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Research Article
Vibration Analysis of Multi-Cracked Beam Traversed by Moving Mass

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Abstract
In this study vibration analysis of multi-cracked beam under moving mass was investigated. Timoshenko beam model was considered and governing equations were found. Assumed-modes method was applied to find eigenvalues and eigenvectors of cracked beam. Cracks in the beam structure were modeled as local flexibility and were replaced by transverse and rotational spring pair. The compliance factors of springs were found based on fracture mechanic theorem. Due to cracks, beam was divided into segments. Compatibility equations were implemented to relate every two segments. Different boundary conditions were considered and deflections were found and debated. Natural frequencies were calculated and compared. It was observed that, crack attenuates beam structure and reduces natural frequency effectively. Mid-span of the beam was found as the critical point for crack location. Mid-span deflection for various mass speeds was calculated and the critical speed was defined. Results were verified against those which were found through Discrete Element Technique (DET) and Finite Element Method (FEM).

Key words: Multi-cracked beam, transfer matrix method, timoshenko beam model, beam under moving mass

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Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Analyzing the free and forced vibration of structures is an interesting and important issue that has been performed widely. Study on the vibration of beam-like structures has been addressed extensively in the literature. Investigations in this area were achieved by Fryba (1999). He worked effectively on the dynamic analysis of the beam under different kinds of loading. Crack attendance in the beam structures is one of the popular phenomenon. Crack may cause serious change in the mechanical behavior of the structure. Many studies are performed to simulate the initiation and propagation of cracks in the beam. Mahmoud and Abou Zaid (2002) studied on the effect of cracked beam under the act of moving mass. They applied iterative modal analysis approach to solve the deflection of cracked beam. Increase in deflection was reported as the result of crack attendance. The initiation of crack in the beam reduces beam rigidity and thereby decreases beam's natural frequencies. Mazanoglu *et al.* (2009) applied energy method to calculate natural frequencies of double cracked beam. Breathing model was implemented to simulate the cracks. Mofid and Akin (1996) introduced new method to study the response of the beam under moving mass. The method was called Discrete Element Technique (DET) and accurate results were found. Ariaei *et al.* (2009) used DET to find the dynamic vibration of cracked beam. They investigated the effect of speed, crack depth and crack location in the beam deflection and found that the beam with breathing crack model results less deflection in compare with open crack one. Crack detection in the beam is also achieved by many researchers. Crack detection in the Timoshenko beam was analyzed by Khaji *et al.* (2009) via closed form solution. In order to detect crack, initially they measured natural frequencies and then found crack location and sectional flexibility through characteristic equation. In another study Nguyen and Tran (2010) used on-vehicle signals and wavelet analysis to detect crack in multi-cracked beam. Transfer Matrix Method (TMM) was implemented by many researchers. Lin (2004) used TMM to solve the direct and inverse problem of cracked beam. The sectional flexibility and location of crack was found through introducing characteristic equations of cracked beam. Moreover, the relationship between sectional flexibility and crack size was implemented to detect crack depth. Crack identification in the beam is presented by Zhang *et al.* (2009). Wavelet analysis is combined with TMM to find the crack depth and crack location. Identification of multiple cracks is reported in another study by Lee (2009). In their study crack is modeled as rotational spring. While forward solution is performed using FEM, the inverse problem is solved iteratively by Newton-Raphson method. In another study Lin and Chang (2006) developed TMM to find the forced response of cracked cantilever beam. They considered two-span beam model and simulated the crack as a rotational spring with sectional flexibility. Cracks identification in the stepped multi-cracked beam was calculated by Attar (2012) using TMM. Several cracks were considered in the beam with different end conditions. Ariaei *et al.* (2010) repaired cracked beam subjected to moving mass using piezoelectric patches. They considered TMM to solve the Timoshenko cracked beam. It was resulted that applying piezoelectric patch at crack area is helpful and natural frequencies of repaired beam in comparison with healthy one have good adjustment. Vibration analysis of the beam with crack was also considered in few studies (Lin *et al.*, 2002; Loya *et al.*, 2006; Mazanoglu and Sabuncu, 2010). To the best knowledge of the author, study on the multi-cracked beam under moving mass has not been yet recorded in the literature. Current study is focused on the evaluating of frequency and forced response of the multi-cracked beam.

MATERIALS AND METHODS

Vibration analysis of multi-cracked beam was performed in this study. Beam was traversed by moving mass with constant speed. Timoshenko beam model was applied to derive the governing beam equations. To simulate the crack transverse and rotational spring pair was used. Characteristic equation of multi-cracked beam was derived and natural frequencies and mode shapes for different numbers of cracks were calculated. Moreover, different speeds of moving mass were considered and compared. All computations were performed through programming in MATLAB software.

