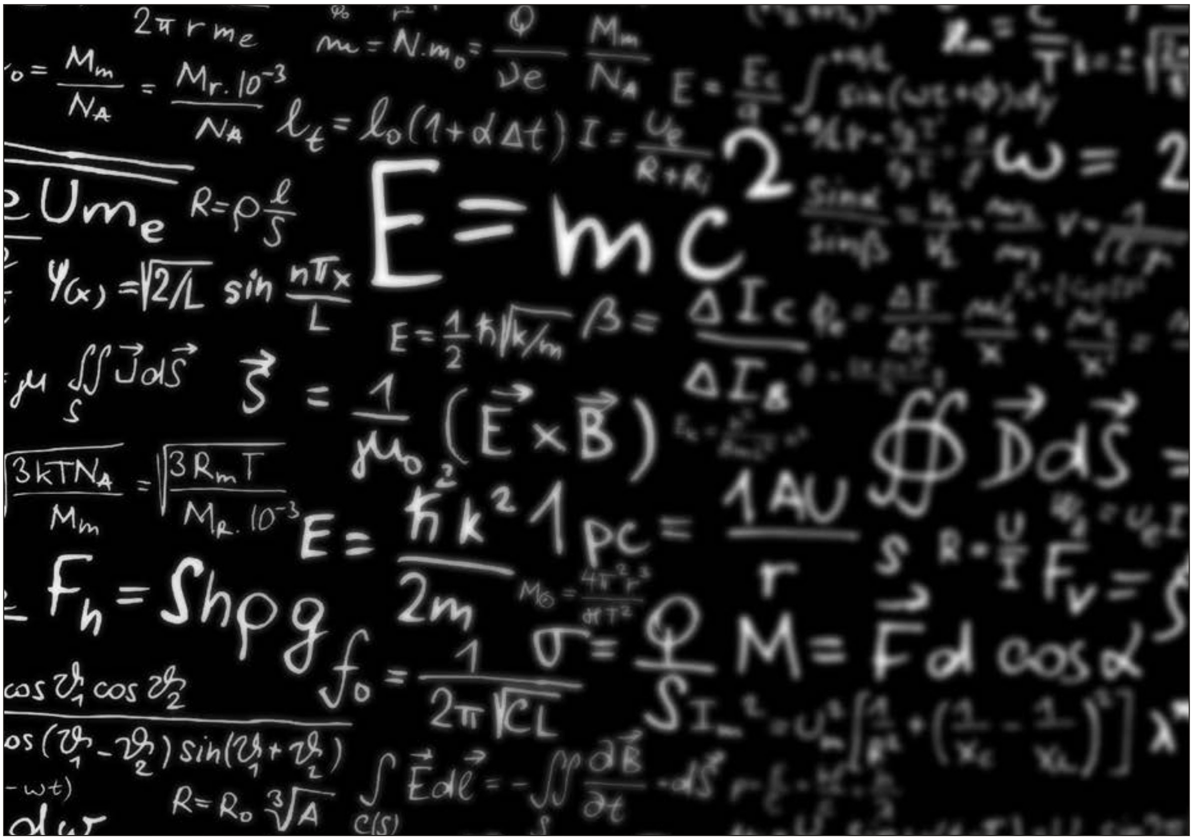

Asian Journal of Mathematics & Statistics

ISSN 1994-5418

science alert

ANSInet
an open access publisher
http://ansinet.com

OPEN ACCESS

Asian Journal of Mathematics and Statistics

ISSN 1994-5418
DOI: 10.3923/ajms.2017.13.17

Research Article

Fixed Point Theorem for Quasi-js-metric Spaces

[1]Aklesh Pariya, [2]Antima Sindersiya, [2]Viquar Husain Badshah and [3]Nirmala Gupta

[1]Department of Mathematics, Medi-Caps University, Indore (M.P.), India
[2]School of Studies in Mathematics, Vikram University, Ujjain (M.P.), India
[3]Department of Mathematics, Govt. Girls P.G. College, Ujjain (M.P.), India

Abstract

Objective: The objective of this study was to prove some fixed point theorems for Ciric type contraction in quasi JS-metric space which generalizes existing results in quasi JS-metric spaces. **Methodology:** For proving the theorem collected some basic concepts and results from the literature. **Results:** This study showed that fixed point theorems for two mappings in quasi JS-metric spaces were proved. **Conclusion:** Therefore, Theorem 2.1 is a generalization of many fixed point results in fixed point literature in metric space, quasi metric space to quasi JS-metric space for Ciric type contraction.

