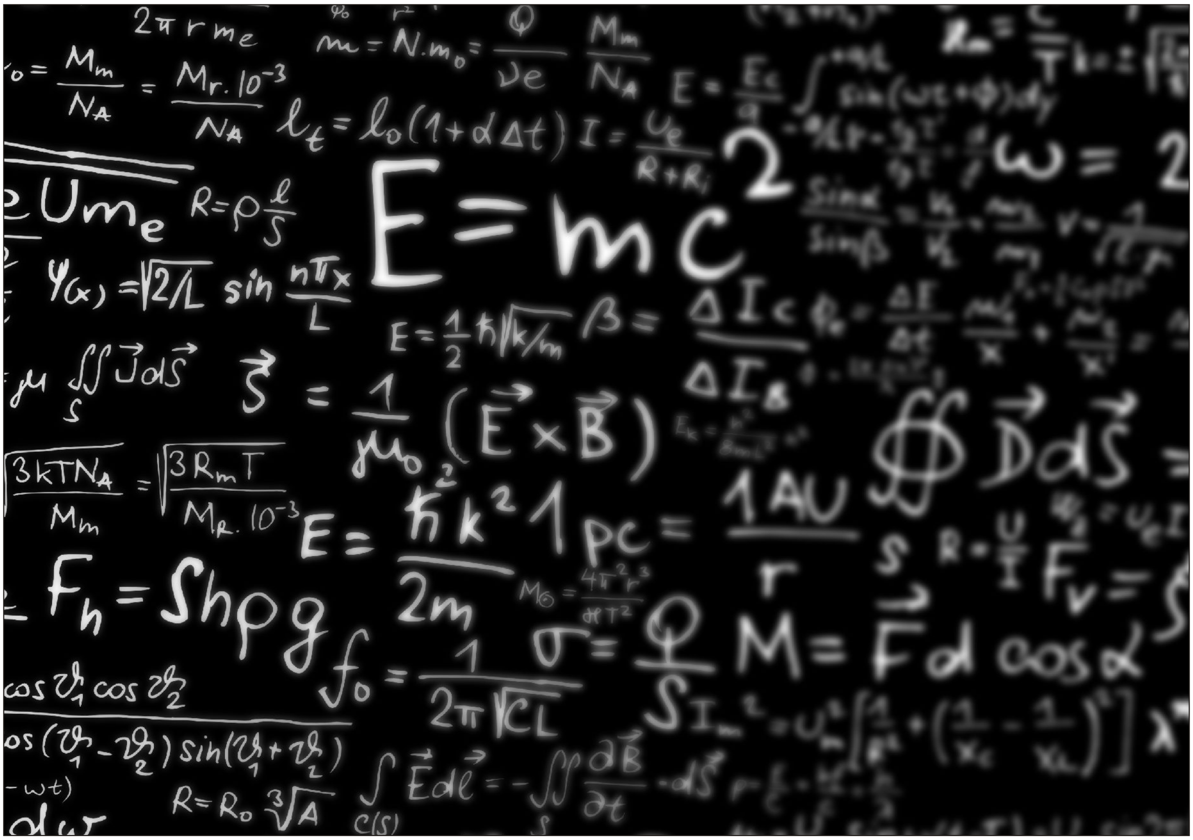

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Research Article

Generating q-Analogue of I-Function Satisfying Truesdell's Ascending F_q-Equation

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Abstract

Various forms of q-analogue of I-function satisfying Truesdell's descending F_q-equation have been studied and various generating functions by the application of these various forms have been derived. The aim of this study was to produce q-analogue of I-function which fulfilling Truesdell's Ascending F_q-equation. In this study, various forms of q-analogue of I-function satisfying Truesdell's ascending F_q-equation have been obtained. Certain generating functions for q-analogue of I-function have been derived by using these forms. Further, some particular cases of these results in terms of q-analogue of H-and G-functions which appear to be new have also been obtained.

Key words: F_q-equation, generating functions, q-analogue of I-function, q-analogue, descending F_q-equation, H-function, q-analogue of G-function, ascending F_q-equation

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INTRODUCTION

The concept of basic hypergeometric functions has been studied by many authors and some new generalized forms of these functions have been derived. So, it is significant to study these functions due to their applications in the fields like engineering and physical sciences[1].