Equations of cracked Timoshenko beam: Cracked beam under moving mass is shown in Fig. 1. Crack is located at the middle and is modeled as transverse and rotational spring pair (Fig. 2). Timoshenko beam model is used and simply supported end conditions are considered. It is assumed that,

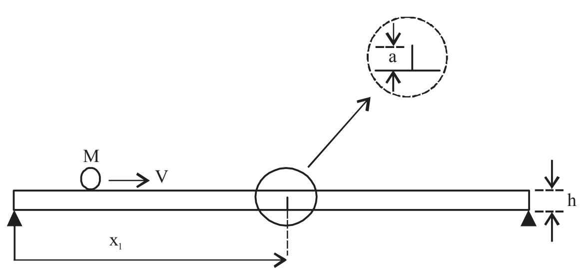

Fig. 1: Cracked beam under moving mass

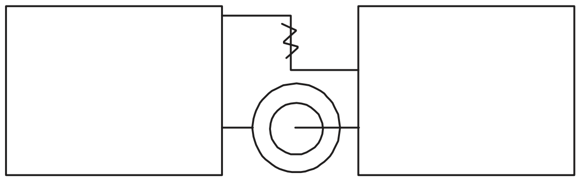

Fig. 2: Crack modelling

mass remains in contact with the beam surface during travelling. Equations of motion for Timoshenko beam model are derived in Eq. 1 and 2:

$$\rho A\frac{\partial^2\Psi_i}{\partial t^2}-\kappa AG\left(\frac{\partial^2\Psi_i}{\partial x^2}-\frac{\partial\varnothing_i}{\partial x}\right)=P(t)\delta(x-\xi(t)) \quad (1)$$

$$EI\frac{\partial^2\varnothing_i}{\partial x^2}+\kappa AG\left(\frac{\partial^2\Psi_i}{\partial x}-\varnothing_i\right)-\rho I\frac{\partial^2\varnothing_i}{\partial t^2}=0,\ x_{i-1}<x<x_i \quad (2)$$

where, Ψ_i and $\varnothing_i$ are deflection and rotation of i-th segment, respectively.

E, ρ, κ, G, I and A are modulus of elasticity, volumetric mass density, shear correction factor, shear modulus, cross section moment of inertia and cross section area, respectively. Term δ(x-ξ(t)) is Dirac delta function and ξ(t) (which is equal to vt) denotes the location of moving mass. The P(t) is the force that is exerted on the beam structure by moving mass and is defined as follow:

$$P(t)=M\{g\frac{\partial^2\Psi_i}{\partial t^2}\,|\,x=\xi(t)\} \quad (3)$$

where, M is the mass of traveler and g is acceleration of gravity. Compatibility requirements in the crack section based on beam continuity in bending moment and shear force are defined and expressed in Eq. 4 and 5; two other equations (Eq. 6, 7) are brought according to discontinuities in transverse and rotational displacements at crack location:

$$EI\varnothing_1'(x_1^-,t)=EI\varnothing_2'(x_1^+,t) \quad (4)$$

$$\kappa AG[\psi_1'(x_1^-,t)-\varnothing_1(x_1^-,t)]=\kappa AG[\psi_2'(x_1^+,t)-\varnothing_2(x_1^+,t)] \quad (5)$$

$$\psi_2(x_1^+,t)-\psi_1(x_1^-,t)=c_v\frac{\kappa hG}{E}[\psi_2'(x_1^+,t)-\varnothing_2(x_1^+,t)] \quad (6)$$

$$\varnothing_2(x_1^+,t)-\varnothing_1(x_1^-,t)=c_\theta h\varnothing_2'(x_1^+,t) \quad (7)$$

where, x_1 is location of crack (Fig. 1) and x_1^- and x_1^+ are those locations which are placed immediately before and after crack position. The c_v and c_θ are non-dimensional function of γ = a/h and for rectangular cross-section have been defined in following statements (Mahmoud and Abou Zaid, 2002):

$$c_v=\left(\frac{\gamma}{1-\gamma}\right)^2\left(-0.22+3.82\gamma+1.54\gamma^2-14.64\gamma^3+9.60\gamma^4\right) \quad (8)$$

$$c_\theta=2\left(\frac{\gamma}{1-\gamma}\right)^2\left(5.93-19.69\gamma+37.1\gamma^2-35.84\gamma^3+13.12\gamma^4\right) \quad (9)$$

The following formulations are brought for different boundary conditions.

Both sides simply supported:

$$\begin{aligned}\psi(0,t)&=\varnothing'(0,t)=0\\ \psi(L,t)&=\varnothing'(L,t)=0\end{aligned} \quad (10)$$

Left side simply supported, right side clamped:

$$\begin{aligned}\psi(0,t)&=\varnothing'(0,t)=0\\ \psi(L,t)&=\varnothing(L,t)=0\end{aligned} \quad (11)$$

Left side clamped, right side simply supported:

$$\begin{aligned}\psi(0,t)&=\varnothing(0,t)=0\\ \psi(L,t)&=\varnothing'(L,t)=0\end{aligned} \quad (12)$$

Both side clamped:

$$\begin{aligned}\psi(0,t)&=\varnothing(0,t)=0\\ \psi(L,t)&=\varnothing(L,t)=0\end{aligned} \quad (13)$$