Key words: Fixed point, Ciric type contraction, quasi metric space, JS-metric space, quasi JS-metric space

Citation: Aklesh Pariya, Antima Sindersiya, Viquar Husain Badshah and Nirmala Gupta, 2017. Fixed point theorem for Quasi-JS-metric spaces. Asian J. Math. Stat., 10: 13-17.

Corresponding Author: Antima Sindersiya, School of Studies in Mathematics, Vikram University, Ujjain (M.P.), India

Copyright: © 2017 Aklesh Pariya *et al.* This is an open access article distributed under the terms of the creative commons attribution License, which permits unrestricted use, distribution and reproduction in any medium, provided the original author and source are credited.

Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Banach[1] introduced the concept of contraction mapping in metric space which is an important tool in the theory of metric spaces. It guarantees the existence and uniqueness of fixed points of certain self-maps of metric spaces and provides a constructive method to find those fixed points. It is also known as Banach contraction principle or Banach fixed point theorem. It states that "Every self contraction mapping of a complete metric space has a unique fixed point". This principle was extended and generalized in various directions. For example, the concepts of Ciric contraction[2], quasi-contraction[3], JS-contraction[4] and JS-Ciric contraction[5] had been introduced and many interesting generalizations of the Banach contraction principle were obtained. Banach contraction principle was extended to a JS-metric space by Jleli and Samet[6] and further extended this principle to a quasi JS-metric space. Noorwali *et al.*[7] combined a number of existing fixed point results by introducing a new distance (that includes, as particular cases, standard metric spaces, b-metric spaces, dislocated metric spaces and modular spaces). The objective was to prove some fixed point theorems for Ciric type contraction in quasi JS-metric space which generalizes existing results in quasi JS- metric spaces.

METHODOLOGY

Basic definitions and preliminaries: For the sake of completeness, collected some basic concepts and results from the literature. Let denote the set $\mathbb{N}\cup\{0\}$ where, $\mathbb{N}$ represent the set of all positive integers. Let X be a nonempty set and let $D:X\times X\rightarrow[0, \infty]$ be a given mapping. For every $x\in X$, define the sets:

$$C_L (D, X, x) = \{\{x_n\}\subseteq X: \lim_{n\to\infty} D(x, x_n) = 0\}$$

$$C_R (D, X, x) = \{\{x_n\}\subseteq X: \lim_{n\to\infty} D(x_n, x) = 0\}$$

Definition 1: Mapping $D:X\times X\rightarrow[0, \infty]$ is a quasi JS-metric space on a nonempty set X if it satisfies the following conditions:

- (D_1) $D(x, y) = D(y, x) = 0 \Rightarrow x = y$, for every $x, y\in X$
- (D_2) There exists $C>0$ such that
- If $x, y\in X$, $\{x_n\}$ C_L (D, X, x), then $D(x, y)\leq C \lim_{n\to\infty}\sup D(x_n, y)$
- If $x, y\in X$, $\{x_n\}\in C_L$ (D, X, x), then $D(y, x)\leq C \lim_{n\to\infty}\sup D(y, x_n)$
- In this case, the X, D is called a quasi JS-metric space

Remark 1 [6]: If in addition to the condition in definition 1, the equality

- (D_3) $D(x, y) = D(y, x)$ is satisfied for each $x, y\in X$, then, (X, D) is called JS-metric space

Definition 2 [6]: Let (X, D) be a quasi JS-metric space. Let $\{x_n\}$ be a sequence in X and $x\in X$. Then:

- $\{x_n\}$ is said to be left D-convergent to x if $\{x_n\}\in C_L$ (D, X, x), in this case x is said to be a left D-limit of $\{x_n\}$
- $\{x_n\}$ is said to be left D-convergent to x if $\{x_n\}\in C_R$ (D, X, x), in this case is said to be a right D-limit of $\{x_n\}$
- $\{x_n\}$ is said to be D-convergent to x if x_n is both left and right D-convergent to x, in this case is said to be a D-limit of $\{x_n\}$

Definition 3 [6]: Let (X, D) be a quasi JS-metric space. Let $\{x_n\}$ be a sequence in X.