Some basic hypergeometric functions are Meijer's G-function, Fox's H-function, Mac-Roberts's E-function, Saxena's I-function and their q-analogues. The q-analogue of I-function have been introduced in terms of Mellin-Barnes type basic contour integral by Saxena *et al.*[2] as:

$$I(z) = I^{m,n}_{A_i,B_i;r}\left[z;q\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right] = \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s})G(q^s)G(q^{1-s})\sin\pi s\right]}\pi z^s ds \tag{1}$$

where, α_j, β_j, α_{ji} β_{ji}, are real and positive, a_j, b_j, a, b_{ji} are complex numbers and:

$$G(q^\alpha) = \prod_{n=0}^{\infty}(1-q^{\alpha+n})^{-1} = \frac{1}{(q^\alpha;q)_\infty}$$

where, L is representing a contour of integration ranging from -i∞ to i∞ in such a way so that all poles of $G(q^{b_j-\beta_j s})$; $1\le j\le m$ are to right and those of $G(q^{1-a_j+\alpha_j s})$; $1\le j\le n$. The integral converges if Re [s log (x)-log sin πs}<0, for large values of |s| on the contour L.

Setting r = 1, $A_i = A$, $B_i = B$ in Eq. 1 we get q-analogue of H-function defined by Saxena *et al.*[2] as follows:

$$H^{m,n}_{A,B}\left[z;q\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,A}\\(b_j,\beta_j)_{1,B}\end{matrix}\right] = \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})}{\prod_{j=m+1}^{B} G(q^{1-b_j+\beta_j s})\prod_{j=n+1}^{A} G(q^{a_j-\alpha_j s})G(q^s)G(q^{1-s})\sin\pi s}\pi z^s ds \tag{2}$$

Additional if we put $\alpha_j = \beta_j = 1$, Eq. 2 reduces to the basic analogue of Meijer's G-function specified by Saxena *et al.*[2]:

$$G^{m,n}_{A,B}\left[z;q\,\middle|\,\begin{matrix}a_1, a_2, \ldots, a_A\\b_1, b_2, \ldots, b_B\end{matrix}\right] = \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m} G(q^{b_j-s})\prod_{j=1}^{n} G(q^{1-a_j+s})}{\prod_{j=m+1}^{B} G(q^{1-b_j+s})\prod_{j=n+1}^{A} G(q^{a_j-s})G(q^s)G(q^{1-s})\sin\pi s}\pi z^s ds \tag{3}$$

Basic analogue of I-function in terms of Gamma function is defined by Ahmad *et al.*[3] as follows:

$$I^{m,n}_{q,A_i,B_i;R}\left[z;q\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,n}\ (a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,m}\ (b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right] = \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma_q(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma_q(1-a_j+\alpha_j s)}{\sum_{i=1}^{R}\left\{\prod_{j=m+1}^{B_i}\Gamma_q(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{A_i}\Gamma_q(a_{ji}-\alpha_{ji}s)\right\}\Gamma_q(s)\Gamma_q(1-s)\sin\pi s}\pi z^s ds \tag{4}$$

Certain expansion formulae for a basic analogue of I-function defined by Ahmad *et al.*[3] in terms of basic analogue of Gamma function have been derived by Ahmad *et al.*[4].

To achieve the unification of special functions, Truesdell[5] has introduced the theory which provided a number of results for special functions satisfying the so called Truesdell's F-equation. Agrawal[6] extended the concept further and derived results for descending F-equation. Various properties like orthogonality, Rodrigue's and Schalafli's formulae for F-equation, which turn out to be special functions have been obtained by Agrawal[6].

The function F(z, α) is supposed to satisfy the ascending F-equation if:

$$D_z^r F(z,\alpha) = F(z,\alpha+r) \tag{5}$$

Truesdell[5] has obtained following generating functions using Taylor's series for F(z, a) satisfying ascending F-equation:

$$F(z+y,\alpha) = \sum_{n=0}^{\infty} y^n \frac{F(z,\alpha+n)}{n!} \tag{6}$$

The q-derivative of Eq. 5 can be written as:

$$D_{q,z}^r F(z,\alpha) = F(z,\alpha+r) \tag{7}$$

Ahmad *et al.*[7] obtained generating functions of q-analogue of I-function satisfying Truesdell's descending F_q-equation. Jain *et al.*[8] derived some generating functions of q-analogue of Mittag-Leffler function and Hermite polynomial satisfying Truesdell's ascending and descending F_q-equation. In this connection, it have obtained various forms of I-function which satisfies Truesdell's F_q-equation and have obtained various generating functions by employing these forms.