Finding eigenvalues and eigenfunction for free vibration: Free vibration of cracked beam can be calculated letting the right side of Eq. 1 to be zero. Using separable solution and substituting $\Psi i(x,t)=\Lambda_i(x)e^{j\Omega t}$ and $\varnothing_i(x,t)=\varphi_i(x)e^{j\Omega t}$ in Eq. 1 and 2 and doing some arrangements result (Ariaei *et al.*, 2010):

$$\Lambda_i^{iv}(x)+(\alpha+\beta)\Lambda_i''(x)-(\eta-\alpha\beta)\Lambda_i(x)=0 \quad (14)$$

$$\varphi_i^{iv}(x)+(\alpha+\beta)\varphi_i''(x)-(\eta-\alpha\beta)\varphi_i(x)=0\ x_{i-1}<x<x_i \quad (15)$$

$$\alpha=\frac{\rho\Omega^2}{E}\ \beta=\frac{\rho\Omega^2}{\kappa G}\ \eta=\frac{A\rho\Omega^2}{EI}$$

Eigenvalues of cracked beam are calculated through solving Eq. 14 and 15 as follow:

$$\Lambda_i(x) = A_i \cosh\lambda_1 (x - x_{i-1}) + B_i \sinh\lambda_1 (x - x_{i-1}) + C_i \cosh\lambda_2 (x - x_{i-1}) + D_i \cosh\lambda_2 (x - x_{i-1}) \quad (16)$$

$$\varphi_i(x) = B_i q_1 \cosh\lambda_1 (x - x_{i-1}) + B_i q_1 \sinh\lambda_1 (x - x_{i-1}) + D_i q_2 \cosh\lambda_2 (x - x_{i-1}) + C_i q_2 \cosh\lambda_2 (x - x_{i-1}) \; x_{i-1} < x < x_i \quad (17)$$

$$\lambda_1 = [((\frac{\alpha-\beta}{2})^2 + \eta)^{1/2} - \frac{\alpha-\beta}{2}]^{1/2}, \; \lambda_2 = [((\frac{\alpha-\beta}{2})^2 + \eta)^{1/2} + \frac{\alpha-\beta}{2}]^{1/2}$$

$$\lambda_3 = (\beta)^{1/2}, \; q_1 = \frac{(\lambda_3^2 + \lambda_1^2)}{\lambda_1}, q_2 = \frac{(\lambda_3^2 + \lambda_2^2)}{\lambda_2} \quad (18)$$

Constants A_i, B_i, C_i and D_i in Eq. 16 and 17 should be determined through applying Eq. 4-7. Constants of every segment are associated to previous segment through transfer matrix method:

$$\begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \quad (19)$$

Elements of matrix Γ for the beam with one crack at midpoint are defined in Appendix 1. Whenever numbers of cracks are more than one, constants of last segment can be calculated via following procedure:

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \\ C_{n+1} \\ D_{n+1} \end{bmatrix} = \Gamma_n \, \Gamma_{n-1} \Gamma_{n-2} \ldots \Gamma_1 \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \quad (20)$$

Equation 20 shows the relation between segments, where, n is the number of cracks. Once there are n cracks in the beam structure and the distance between cracks are equal then Eq. 20 reduces to:

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \\ C_{n+1} \\ D_{n+1} \end{bmatrix} = \Gamma^n \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \quad (21)$$

Applying this method, all unknown constants reduce to four. The four unknowns can be calculated through contributing boundary conditions. Determinant of resultant matrix leads to characteristic equation that is defined as follow:

$$2\left(q_2\lambda_2)(q_1 - \lambda_1)\right)\kappa_1 c_\theta \sin(\lambda_2 l) \cosh(\lambda_1 l) \sin(\lambda_1 l) \sin(\lambda_2 l) + \left(q_2\lambda_2)(q_1 - \lambda_1)\right)^2 c_v c_\theta \cosh^2(\lambda_1 l) \sin^2(\lambda_2 l)$$

$$-2\left(q_1\lambda_1)(q_2 - \lambda_2)\right)\kappa_2 c_v \sin(\lambda_1 l) \cosh(\lambda_1 l) \cos^2(\lambda_2 l) - \left(q_1\lambda_1)(q_2 - \lambda_2)\right)^2 c_v c_\theta \sinh^2(\lambda_1 l) \cos^2(\lambda_2 l) = 0 \quad (22)$$

Where:

$$\kappa_1 = q_1\lambda_1 - q_2\lambda_2, \; \kappa_2 = q_2\lambda_1 + q_1\lambda_2$$

One can find eigenvalues solving Eq. 22.