- $\{x_n\}$ is said to be left D-Cauchy sequence if:

$$\lim_{n\to\infty} D(x_{n+m}, x_n) = 0$$

- $\{x_n\}$ is said to be right D-Cauchy sequence if:

$$\lim_{n\to\infty} D(x_n, x_{n+m}) = 0$$

- $\{x_n\}$ is said to be D-Cauchy sequence if it is both left and right D-Cauchy sequence

Definition 4 [6]: Let (X, D) be a quasi JS-metric space.

- X is said to be left D-complete if every left D-Cauchy sequence in X is left D-convergent to some element in X
- X is said to be right D-complete if every left D-Cauchy sequence in is right -convergent to some element in X
- X is said to be D-complete if and only if it is left and right D-complete, so that every D-Cauchy sequence in X is D-convergent to some element in X

Definition 5: Let X be a nonempty set. A mapping $q: X\times X\rightarrow [0, \infty)$ is called quasi metric on X, if the following conditions are fulfilled:

- (Q_1) for every $x, y\in X$, have $q(x, y) = 0 = q(y, x)\Leftrightarrow x=y$
- (Q_2) for every $x, y\in X$, have $q(x, y)\leq q(x, z)+q(z, y)$

Pair (X, q) is called quasi-metric space.

Proposition 1.1: Any quasi-metric space is a quasi-JS-metric space with C=1.

RESULTS

In this section, the fixed point theorem for Ciric type contraction with two mappings in quasi JS-metric space was proved.

Definition 1: Let f, g: X→X be a two functions and k∈(0, 1). The f is generalized k-quasi contraction mapping with respect g. If it satisfies the following condition:

$$D\big(f(x),f(y)\big)\le k\ \max\left\{\begin{matrix} D\big(g(x),g(y)\big),\ D\big(g(x),f(x)\big), \\ D\big(g(y),f(y)\big),\ D\big(g(x),f(y)\big), \\ D\big(g(y),\ f(x)\big) \end{matrix}\right\} \qquad (1)$$

for every x, y∈X.

Proposition 2.1: Suppose that f is generalized-quasi contraction mapping with respect to g for some k∈(0, 1). If point ω∈X with D(ω, ω)<∞, then D(ω, ω) = 0.

For each x∈X, let us define:

$$\delta\ (D, f, x) = \sup\ \{D(f^{\,i}(x),f^{\,j}(x)):\ i,j\in\mathbb{N}_0\}$$

where, $f^0(x) = x$.

Theorem 2.1: Let (X, D) be a D-complete quasi JS-metric space with constant C and let f, g: X→X be generalized k-quasi contraction mapping with respect for some and . Suppose that there exists $x_0\in X$ such that $\delta(D, f, x_0)<\infty$. Then $\{f^n(x_0)\}$ converges to some ω∈X. If $D(x_0, f(\omega)<\infty)$, $D(\omega, f(\omega))<\infty$ and C<1, then ω is a fixed point of f and g. Moreover, if ω' is another fixed point of f and g with D(ω, ω')<∞, D(ω', ω)<∞ then ω = ω'.

Proof: Since f⊂g such that $f^0 = g^1$ and continuing in this similar manner $f^n = g^{n+1}$ for every n∈ℕ.