Following results have been used of multiplication formulae for q-analogue of Gamma functions to obtain main the consequences of this study:

$$\prod_{k=0}^{m-1}\Gamma_q\left(\frac{\alpha+r+k}{m}\right)=\frac{(q^{\alpha};q)_r}{(1-q)^r}\prod_{k=0}^{m-1}\Gamma_q\left(\frac{\alpha+k}{m}\right) \tag{8}$$

$$\prod_{k=0}^{m-1}\Gamma_q\left(1-\frac{\alpha+r+k}{m}\right)=\frac{q^{\frac{2r\alpha+r^2-r}{2m}}(1-q)^r}{(-1)^r(q^{\alpha};q)_r}\prod_{k=0}^{m-1}\Gamma_q\left(1-\frac{\alpha+k}{m}\right) \tag{9}$$

In the results that follow, by $\Delta(\mu, \alpha)$ we shall mean the array of μ parameters:

$$\frac{\alpha}{\mu}, \frac{\alpha+1}{\mu}, \ldots, \frac{\alpha+\mu-1}{\mu}; \qquad (\mu=1,2,3,\ldots) \tag{10}$$

and:

$$(\Delta(\mu,\alpha),\beta) \text{ stands for } \left(\frac{\alpha}{\mu},\beta\right),\left(\frac{\alpha+1}{\mu},\beta\right),\ldots,\left(\frac{\alpha+\mu-1}{\mu},\beta\right) \tag{11}$$

The main objective of this study was to generate functions of q-analogue of I-function that satisfying Truesdell's ascending F_q-equation. In this study, it have extended Truesdell's F-equation to its q-analogue and named the corresponding equation as F_q-equation. It have further derived various forms of q-analogue of I-function satisfying Truesdell's ascending F_q-equation. These forms have been employed to arrive at certain generating functions for q-analogue of I-function. Some particular cases of these results which appear to be new have also been obtained.

GENERATING FUNCTIONS FOR Q-ANALOGUE OF I-FUNCTION

Various forms of I-function satisfying Truesdell's ascending F-equation have been obtained by Jain *et al.*[9] and using this, different forms of q-analogue of I-function which satisfy Truesdell's ascending F_q-equation have been established:

$$\text{(I)}\ \left(q^{\frac{\alpha-1}{2}\left[\frac{\rho-1}{\rho}\right]}z\right)^{-\alpha} I_{p_i,q_i,1}^{m,n}\left[q^{\alpha h(\lambda-1)}z^{h\lambda};q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\rho}\},\{\Delta(\rho,\alpha),h\}\\ \{\Delta(\lambda,\alpha),h\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\rho}\},\{\Delta(\rho,\alpha),h\}\end{matrix}\right.\right] \tag{12}$$

$$\text{(II)}\ \left(\frac{q^{\frac{1}{2}\left[\frac{\alpha}{\lambda}+\alpha-1\right]}z}{(1-q)}\right)^{-\alpha} I_{p_i,q_i,1}^{m,n}\left[q^{\alpha h(\lambda+1)}z^{h\lambda};q\left|\begin{matrix}\left\{\Delta(\lambda,\alpha+\frac{1}{2}),h\right\}\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(2\lambda,2\alpha),h\},\{(b_j,\beta_j)_{2\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{13}$$

$$\text{(III)}\ \left(\frac{q^{\frac{1}{2}\left[\frac{3\alpha+1}{3\lambda}+\alpha-1\right]}z}{(1-q)}\right)^{-\alpha} I_{p_i,q_i,1}^{m,n}\left[q^{\alpha h(\lambda+1)}z^{h\lambda};q\left|\begin{matrix}\left\{\Delta(\lambda,\alpha+\frac{2}{3}),h\right\}\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\left\{\Delta(\lambda,\alpha+\frac{1}{3}),h\right\}\\ \{\Delta(3\lambda,3\alpha),h\},\{(b_j,\beta_j)_{3\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{14}$$

