Asian Journal of Plant Sciences

ISSN 1682-3974

Asian Journal of Plant Sciences 6 (5): 854-858, 2007
ISSN 1682-3974

Redistribution of Water Injected into the Soil

[1]M. Mahmoodian-Shooshtari, [2]Zahra Izadpanah and [3]Seyed A.A. Jafari Moosavi
[1]Department of Civil Engineering, College of Engineering, Shahid Chamran University, Ahwaz, Iran
[2]College of Water Science Engineering, Shahid Chamran University, Ahwaz, Iran
[3]Enviromental Health Faculty, Jndi-Shapoor Medical Science University, Ahwaz, Iran

Abstract: In 1984 an analytically response function for one dimensional vertical redistribution of water injected into the soil was drived. In this study a theoretical $h_0(t)$ solution (the time variation of capillary pressure) for radial redistribution of water injected into the soil is developed and compared to the corresponding one for the vertical case. The compression of theoretical solution and excremental results shows that Except for the slightly more rapid redistribution, no advantages for radial redistribution over the vertical one is observed.

Key words: Theoretical, vertical, radial redistribution, capillary pressure head

INTRODUCTION

Sap flows between soil layers of different moisture content (termed hydraulic redistribution) have the potential to influence water budgets and species interaction (Peek *et al*., 2005). Using a great deal of excremental data, Brooks and Corey, (1966) showed that the relationship between water content θ and capillary pressure (h) and between hydraulic conductivity (k) and capillary pressure (h) can be expressed by

$$\begin{aligned} k(h) &= k_m\left(\frac{h}{h_b}\right)^{-(2+3\lambda)} && \text{For } h \geq h_b \\ k(h) &= k_m && \text{For } h \leq h_b \end{aligned} \qquad (1)$$

And

$$\begin{aligned} \theta(h) &= (\theta_m - \theta_r)\left(\frac{h}{h_b}\right)^{-\lambda} + \theta_r && ,h \geq h_b \\ \theta(h) &= \theta_m && ,h \leq h_b \end{aligned} \qquad (2)$$

In these equations, θ_m is the maximum volumetric water content, k_m is the corresponding hydraulic conductivity, λ is a dimentionless exponent, h_b is the displacement suction (bubbling pressure head) and θ_r is the residual water content. Mahmoodian-Shooshtari *et al*. (1984) investigated the parameters in the Brooks and Corey (1966) k(h) and h(θ) relationships analytically and experimentally. The method is based upon fitting an analytically derived response function to the corresponding measured one for one dimensional vertical redistribution of water injected into the soil. The theoretical response function used is:

$$h_0(t) = h_b(at+1)^{\frac{1}{4\lambda+1}} \qquad (3)$$

where
h_0 (t) = Capillary pressure head at r = 0, t = time

$$a = \frac{\pi(4\lambda+1)(\theta_m - \theta_i)h_b k_m}{2\lambda(1+3\lambda)V^2} \qquad (4)$$

θ_1 = Initial water content
h_b, λ, θ_m and k_m are defined as before and
V = volume of injected water per unit area.

Equation 3, which was derived from Richards' Equation by neglecting gravity forces, assumes that k and θ are single valued function of h. It was also necessary to use an assumed distribution of water content to derive Eq. 3. The water content distribution was approximated by a series of step functions (Spiegel, 1999).

This study presents a theoretical $h_0(t)$ solution for radial redistribution of water injected into the soil and then compared with the Eq. 3. The reasons for investigating radial redistribution was to ascertain if there are any inherent advantages to this geometry.