Forced response of multi-cracked beam: In order to find forced vibration of cracked beam Eq. 1 and 2 are rewritten here:

$$\rho A \frac{\partial^2 \Psi_i}{\partial t^2} - \kappa AG\left(\frac{\partial^2 \Psi_i}{\partial t^2} - \frac{\partial \varnothing_i}{\partial x}\right) = P(t)\delta(x - \xi(t)) \quad (23)$$

$$EI\frac{\partial^2 \varnothing_i}{\partial x^2} + \kappa AG\left(\frac{\partial^2 \Psi_i}{\partial x} - \varnothing_i\right) - \rho I \frac{\partial^2 \varnothing_i}{\partial t^2} = 0, \; x_{i-1} < x < x_i \quad (24)$$

Identical modal amplitudes for deflection and rotation are introduced as:

$$\Psi_i(x,t) = \sum_{k=1}^{N} \Lambda_{ik}(x) T_k(t) \quad (25)$$

$$\phi_i(x,t) = \sum_{k=1}^{N} \varphi_{ik}(x) T_k(t) \quad (26)$$

where, $\Lambda_{ik}(x)$ and $\varphi_{ik}(x)$ are eigenfunction (modal shape) and $T_k(t)$ is generalized time function. Substituting Eq. 25-26 in Eq. 23-24 leads to:

$$\sum_{k=1}^{N} \{\rho A \, \Lambda_{ik}(x) \ddot{T}_k(t) - \kappa AG(\Lambda'_{ik} - \varphi'_{ik}) T_k(t)\} = P(t)\delta[x - \xi(t)] \quad (27)$$

$$\sum_{k=1}^{N} \{EI\varphi'_{ik}(x) T_k(t) + \kappa AG(\Lambda'_{ik} - \varphi_{ik}) T_k(t) - \rho I \varphi_{ik}(x) \ddot{T}_k(t)\} = 0 \; x_{i-1} < x < x_i \quad (28)$$

From free vibration analysis it can be concluded:

$$-\kappa AG\left(\Lambda'_{ik}(x) - \varphi'_{ik}(x)\right) = \rho A \, \Omega_k^2 \, \Lambda_{ik}(x) \quad (29)$$

$$EI\varphi''_{ik}(x)+\kappa AG\left(\Lambda'_{ik}(x)-\varphi_{ik}(x)\right)=-\rho I\Omega_k^2\varphi_{ik}(x) \qquad (30)$$

where, Ω_k is k-th natural frequency of cracked beam. Substituting Eq. 29-30 in Eq. 27-28 results in:

$$\rho A\sum_{k=1}^{N}\Lambda_{ik}(x)[\ddot{T}_k(t)+\Omega_k^2T_k(t)]=P(t)\delta[x-\xi(t)] \qquad (31)$$

$$\rho I\sum_{k=1}^{N}\varphi_{ik}(x)[\ddot{T}_k(t)+\Omega_k^2T_k(t)]=0 \quad x_{i-1}<x<x_i \qquad (32)$$

In order to solve Eq. 31 and 32 for time-function ($T_k(t)$), orthogonality conditions are used (Dadfarnia *et al*., 2005):

$$\sum_{i=1}^{n+1}\int_0^L[\rho A\Lambda_{ik}(x)\Lambda_{ij}(x)+\rho I\varphi_{ik}(x)\varphi_{ij}(x)]dx=Z_k\delta_{kj}\quad k,\ j=1,2,\ldots,N \qquad (33)$$

Where:

$$Z_k=\sum_{i=1}^{n+1}\int_0^L[\rho A\Lambda_{ik}^2(x)+\rho I\varphi_{ik}^2(x)]dx \qquad (34)$$

where, δ_{kj} is Kronecker delta and L is the entire length of the beam. Multiplying Eq. 31 by $\Lambda_{jk}(x)$ and Eq. 32 by $\varphi_{jk}(x)$ and then adding each side together and integrating over entire length of the beam and finally using orthogonality conditions result:

$$\begin{aligned}\ddot{T}_k(t)\Omega_k^2T_k(t)&=\frac{P(t)}{Z_k}\int_0^L\Lambda_{ik}(x)\delta[x-\xi(t)]dx=\frac{P(t)}{Z_kL}\Lambda_{ik}(\xi)\\&=\frac{\Lambda_{ik}(\xi)}{Z_kL}M\left\{g-\sum_{k=1}^{N}\Lambda_{ik}(\xi)\ddot{T}_k\right\}=Q_k(t)\end{aligned} \qquad (35)$$

where, M is traveling mass. Equation 35 is a coupled second orders linear differential equation. Solution of above equation is presented by Ariaei *et al*. (2010); time function can be determined by solving this equation. Deflection and rotation of cracked beam are calculated through substituting generalized time function in Eq. 25 and 26 for every segment of the beam.

RESULTS

Two case studies were defined and maximum deflection of the beam with different numbers of crack was calculated.

First case study: Simply supported beam with length L = 50 m, density p = 7860 kg m^{-3}, modulus of elasticity $E = 2.1\times10^{11}$ N m^{-2}, cross sectional height h = 1 m, cross sectional width D = 0.5 m, crack depth $\alpha = 0.5$ m and shear

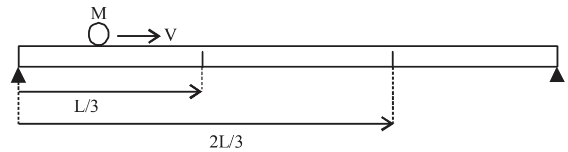

Fig. 3: Beam with two cracks

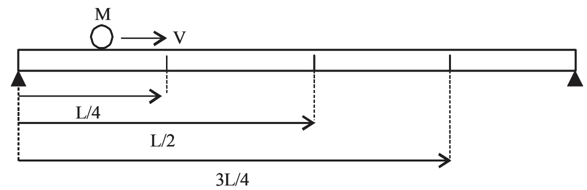

Fig. 4: Beam with three cracks

Table 1: Compare of maximum deflection in cracked beam

Maximum deflection	One-crack Current study	One-crack Ariaei *et al*. (2010)	Two-crack Current study	Three-crack Current study
TMM	2.037	2.051	2.075	2.249
DET	2.018	2.059	2.072	2.216
FEM	1.937	-	2.011	2.157