$$\text{(IV)}\ \left(q^{\frac{\alpha-1}{2}}z\right)^{-\alpha} e^{\pi i\alpha} I_{p_i,q_i,1}^{m,n}\left[(q^{\alpha}z)^{h\lambda};q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(\lambda,\alpha),h\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{15}$$

$$\text{(V)}\ \left(q^{\frac{1}{2}(\alpha-1)(\frac{\lambda-1}{\lambda})}z\right)^{-\alpha} I_{p_i,q_i,1}^{m,n}\left[q^{h\alpha(\lambda-1)}z^{h\lambda};q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\end{matrix}\right.\right] \tag{16}$$

$$\text{(VI)}\ \left(q^{\frac{1}{2}(\alpha-1)\left(\frac{\lambda-1}{\lambda}+\frac{1}{\rho}\right)}z\right)^{-\alpha} e^{\pi i\alpha} I_{p_i,q_i,1}^{m,n}\left[\begin{matrix}(q^{\alpha}z)^{h\lambda};\\ q\left|\begin{matrix}\{\Delta(\rho,\alpha),h\}\{(a_j,\alpha_j)_{\rho+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(\rho,\alpha),h\}\{(b_j,\beta_j)_{\rho+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\end{matrix}\right.\end{matrix}\right] \tag{17}$$

Suppose that the form Eq. 12 is A (z, α), then replace q-analogue of I-function by the definition (Eq. 4) and interchanging order of integration and differentiation, which is justified under the convergence conditions (Eq. 3), it observed that:

$$D_{q,z}^{r}A(z,\alpha)=\frac{1}{2\pi i}\int_{L}\frac{\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+k}{\lambda}-hs\right)\prod_{j=\lambda+1}^{m}\Gamma_q(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma_q(1-a_j+\alpha_j s)\,q^{\frac{\alpha(\alpha-1)}{2\rho}-\frac{\alpha(\alpha-1)}{2}}q^{\alpha h(\lambda-1)s}D_{q,z}^{r}\left\{z^{h\lambda s-\alpha}\right\}\pi ds}{\sum_{i=1}^{1}\left\{\prod_{j=m+1}^{q_i-\rho}\Gamma_q(1-b_{ji}+\beta_{ji}s)\prod_{k=0}^{\rho-1}\Gamma_q\left(1-\frac{\alpha+k}{\rho}+hs\right)\prod_{j=n+1}^{p_i-\rho}\Gamma_q(a_{ji}-\alpha_{ji}s)\prod_{k=0}^{\rho-1}\Gamma_q\left(\frac{\alpha+k}{\rho}-hs\right)\right\}\Gamma_q(s)\Gamma_q(1-s)\sin\pi s}\tag{18}$$

Now results Eq. 8 and 9 lead to two very important identities:

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+k}{\lambda}-hs\right)=\frac{(1-q)^r}{(q^{\alpha-h\lambda s};q)_r}\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+r+k}{\lambda}-hs\right)\tag{19}$$

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha+k}{\lambda}+hs\right)=\frac{(-1)^r(q^{\alpha-h\lambda s};q)_r}{q^{\frac{2r(\alpha-h\lambda s)+r^2-r}{2\lambda}}(1-q)^r}\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha+r+k}{\lambda}+hs\right)\tag{20}$$

Using these identities Eq. 19 and 20 we see that Eq. 18 takes the form:

$$D_{q,z}^{r}A(z,\alpha)=\left[A(z,\alpha+r)\right]\tag{21}$$

This is the Truesdell's form of ascending F_q-equation.

In the same way forms Eq. 13-17 can be exposed to satisfy Truesdell's ascending F_q-equation.