MATERIALS AND METHODS

Richard's equation: Richards' equation for radial flow without gravity effect is:

Corresponding Author: M. Mahmoodian-Shooshtari, Department of Civil Engineering, College of Engineering, Shahid Chamran University, Ahwaz, Iran

$$\frac{1}{r}\frac{\partial}{\partial r}\{rk(h)\frac{\partial h}{\partial r}\} = \frac{-\partial\theta}{\partial t} \quad (5)$$

Where r is the radial coordinate in a horizontal plane. Two boundry conditions and one initial condition are required to obtain solution. There are no exact solution available for this equation, but approximate analytical and numerical solutions have been developed by various investigators. Regardless of the method selected for solution, realistic data or mathematical expressions are needed for k(h) and θ (h).

Radial redistribution: The flow phenomenon under consideration is the radial redistribution of a slug of water that is rapidly injected on the cylindrical surface ($r = r_0$) into a semi infinite soil with uniform initial water content. Figure 1 shows a homogeneous soil thinckness b with uniform initial water content θ_1, which is situated between two horizontal impervious boundaries having a central bore of radius r_0. A known volume of water, say V, is injected instantaneously into the soil through the openings of the bore walls. Immediately after injection of water, the moisture begins to move outward from the moist shell under the influence of capillary-pressure gradients. The boundary conditions for this model are:

$$\frac{\partial h}{\partial r} = 0 \text{ at } r = r_0 \text{ and } r = \infty, \quad t \geq 0 \quad (6)$$

Which means no water flux at $\frac{\partial h}{\partial r} = 0$ and $r = \infty$

The initial conditions are described by

$$\theta = \theta_m \quad \text{for} \quad r_0 \leq r \leq r_{f0}, \quad t = 0^+ \quad (7)$$

And

$$\theta = \theta_i \quad \text{for } r \rangle r_0, t = 0 \quad (8)$$

Also, continuity requires that

$$\int_{r_0}^{\infty} 2\pi b(\theta - \theta_i) r dr = V \quad (9)$$

The coordinate of the wetting front at t = 0 (r_{fo}) is

$$r_{fo} = \left[\frac{V}{\pi b(\theta_m - \theta_i)} + r_0^2\right]^{1/2} \quad (10)$$

Theoretical response function for radial redistribution: Integrating Eq. 5 over r, from r_0 to r, subject to boundary condition (6), gives

$$rk(h)\frac{\partial h}{\partial r} = -\int_{r_0}^{r} r\frac{\partial\theta}{\partial t}dr = \frac{-d}{dt}\int_{r_0}^{r}\theta r\, dr \quad (11)$$

It is assumed that the distribution of water content at any stage during the flow process is given by the step function (Spiegel, 1999) (Fig. 2) distribution which is

$$\begin{aligned}&\theta(r,t) = \theta_0(t) = \theta_0 \quad \text{for} \quad r_0 \leq r \leq r_f\\ &\theta(r,t) = \theta_i = \text{constant} \quad \text{for } r \rangle r_f\end{aligned} \quad (12)$$

Where r_f is the coordinate of the wetting front and θ_0 is the water content at $r = r_0$. Substituting for θ from Eq. 12 into the right hand side of Eq. 11 carrying out the indicated integration gives

$$k(h)\frac{\partial h}{\partial r} = -\frac{1}{2}\frac{d\theta_0}{dt}(r - \frac{r_0^2}{r}) \quad (13)$$

Again, integrating both sides of the results with respect to r over the interval and $r_0 \leq r \leq r_f$ yields