TMM: Transfer Matrix Method, DET: Discrete Element Technique, FEM: Finite Element Method

Table 2: Three natural frequencies of cracked beam

Natural frequencies (Hz)	Without crack (Present study)	One crack in the middle Present study	One crack in the middle Ariaei *et al*. (2010)
f_1	46.85	43.94	43.94
f_2	187.02	187.01	187.01
f_3	395.98	395.98	395.96

modulus $G = 8.077\times10^{10}$ N m^{-2} was considered. Mass (M = 0.2 m) is moving with constant speed V = 40 $msec^{-2}$. Crack locations are shown in Fig. 1-4 for the beam with one, two and three cracks, respectively. The deflection of the mid-span of the beam for one and three cracks was compared in Fig. 5. It can be seen that more cracks in the beam may cause more displacement in the mid-span. Maximum deflection of the beam with one, two and three cracks are tabulated in Table 1. Two alternative methods of Discrete Element Technique (DET) and Finite Element Method (FEM) were implemented and results were compared against them. Results in Table 1 shows trivial discrepancy between applied methods. Moreover, the comparison of results with available studies in the literature shows less than 2% differences (Table 1).

Second case study: In the second case a beam with total length L = 1 m was introduced. The height and width of the beam are h = 0.02, D = 0.02, respectively. Other mechanical properties are similar to earlier case. Natural frequencies of the cracked beam were calculated and arranged in Table 2.

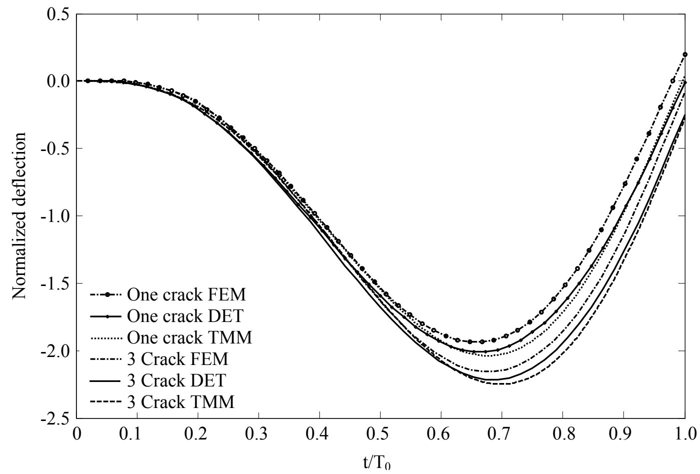

Fig. 5: Comparison of beam midpoint deflection for one and three cracks, TMM: Transfer matrix method, DET: Discrete element technique, FEM: Finite element method

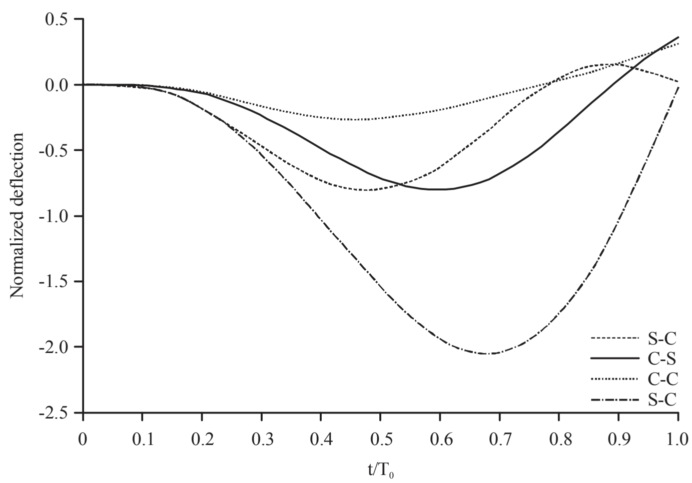

Fig. 6: Normalized deflection of mid-span for different boundary conditions in the beam with one crack in the middle

Table 3: Natural frequencies for different boundary conditions

Natural frequencies (Hz)	Different end conditionsss					
	Fixed-fixed		Hinged-fixed		Hinged-hinged	
	No-crack	One-crack	No-crack	One-crack	No-crack	One-crack
f_1	105.98	101.30	73.11	70.06	46.85	43.94
f_2	291.09	291.09	236.29	234.43	187.02	187.01