In this part it utilized forms Eq. 12-17, to found the following generating functions for q-analogue of I-functions using Truesdell's ascending F_q-equation technique:

$$\text{(I)}\ (1+q^{\alpha})^{-\alpha}I_{p_i,q_i,1}^{m,n}\left[(1+q^{\alpha})^{h\lambda}x;q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\rho}\},\{\Delta(\rho,\alpha),h\}\\ \{\Delta(\lambda,\alpha),h\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\rho}\},\{\Delta(\rho,\alpha),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\frac{(-2\alpha-r+1)(\frac{\rho-1}{\rho})}{2}+2\alpha}\right)^r}{r!}I_{p_i,q_i,1}^{m,n}\left[q^{rh(\lambda-1)}x;q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\rho}\},\{\Delta(\rho,\alpha+r),h\}\\ \{\Delta(\lambda,\alpha+r),h\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\rho}\},\{\Delta(\rho,\alpha+r),h\}\end{matrix}\right.\right]\tag{22}$$

$$\text{(II)}\ (1+q^{\alpha})^{-\alpha}I_{p_i,q_i,1}^{m,n}\left[(1+q^{\alpha})^{h\lambda}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha+\frac{1}{2}),h\}\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(2\lambda,2\alpha),h\},\{(b_j,\beta_j)_{2\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\frac{-1}{2}\left(\frac{2\alpha}{\lambda}+\frac{r}{\lambda}+r-1\right)}(1-q)\right)^r}{r!}I_{p_i,q_i,1}^{m,n}\left[q^{rh(1+\lambda)}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha+r+\frac{1}{2}),h\}\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(2\lambda,2(\alpha+r)),h\},\{(b_j,\beta_j)_{2\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]\tag{23}$$

$$\text{(III)}\ (1+q^{\alpha})^{-\alpha}I_{p_i,q_i,1}^{m,n}\left[(1+q^{\alpha})^{h\lambda}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha+\frac{2}{3}),h\}\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\}\{\Delta(\lambda,\alpha+\frac{1}{3}),h\}\\ \{\Delta(3\lambda,3\alpha),h\},\{(b_j,\beta_j)_{3\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\frac{-1}{2}\left(\frac{\alpha}{\lambda}+\frac{r}{2\lambda}+\frac{r}{2}+\frac{1}{6\lambda}-\frac{1}{2}\right)}(1-q)\right)^r}{r!}I_{p_i,q_i,1}^{m,n}\left[q^{rh(1+\lambda)}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha+r+\frac{2}{3}),h\}\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\}\{\Delta(\lambda,\alpha+r+\frac{1}{3}),h\}\\ \{\Delta(3\lambda,3(\alpha+r)),h\},\{(b_j,\beta_j)_{2\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]\tag{24}$$

$$\text{(IV)}\quad (1+q^{\alpha})^{-\alpha} I^{m,n}_{p_i,q_i,1}\left[(1+q^{\alpha})^{h\lambda}x;q \left| \begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(\lambda,\alpha),h\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{(-1)^r\left(q^{\frac{1-r}{2}}\right)^r}{r!} I^{m,n}_{p_i,q_i,1}\left[q^{rh\lambda}x;q \left| \begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(\lambda,\alpha+r),h\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{25}$$

$$\text{(V)}\quad (1+q^{\alpha})^{-\alpha} I^{m,n}_{p_i,q_i,1}\left[(1+q^{\alpha})^{h\lambda}x;q \left| \begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\left(\frac{\lambda-1}{\lambda}\right)\left[-\alpha-\frac{r}{2}+\frac{1}{2}\right]+\alpha}\right)^r}{r!} I^{m,n}_{p_i,q_i,1}\left[q^{rh(\lambda-1)}x;q \left| \begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\{\Delta(\lambda,\alpha+r),h\}\end{matrix}\right.\right] \tag{26}$$

$$\text{(VI)}\quad (1+q^{\alpha})^{-\alpha} I^{m,n}_{p_i,q_i,1}\left[(1+q^{\alpha})^{h\lambda}x;q \left| \begin{matrix}\{\Delta(\rho,\alpha),h\},\{(a_j,\alpha_j)_{\rho+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(\rho,\alpha),h\},\{(b_j,\beta_j)_{\rho+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{(-1)^r\left(q^{\left(\frac{1}{\lambda}-\frac{1}{\rho}-1\right)\left[\alpha+\frac{r}{2}-\frac{1}{2}\right]+\alpha}\right)^r}{r!} I^{m,n}_{p_i,q_i,1}\left[q^{rh\lambda}x;q \left| \begin{matrix}\{\Delta(\rho,\alpha+r),h\},\{(a_j,\alpha_j)_{\rho+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{\Delta(\rho,\alpha+r),h\},\{(b_j,\beta_j)_{\rho+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\{\Delta(\lambda,\alpha+r),h\}\end{matrix}\right.\right] \tag{27}$$

Proof: To found Eq. 22 it substitute the form Eq. 12 in Truesdell's ascending F-equation (Eq. 6) and replace by $\frac{y}{q^{\alpha}}$ and $q^{-\alpha h}y^{h\lambda}$ by x in sequence to obtain the necessary consequence. Likewise, consequence Eq. 26 can be proved by substituting the form Eq. 16 in Truesdell's ascending F-equation (Eq. 6) and with same substitution.