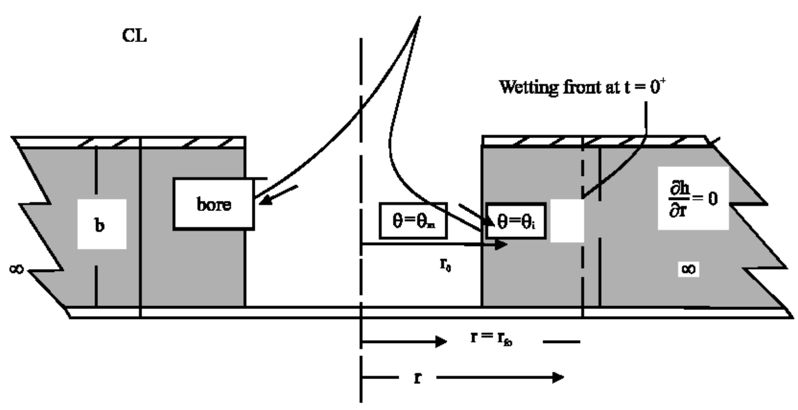

Fig. 1: Schematic diagram of physical model for radial redistribution

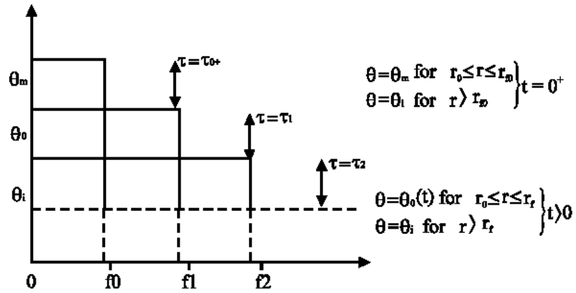

Fig. 2: Step function for moisture distribution

$$\int_{h_o}^{h_i} k(h)dh = -\frac{1}{2}\frac{d\theta_o}{dt}\left[\frac{1}{2}(r_f^2 - r_0^2) - r_0^2 \ln\frac{r_f}{r_0}\right] \tag{14}$$

On the other hand, for Eq. 12 and 9 requires that

$$r_f = \left[\frac{V}{(\pi b(\theta_0 - \theta_i)} + r_0^2\right]^{\frac{1}{2}} \tag{15}$$

Introducing r_f from Eq. 15 into Eq. 14 yields

$$\int_{h_o}^{h_i} k(h)dh = -\frac{1}{2}\frac{d\theta_0}{dt} \left[\frac{V}{2\pi b(\theta_0 - \theta_i)} - \frac{r_0^2}{2}\ln\left(\frac{V}{r_0^2 \pi b(\theta_0 - \theta_i)} + 1\right)\right] \tag{16}$$

In terms of capillary pressure head h_0 at r = 0, Eq. 2 becomes

$$\theta_0 = (\theta_m - \theta_r)(\frac{h_0}{h_b})^{-\lambda} + \theta_r \tag{17}$$

And, thus

$$\frac{d\theta_0}{dh_0} = \frac{-\lambda(\theta_m - \theta_r)}{h_b}\left(\frac{h_0}{h_b}\right)^{-(1+\lambda)} \tag{18}$$

Also,

$$\frac{d\theta_0}{dt} = \frac{d\theta_0}{dh_0}\frac{dh_0}{dt} = \frac{-\lambda(\theta_m - \theta_r)}{h_b}\left(\frac{h_0}{h_b}\right)^{-(1+\lambda)}\frac{dh_0}{dt} \tag{19}$$

Substituting Eq. 1, 19 and 17 into Eq. 16 for k(h), $\frac{d\theta_0}{dt}$ and θ_0, respectively and performing the required operations with $\theta_I = \theta_r$ yields

$$\frac{k_m h_b}{1+3\lambda}\left(\frac{h_0}{h_b}\right)^{-(1+3\lambda)} = \frac{\lambda(\theta_m - \theta_r)}{4h_b}\left(\frac{h_0}{h_b}\right)^{-(1+\lambda)} \left[\frac{V\left(\frac{h_0}{h_b}\right)^{\lambda}}{\pi b(\theta_m - \theta_r)} - r_0^2 \,\mathrm{Ln}\left(\frac{V\left(\frac{h_0}{h_b}\right)^{\lambda}}{r_0^2 \pi b(\theta_m - \theta_r)} + 1\right)\right]\frac{dh_0}{dt} \tag{20}$$

Letting $\hat{h}_0 = \frac{h_0}{h_b}$, separating variables, integrating over time from 0 to t and over $\hat{h}$ from 1.0 to $\hat{h}$, Eq. 20 becomes.