Since f is generalized k-quasi contraction mapping, for all $j\in\mathbb{N}_0$, have:

$$D\big(f^{n+i}(x_0),f^{n+j}(x_0)\big)$$
$$\le k\ \max\left\{\begin{matrix} D\big(g^{n+i}(x_0),g^{n+1+j}(x_0)\big),\ D(g^{n+i}(x_0), \\ f^{n+i}(x_0)),\ D(g^{n+1+j}(x_0),f^{n+1+j}(x_0)), \\ D\big(g^{n+i}(x_0),f^{n+1+j}(x_0)\big),\ D(g^{n+1+j}(x_0),f^{n+i}(x_0), \end{matrix}\right\} \qquad (2)$$
$$\le k\ \max\left\{\begin{matrix} D(f^{n-1+i}(x_0),f^{n+j}(x_0)),\ D(f^{n-1+i}(x_0), \\ f^{n+i}(x_0)),\ D(f^{n+j}(x_0),f^{n+1+j}(x_0)), \\ D\ \big(f^{n-1+i}(x_0),f^{n+1+j}(x_0)\big),\ D(f^{n+j}(x_0),f^{n+i}(x_0)\big) \end{matrix}\right\}$$

Also:

$$\delta(D, f, f^n(x_0)) = \sup\ \{D(f^{n+i}(x), f^{n+j}(x)):\ i,j\in\mathbb{N}_0\} \qquad (3)$$

From Eq. 2 and 3 have:

$$\text{Max}\left\{\begin{matrix} D\big(f^{n-1+i}(x_0),f^{n+j}(x_0)\big),D(f^{n-1+i}(x_0),f^{n+i}(x_0)), \\ D\big(f^{n+j}(x_0),f^{n+1+j}(x_0)\big), \\ D\big(f^{n-1+i}(x_0),f^{n+1+j}(x_0)\big),D(f^{n+j}(x_0),f^{n+i}(x_0), \end{matrix}\right\}\le\delta\big(D,\ f,\ f^n(x_0)\big)$$

$$\text{Max}\left\{\begin{matrix} D\big(g^{n+i}(x_0),g^{n+1+j}(x_0)\big), \\ D(g^{n+i}(x_0),f^{n+i}(x_0)), \\ D(g^{n+1+j}(x_0),f^{n+1+j}(x_0)), \\ D\big(g^{n+i}(x_0),f^{n+1+j}(x_0)\big), \\ D(g^{n+1+j}(x_0),f^{n+i}(x_0), \end{matrix}\right\}\le\delta\big(D,\ f,\ f^n(x_0)\big) \qquad (4)$$

Therefore, Eq. 2 yields $\delta(D, f, f^n(x_0))\underline{<}k\ \delta(D, f, f^{n-1}(x_0))$. Hence, for any $n\in\mathbb{N}_0$ have:

$$\delta(D, f, f^n(x_0))\underline{<}\ k^n\ \delta(D,f,x_0) \qquad (5)$$

Then for every m, $n\in\mathbb{N}_0$ find:

$$D(f^n(x_0), f^{n+m}(x_0))\underline{<}\delta(D, f, f^n(x_0))\underline{<}k_n\delta(D, f, x_0)$$

and:

$$D\ (f^{n+m})\ (x_0), f^n(x_0))\underline{<}\delta(D, f, f^n(x_0))\underline{<}k^n\delta(D, f, x_0) \qquad (6)$$

Since $\delta(D, f, x_0)<\infty$ and k<1, have:

$$\lim_{m,\ n\to\infty} D\ (f^n(x_0),f^{n+m}(x_0)) = \lim_{m,\ n\to\infty} D\ (f^{n+m}(x_0), f^n(x_0)) = 0 \qquad (7)$$

This implies that $\{f^n(x_0)\}$ is both left and right D-Cauchy sequence and hence D-Cauchy sequence. Since (X, D) is a D-Complete, then there exists ω∈X such that:

$$\lim_{n\to\infty} f^n(x_0) = \lim_{n\to\infty} g^{n+1}(x_0) = \omega$$

That is:

$$\lim_{n\to\infty} D\ (f^n(x_0), \omega) = \lim_{n\to\infty} D\ (f^n(x_0)) = 0$$

$$\lim_{n\to\infty} D\ (g^{n+1}(x_0), \omega) = \lim_{n\to\infty} D\ (g^{n+1}(x_0)) = 0$$