To found Eq. 23 it substitute the form Eq. 13 in Truesdell's ascending F-equation (Eq. 6) and substitute by $\frac{y}{q^{\alpha}}$ and $q^{\alpha h}y^{h\lambda}$ by x in series to get the mandatory result. Similarly, result (Eq. 24) can be proved by substituting the form Eq. 14 in Truesdell's ascending F-equation (Eq. 6) and by same substitution.

To establish Eq. 25 we substitute the form Eq. 15 in Truesdell's ascending F-equation (Eq. 6) and alternate by $\frac{y}{q^{\alpha}}$ and $y^{h\lambda}$ by x in series to get the necessary outcome. Correspondingly, result (Eq. 27) can be established by substituting the form Eq. 17 in Truesdell's ascending F-equation (Eq. 6) and via same substitution.

Special cases: These consequences capitulate as special cases of certain generating function for q-analogue of Fox's H-function[2] and q-analogue of Meijer's G-function[2].

- If we supposed then the sequence (Eq. 22) diminishes to generating function of q-analogue of Fox's H-function:

$$(1+q^{\alpha})^{-\alpha} H^{m,n}_{P,Q}\left[(1+q^{\alpha})^{h\lambda}x;q \left| \begin{matrix}\{(a_j,\alpha_j)\},\{\Delta(\rho,\alpha),h\}\\ \{\Delta(\lambda,\alpha),h\},\{(b_j,\beta_j)\},\{\Delta(\rho,\alpha),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\frac{(-2\alpha-r+1)(\frac{\rho-1}{\rho})}{2}+2\alpha}\right)^r}{r!} H^{m,n}_{P,Q}\left[q^{rh(\lambda-1)}x;q \left| \begin{matrix}\{(a_j,\alpha_j)\},\{\Delta(\rho,\alpha+r),h\}\\ \{\Delta(\lambda,\alpha+r),h\},\{(b_j,\beta_j)\},\{\Delta(\rho,\alpha+r),h\}\end{matrix}\right.\right] \tag{28}$$

Again taking in Eq. 28, it gives Meijer's G-function as:

$$(1+q^{\alpha})^{-\alpha} G^{m,n}_{P,Q}\left[(1+q^{\alpha})^{\lambda}x;q \left| \begin{matrix}a_j,\Delta(\rho,\alpha)\\ \Delta(\lambda,\alpha),b_j,\Delta(\rho,\alpha)\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\frac{(-2\alpha-r+1)(\frac{\rho-1}{\rho})}{2}+2\alpha}\right)^r}{r!} G^{m,n}_{P,Q}\left[q^{r(\lambda-1)}x;q \left| \begin{matrix}a_j,\Delta(\rho,\alpha+r)\\ \Delta(\lambda,\alpha+r),b_j,\Delta(\rho,\alpha+r)\end{matrix}\right.\right] \tag{29}$$

In the same way, Eq. 23-27 can be used to acquiesce apparently new and interesting consequences for q-analogue of Fox's H-function and Meijer's G-function[2].

CONCLUSION

This study presented various forms of q-analogue of I-function satisfying Truesdell's ascending F_q-equation. These forms have been employed to arrive at certain generating functions for q-analogue of basic analogue of I-function.

The results proved in this study along with their particular cases are believed to be new. As these functions have well established as applicable functions these results are likely to contribute significantly in certain application of the theory of q-calculus.

SIGNIFICANCE STATEMENT

This study discovers various forms of q-analogue of I-function satisfying Truesdell's ascending F_q-equation. These forms have been employed to arrive at certain generating functions for q-analogue of basic analogue of I-function. As these functions have well established as applicable functions these results are likely to contribute significantly in certain application of the theory of q-calculus.

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