$$\frac{4K_m t}{\lambda(1+3\lambda)h_b(\theta_m - \theta_r)} = \frac{V\left(\hat{h}_o^{(1+3\lambda)} - 1\right)}{\pi b h_b^2(\theta_m - \theta_r)(1+3\lambda)} - \frac{r_o^2}{h_b^2}\int_{1.0}^{\hat{h}_o} x^{2\lambda} \ln\left(\frac{Vx^{\lambda}}{r_o^2 \pi b(\theta_m - \theta_r)} + 1\right)dx \tag{21}$$

Defining

$$\hat{t} = \frac{4k_m h_b}{r_0^2(\theta_m - \theta_r)}, \tag{22}$$

and

$$\hat{V} = \frac{V}{r_0^2\, \pi b(\theta_m - \theta_r)}, \tag{23}$$

Eq. 21 in dimensionless form is

$$\hat{t} = \lambda\,\hat{V}(\hat{h}_0^{(1+3\lambda)} - 1) - \lambda(1+3\lambda)\int_{1.0}^{\hat{h}_0} x^{2\lambda}\ \mathrm{Ln}(\hat{V}x^{\lambda} + 1)dx \tag{24}$$

Effect of λ and $\hat{V}$ on theoretical curves for radial redistribution: To demonstrate the influence of each of the parameters λ and $\hat{V}$ on the theoretical response, $h_0(t)$ was calculated from Eq. 24 using three different values of each parameter with the other parameter held constant. The results of these computations are shown is Fig. 3 and 4 where $\hat{h}_0$ is plotted as a function of $\hat{t}$.

The information shown in Fig. 3 indicates that the shape (slope) of the $\hat{h}_0$ (t) curves is dependent on the values of λ. Figure 3 also shows that the effect of λ becomes less pronounced as λ increases. The difference in the slope for λ = 1.5 and λ = 2.25 is greater than the difference between λ = 2.25 and λ = 3.0. Support is given

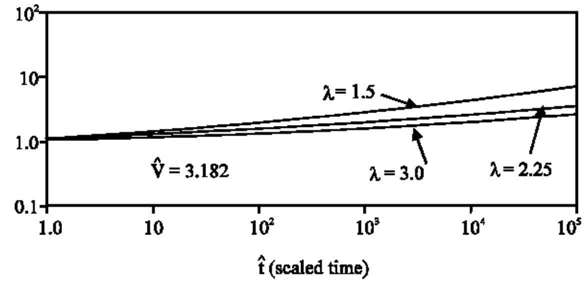

Fig. 3: Compartion of radial solutions (theoretical) for different values of λ with other parameters held constant

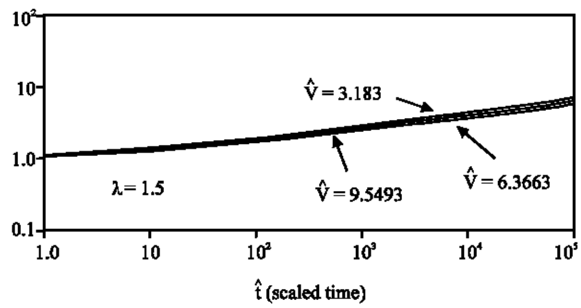

Fig. 4: Compartion of radial solutions (theoretical) for different values of $\hat{v}$ with the other parameters held constant

for this observation by the fact that the permeability reduced tremendously with small increases in capillary-pressure head for a porous medium of high λ value. Indeed, the saturated voids of a medium with high λ value empty suddenly when the capillary pressure head reaches a value somewhat greater than h_b, after which the moisture redistribution is greatly slowed due to the rapid drop in permeability associated with the small increase of capillary-pressure head above h_b. Figure 4 indicates that the shape and vertical location of the curves are also influenced slightly by $\hat{V}$. Because $\hat{V}$ is a function of V, r_0, b, θ_m and θ_r the change indicated in the curves is due to variations in any one, or a combination of these parameters.