For any m, $n\in\mathbb{N}_0$ and condition (Eq. 6) have:

$$D(f^n(x_0), f^m(x_0)) \leq \delta(D, f, fn(x_0)) \leq k^n \delta(D, f, x_0)$$

and:

$$D(f^m(x_0), f^n(x_0)) \leq (D, f, f^n(x_0)) \leq k^n \delta(D, f, x_0) \quad (8)$$

Now assume that $D(x_0, f(\omega)<\infty$ and $D(\omega, f(\omega))<\infty$. Then by using Eq. 8 and condition (D_2) there exists $C>0$ such that:

$$D(\omega, f^n(x_0)) \leq C \lim_{m\to\infty} \sup D(f^m(x_0), f^n(x_0)) \leq Ck^n \delta(D, f, x_0)$$

$$D(f^n(x_0), \omega) \leq C \lim_{m\to\infty} \sup D(f^n(x_0), f^m(x_0)) \leq Ck^n \delta(D, f, x_0) \quad (9)$$

for every $n\in\mathbb{N}_0$.

Since f is generalized k-quasi contraction mapping with respect to g and condition (Eq. 1) have:

$$D\left(f(x_0), f(\omega)\right) \leq k \max\begin{Bmatrix} D\left(g(x_0), g(\omega)\right), D\left(g(x_0), f(x_0)\right), \\ D\left(g(\omega), f(\omega)\right), \\ D\left(g(x_0), f(\omega)\right), D\left(g(\omega), f(x_0)\right) \end{Bmatrix} \quad (10)$$

From Eq. 8 and 9 have:

$$\begin{cases} D(g(x_0), g(\omega)) = D(f^0(x_0), f^0(\omega)) < Ck^0 \delta(D, f, x_0) = \\ \qquad C\,\delta(D, f, x_0) \\ D(g(x_0), f(x_0)) = D(f^0(x_0), f(x_0)) < k^0 \delta(D, f, x_0) = \\ \qquad \delta(D, f, x_0) \\ D(g(\omega), f(x_0)) = D(f_0(\omega), f(x_0)) < Ck\,\delta(D, f, x_0) < \\ \qquad C\,\delta(D, f, x_0) \end{cases} \quad (11)$$

Hence:

$$D\left(f(x_0), f(\omega)\right) \leq \max\begin{Bmatrix} kC\delta(D, f, x_0), k\delta(D, f, x_0), \\ kD\left(g(\omega), f(\omega)\right), \\ kD\left(g(x_0), f(\omega)\right), kC\delta(D, f, x_0) \end{Bmatrix} \quad (12)$$

Again since f is generalized k-quasi contraction mapping with respect to g have:

$$D\left(f^2(x_0), f(\omega)\right) \leq \max\begin{Bmatrix} k^2C\delta(D, f, x_0), k^2\delta(D, f, x_0), \\ kD\left(g(\omega), f(\omega)\right), \\ kD\left(g(x_0), f(\omega)\right), k^2C\delta(D, f, x_0) \end{Bmatrix} \quad (13)$$

Continuing in the same manner, have:

$$D\left(f^n(x_0), f(\omega)\right) \leq \max\begin{Bmatrix} k^nC\delta(D, f, x_0), k^n\delta(D, f, x_0), \\ kD\left(g(\omega), f(\omega)\right), \\ kD\left(g(x_0), f(\omega)\right), k^nC\delta(D, f, x_0) \end{Bmatrix} \quad (14)$$

for every $n\in\mathbb{N}$. Now $D(x_0, f(\omega)<\infty$ and $\delta(D, f, x_0)<\infty$, have:

$$\lim_{n\to\infty} \sup D(f^n(x_0), f(\omega)) \leq k\, D(g(\omega), f(\omega))$$

$$\leq k\, D(f^0(\omega), f(\omega)) = kD(\omega, f(\omega)) \quad (15)$$

From the condition (D_2) and $C<1$ and $k<1$, get:

$$D\left(\omega, f(\omega)\right) \leq C \limsup_{n\to\infty} D\left(f^n(x_0), f(\omega)\right)$$

$$\leq Ck\, D(\omega, f(\omega)) < D(\omega, f(\omega)) \quad (16)$$

which is contradiction. Hence, $D(\omega, f(\omega)) = kD(f(\omega), \omega) = 0$. Therefore, get the $\omega = f(\omega)$.