Comparison between frequencies of current study and available studies in the literature shows acceptable adjustment. It can be observed that there is reduction in natural frequencies of cracked beam. Maximum deflection of cracked beam for different boundary conditions is illustrated in Fig. 6. Type of boundary conditions changes natural frequencies dramatically and affects beam deflection. Natural frequencies of cracked beam for different boundary conditions were tabulated in Table 3. Maximum natural frequencies in Table 3 are belonged to fixed-fixed beam with minimum mid-span deflection (Fig. 6). On the contrary, hinged-hinged beam with minimum natural frequencies has maximum deflection. First natural frequency of the beam with different numbers of crack was shown in Fig. 7. Increasing crack numbers reduces frequency additionally. The effect of crack location in the beam was studied by changing the position of crack along axial direction. Figure 8 shows the first natural frequency of single cracked beam; the position of crack is changed along axial direction and frequency is then recorded. Natural frequency decreases gradually as crack location moves toward middle. There is a massive reduction in the natural frequency at the mid-span. It declares that the existence of crack at the middle of the beam is so critical. First and second

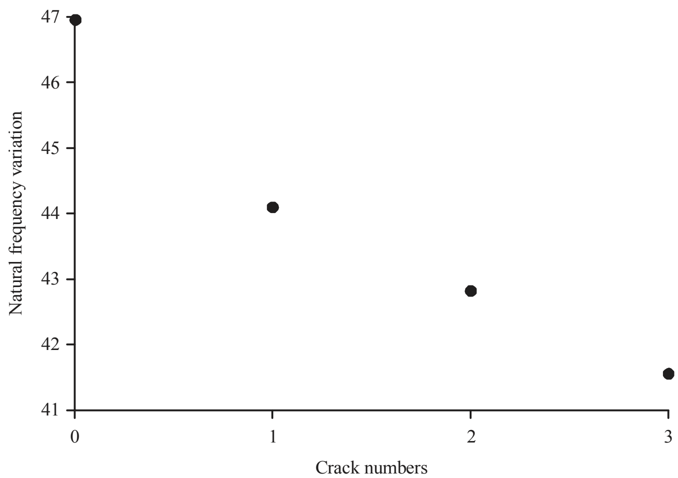

Fig. 7: Decrease of natural frequency with increasing numbers of crack

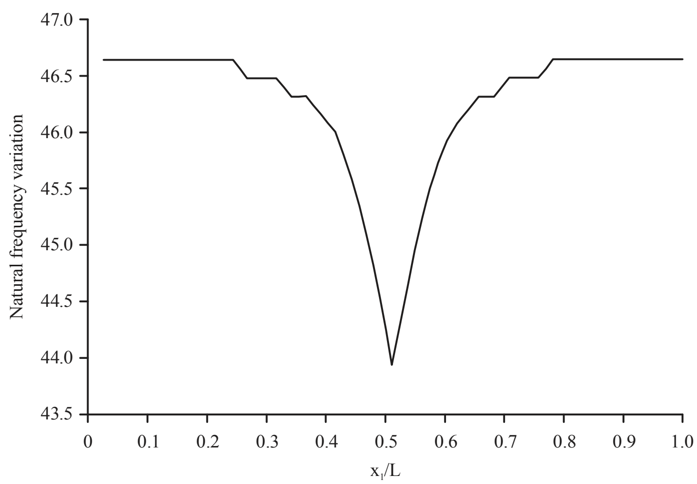

Fig. 8: Recording of first natural frequency for different crack locations

mode shapes of the beam with one, two and three cracks were found in Fig. 9a-c. Due to crack, discontinuity happens in the mode shape diagrams. The locations of cracks can be detected easily by searching discontinuities in every diagram. Figure 10 shows the maximum deflection of single cracked beam; crack location was changed along the axial axis and deflection was then obtained. Different crack depths were considered and deflection was then found. The presence of crack at the middle of the beam gives maximum deflection. For those crack location away of the mid-point maximum deflection decreases considerably. Study on crack depth in Fig. 10 discovered that, it can cause drastic effect in the beam deflection. Maximum deflection of the beam was compared considering various mass speeds (Fig. 11). The recorded results demonstrate that critical speed decreases by increasing numbers of cracks in the beam.

DISCUSSION

Maximum displacement of cracked beam at mid-span showed higher values at present of multiple cracks. Deflection in the beam with three cracks in comparison with single crack revealed 10% increment (Table 1). However double cracked beam displayed just 2% more deflection in comparison with single cracked one. It can be explained that, in the beam with three cracks (Fig. 4) the attendance of crack at mid-span causes significant effect in the beam deflection under moving mass. Natural frequencies analysis presented that, crack may attenuate frequencies in the beam and cause more deflection under loading. Moreover, a simple comparison between natural frequencies of cracked beam with different boundary conditions discovered, the frequency of simply-supported beam reduces 2% more than other kind of boundary