Comparisons of solutions to Eq. 3 and 21: As mentioned before, the purpose of developing the radial response function $h_0(t)$ was to investigate the possible advantages inherent in the radial redistribution over the vertical one. Therefore a comparison of the vertical and radial response function $h_0(t)$ is given here. Using Eq. 3 and 21 and the parameter values in Table 1, two solutions to the $h_0(t)$ function were obtained. These solutions are shown in Fig. 5 the values of the integral in Eq. 21 were obtained numerically using the Simpson's rule of integration.

Table 1: Values of parameters used in Eq. 3 and 21 for calculation of time (Mahmoodian Shooshtari, 1982)

Parameter	Values
K_m (cm sec^{-1})	2.5×10^{-3}
h_b (cm)	65.0
θ_m-θ_r (cm^3 cm^{-3})*	0.20
V (cm^3)	4.0
A (cm^2)	11.40
b (cm)	2.0
r0 (cm)	1.0
λ	1.5

* Assuming $\theta_i = \theta_r$ in Eq. 4

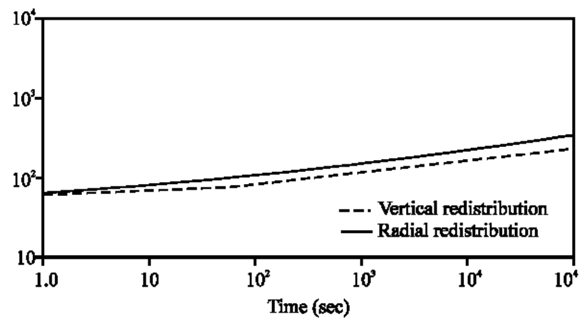

Fig. 5: Compartion of vertical and radial solutions to the moisture redistribution process

Figure 5 shows that, for the same soil parameters and volume of injected water, radial redistribution occur at a faster rate than one dimensional redistribution. This is to be expected, since for the radial case the moisture redistribution is multi- directional rather than in one direction for the vertical case.

The slight advantage that is gained by more rapid redistribution for the radial case is more than offset by other disadvantages, however. For example, Eq. 2 2 must be evaluated by numerical integration while Eq. 3 is an analytical closed from solution. The advantage of an analytical solution over a numerical solution for the purposes of matching calculated and measured response function is obvious. However, the authors suspect that repeated numerical solutions to Eq. 21, coupled with a search procedure, would be required to determine the values of parameters given the minimum different between measured and calculated $h_0(t)$ relationships. As it is apparent the form of Eq. 3 greatly facilitates computation of the parameter values. Furthermore, the flow process for the radial case is not as simple to initiate and monitor experimentally as is the vertical case.

CONCLUSIONS

The function $h_0(t)$ is the most easily measured response variable at the soil surface. Except for a slightly more rapid redistribution, no advantages for radial redistribution over the vertical one were observed.

REFERENCES

Brooks, R.H. and A.T. Corey, 1966. Properties of porous media affecting fluid flow. J. Irrig. Drainage Div., Proc. ASCE 92 (IR2): 61-88.

Mahmoodian-Shooshtari, M., 1982. Unsaturated soil hydraulic properties from redistribution of injected water. Ph.D Thesis, Colorado State University, Fort Collins, pp: 144.

Mahmoodian Shooshtari, M., L.A. Davis, D.B. Mcwhorter and A. Klute, 1984. Unsaturated soil hydraulic properties from redistribution of injected water. Trans. ASAE., 27: 784-789 and 796.

Peek, M.S., R.R. Yel, C.Y. Irans and M.M. Caldwell, 2005. Hydrauulic Distribution through the root systems of senesced plants A. Joshural Effler, Ecol., 86: 633-642.

Spiegel, M.R., 1999. Advanced Mathematics for Engineers and Scientists. Mcgraw. Hill Book Company 26th Printing.