Similarly, showed that $\omega = g(\omega)$. Hence, ω is a fixed point of f and g.

Uniqueness: Suppose that ω′ is another fixed point of f and g with $D(\omega, \omega')<\infty$, $D(\omega', \omega)<\infty$ and $D(\omega', \omega')<\infty$, then f is generalized k-quasi contraction mapping with respect to we have:

$$D\left(\omega', \omega\right) = \mathfrak{D}\left(f(\omega'), f(\omega)\right) \leq k \max\begin{Bmatrix} D(g(\omega'), g(\omega)), D\left(g(\omega'), f(\omega')\right), \\ D\left(g(\omega), f(\omega)\right), \\ D\left(g(\omega'), f(\omega)\right), D\left(g(\omega), f(\omega')\right) \end{Bmatrix}$$

$$\leq k \max\begin{Bmatrix} D(\omega', \omega), D\left(\omega'\omega'\right), D\left(\omega, \omega\right), \\ D\left(\omega', \omega\right), D(\omega, \omega') \end{Bmatrix}$$

$$\leq k \max\{D(\omega', \omega), 0, 0, D(\omega', \omega), D(\omega, \omega')\}$$

By proposition 2.1, which implies that $D(\omega', \omega) \leq k \max\{D(\omega', \omega), D(\omega, \omega')\}$.

Similarly, $D(\omega, \omega') \leq k \max\{D(\omega', \omega), D(\omega, \omega')\}$.

So that:

$$\max\{D(\omega', \omega), D(\omega, \omega')\} \leq k \max\{D(\omega', \omega), D(\omega, \omega')\}$$

Since $D(\omega, \omega')<\infty$, $D(\omega', \infty)<\infty$ and $k\in(0, 1)$, $D(\omega', \omega)$ which yields $\omega' = \omega$.

SIGNIFICANCE STATEMENTS

This study discovers the fixed point results in quasi JS-metric space for two mappings .The research work has been done previously on the fixed point theorems in quasi JS-metric space for single mapping while the present paper discovers the research for two mappings which is new and covers many existing fixed point theorems in abstract metric spaces (such as standard quasi-metric space, quasi-b-metric spaces, dislocated quasi-metric spaces and quasi modular spaces). This study proves to be beneficial for the researchers who are working in the field of fixed point theory to introduce a number of contraction type mapping in quasi JS-metric spaces.

CONCLUSION

This study proposed fixed point theorem for Ciric type contraction in quasi JS-metric space for two mappings. It generalizes fixed point theorem in quasi JS-metric space from single mapping to two mappings and proves that Ciric type contraction for two mappings in quasi JS-metric space exists in fixed point literature. Theorem 2.1 is a generalization of many fixed point results in fixed point literature in metric space, quasi metric space to quasi JS-metric space for ciric type contraction.

REFERENCES

1. Banach, S., 1922. Sur les operations dans les ensembles abstraits et leurs application aux equations integrales. Fund. Math., 3: 133-181.
2. Ciric, L.B., 1971. Generalized contractions and fixed-point theorems. Publ. Inst. Math., 12: 19-26.
3. Ciric, L.B., 1974. A generalization of Banach's contraction principle. Proc. Am. Math. Soc., 45: 267-273.
4. Jleli, M. and B. Samet, 2014. A new generalization of the Banach contraction principle. J. Inequal. Applic., Vol. 38. 10.1186/1029-242X-2014-38.
5. Hussain, N., V. Parvaneh, B. Samet and C. Vetro, 2015. Some fixed point theorems for generalized contractive mappings in complete metric spaces. Fixed Point Theory Applic., Vol. 185. 10.1186/s13663-015-0433-z.
6. Jleli, M. and B. Samet, 2015. A generalized metric space and related fixed point theorems. Fixed Point Theory Applic., Vol. 15. 10.1186/s13663-015-0312-7.
7. Noorwali, M., H.H. Alsulami and E. Karapınar, 2016. Some extensions of fixed point results over quasi-JS-spaces. J. Funct. Spaces, 10.1155/2016/6963041.