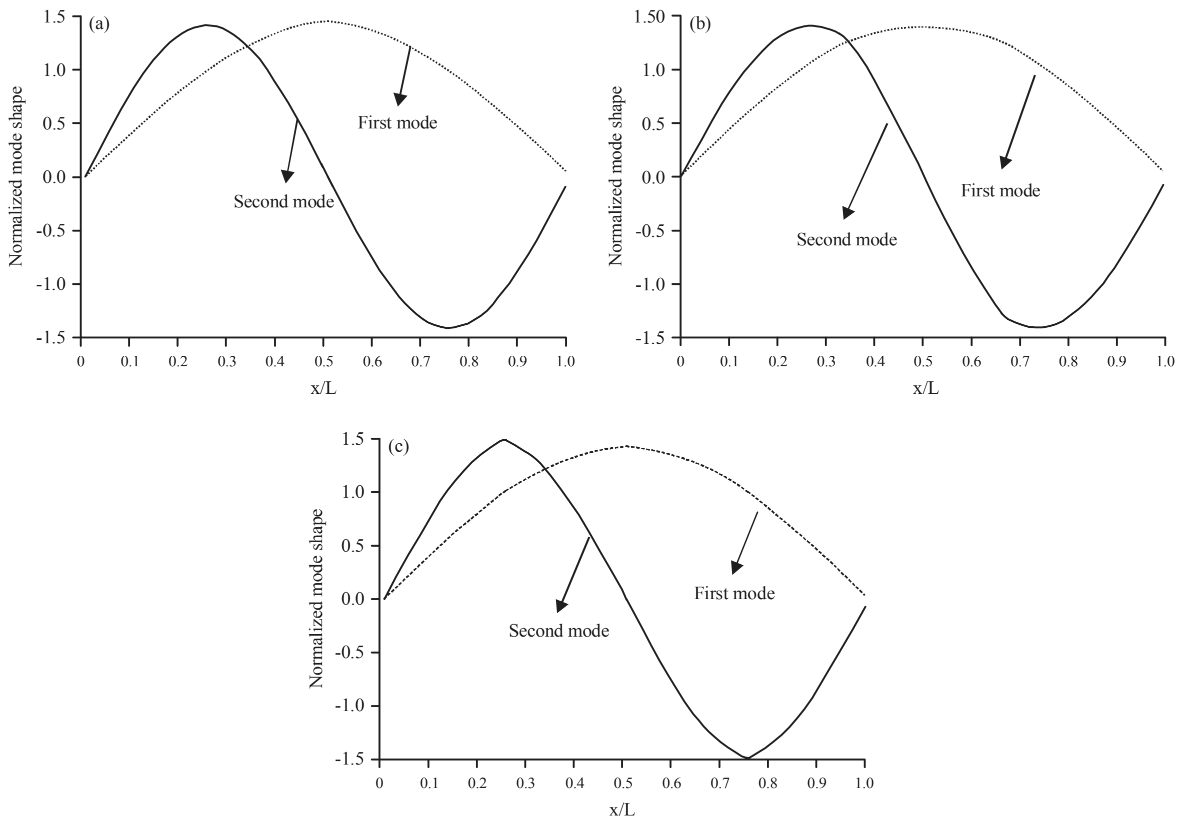

Fig. 9(a-c): First and second mode shapes of the beam with, (a) One crack in the middle, (b) Two cracks and (c) Three cracks

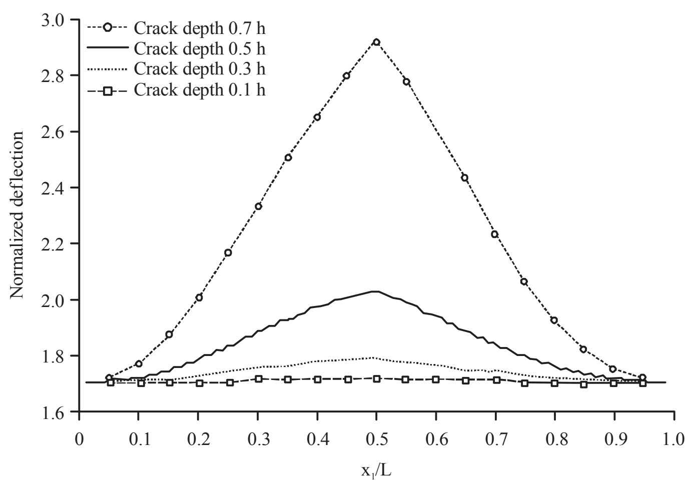

Fig. 10: Maximum normalized deflection versus crack locations for different crack depths

conditions. Study on crack depth revealed that, it can effectively change beam behavior. The deflection of the beam for two crack depths 0.7 and 0.5 h was compared and observed that, the former cause 46% more deflection. Different speeds of moving mass was investigated and concluded critical speed decreases as cracks numbers in the beam increase. The critical speed of moving mass in the beam with three cracks in comparison with healthy one showed 11.8% reduction. Finally it can be concluded that, crack in the beam may cause local flexibility and reduce the stiffness. Under the influence of moving mass, the maximum deflection of cracked beam was increased and retarded. Natural

