k+2)
0	2	11/2
1	3	-79/6
2	4	479/2

Table 3: Numerical results for example 3 by DTM

k	X (k)	Y (k)
0	8	-14
1	-2	½
2	-7/3	10/3
3	-1/12	3/8

$$x(t) = \sum_{k=0}^{k=3} X(k)t^k \quad (23)$$

$$= X(0)+X(1)t+X(2)t^2+X(3)t^3+... \quad (24)$$

$$= 2+8t-2t^2-\frac{7}{3}t^3+... \quad (25)$$

and approximate solution for y(t) is given by:

$$y(t) = \sum_{k=0}^{k=3} Y(k)t^k \quad (26)$$

$$= Y(0)+Y(1)t+Y(2)t^2+Y(3)t^3+... \quad (27)$$

$$= 1\text{-}14t+1/2t^2+10/3t^3 \quad (28)$$

Using Laplace transform the exact solution is given by:

$$x(t) = 2\cos t+8\sin t,\ y(t) = \cos t-13\sin t+\sinh t \quad (29)$$

Example 4: Consider the following simultaneous differential equation:

$$\frac{dx}{dt}+\frac{1}{2}\frac{dy}{dt}+x=1,\ \frac{1}{2}\frac{dx}{dt}+\frac{dy}{dt}+y=0 \quad (30)$$

with initial conditions:

$$x(0) = 0 = y(0) \quad (31)$$

Taking the differential transform of both the given equations:

$$(k+1)X(k+1)+\frac{1}{2}(k+1)Y(k+1)+X(k)=1 \quad (32)$$

$$\frac{1}{2}(k+1)X(k+1)+(k+1)Y(k+1)+Y(k)=0 \quad (33)$$

The numerical results for example 4 by DTM are presented in Table 4. Therefore the approximate solution for x(t) is:

$$x(t) = \sum_{k=0}^{k=3} X(k)t^k \quad (34)$$

$$= X(0)+X(1)t+X(2)t^2+X(3)t^3+... \quad (35)$$

Table 4: Numerical results for example 4 by DTM

K	X (k)	Y (k)
0	0	0
1	4/3	-2/3
2	-4/9	5/9
3	62/135	2/45

$$= \frac{4}{3}t-\frac{4}{9}t^2+\frac{62}{135}t^3+... \quad (36)$$

and the approximate solution for y(t) is:

$$y(t) = \sum_{k=0}^{k=3} Y(k)t^k \quad (37)$$

$$= Y(0)+Y(1)t+Y(2)t^2+Y(3)t^3+... \quad (38)$$

$$= \frac{-2}{3}t+\frac{5}{9}t^2+\frac{2}{45}t^3+... \quad (39)$$

Using Laplace transform the exact solution is given by:

$$x(t) = \left[1-\frac{1}{2}\left(e^{-2t/3}+e^{-2t}\right)\right]$$

and

$$y(t) = \frac{1}{2}\left[\left(e^{-2t}-e^{-2t/3}\right)\right] \quad (40)$$

CONCLUSION

The solution obtained by DTM and Laplace transform are compared. It is observed that DTM save time and space and easy to implement. Differential transform method gives solution very fast and does not require linearization, perturbation or any other assumption.

ACKNOWLEDGMENT

Author is very thankful to the reviewer and editor of the Journal for their valuable suggestions to improve the research paper.

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