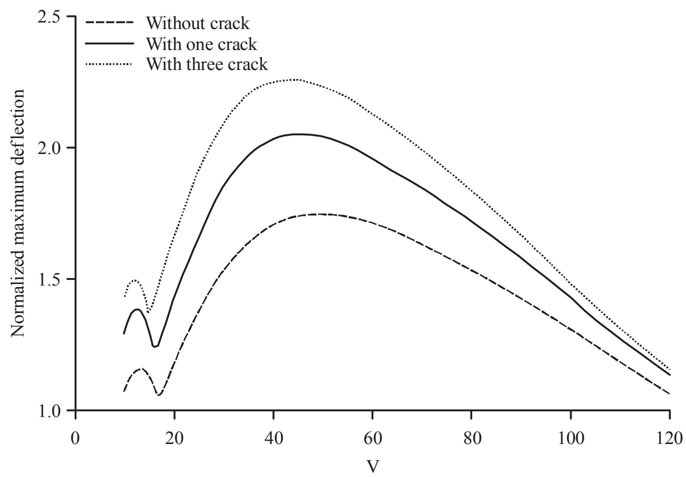

Fig. 11: Maximum normalized deflection versus mass speed for different numbers of cracks in the beam

frequency was reduced by crack and more crack introduced more reduction in the beam's natural frequency. The mid-span of the beam was known as the critical area for crack location and critical speed decreased by introducing crack to the beam structure.

CONCLUSION

Vibration analysis of multi-cracked beam was investigated in this study. Timoshenko beam model was considered and Transfer Matrix Method used to solve the deflection of the beam with multiple cracks. Beam was traversed by moving mass with constant speed. Natural frequencies and mode shapes of cracked beam were calculated and the effect of crack location in natural frequencies was investigated. Following conclusions were received:

- The attendance of crack at mid-span causes significant effect in the beam deflection under moving mass
- Deflection in the beam with three cracks in comparison with single crack revealed 10% increment
- Natural frequency was reduced by crack and more crack introduced more reduction in the beam's natural frequency
- Comparison between natural frequencies of cracked beam with different boundary conditions revealed, the frequency of simply-supported beam reduces 2% more than other kind of boundary conditions
- The critical speed of moving mass in the beam with three cracks in comparison with healthy one showed 11.8% reduction

Finally, it can be concluded that, numbers of crack, speed of traveler, location of crack and type of boundary conditions play important roles in the natural frequency and deflection of cracked beam.

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Appendix 1: Determination of elements of matrix Γ

Elements of transfer matrix Γ were determined as follow:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} \end{bmatrix}$$

Where:

$$\Gamma_{11} = \cosh(\lambda_1 l) - \frac{\left(q_2\lambda_2(\lambda_1 - q_1)\right)}{\kappa_1} c_v \sinh(\lambda_1 l)$$

$$\Gamma_{12} = \sinh(\lambda_1 l) - \frac{\left(q_2\lambda_2(\lambda_2 - q_1)\right)}{\kappa_1} c_v \cosh(\lambda_i l)$$

$$\Gamma_{13} = \frac{\left(q_2\lambda_2(q_2 - \lambda_2)\right)}{\kappa_1} c_v \sin(\lambda_2 l)$$

$$\Gamma_{14} = -\frac{\left(q_2\lambda_2(q_2 + \lambda_2)\right)}{\kappa_1} c_v \cos(\lambda_2 l)$$

Appendix 1: Continue

$$\Gamma_{21} = \sinh(\lambda_1 l) + \frac{(q_1\lambda_1(\lambda_2 - q_2))}{\kappa_2} c_\theta \cosh(\lambda_1 l)$$

$$\Gamma_{22} = \cosh(\lambda_1 l) + \frac{(q_1\lambda_1(\lambda_2 + q_2))}{\kappa_2} c_\theta \sinh(\lambda_1 l)$$

$$\Gamma_{23} = \frac{(q_2\lambda_2(q_2 + \lambda_2)))}{\kappa_2} c_\theta \cos(\lambda_2 l)$$

$$\Gamma_{24} = \frac{(q_2\lambda_2(q_2 + \lambda_2))}{\kappa_2} c_\theta \sin(\lambda_2 l)$$

$$\Gamma_{31} = \frac{(q_1\lambda_1(q_1 - \lambda_1))}{\kappa_1} c_v \sin(\lambda_1 l)$$

$$\Gamma_{32} = \frac{(q_1\lambda_1(q_1 - \lambda_1))}{\kappa_1} c_v \cosh(\lambda_1 l)$$

$$\Gamma_{33} = -\cos(\lambda_2 l) + \frac{(q_1\lambda_1(q_2 + \lambda_2))}{\kappa_1} c_v \sin(\lambda_2 l)$$

$$\Gamma_{34} = -\sin(\lambda_2 l) - \frac{(q_1\lambda_1(q_2 + \lambda_2))}{\kappa_1} c_v \cos(\lambda_2 l)$$

$$\Gamma_{41} = \frac{(q_1\lambda_1(q_1 - \lambda_1))}{\kappa_2} c_\theta \cosh(\lambda_1 l)$$

$$\Gamma_{42} = \frac{(q_1\lambda_1(q_1 - \lambda_1))}{\kappa_2} c_\theta \sinh(\lambda_1 l)$$

$$\Gamma_{43} = -\sin(\lambda_2 l) + \frac{(q_2\lambda_2(q_1 - \lambda_1))}{\kappa_2} c_\theta \cos(\lambda_2 l)$$

$$\Gamma_{44} = \cos(\lambda_2 l) + \frac{(q_2\lambda_2(q_1 - \lambda_1))}{\kappa_2} c_\theta \sin(\lambda_2 l)$$

l: Length of the every segment

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