Asian Journal of Scientific Research

ISSN 1992-1454

Asian Journal of Scientific Research 1 (3): 193-202, 2008
ISSN 1992-1454

Boundary Condition Effect on Dynamic Behaviour of a Uniform Straight Composite and Isotropic Beam Due to Moving Force

Kambiz Bakhshandeh and Bahador Saranjam
Air Naval Research Center, MUT University, Postal Code 7194915685, Shiraz, Iran

Abstract: In this study, the impact of boundary condition in dynamic behaviour of a straight beam under action of moving force is investigated. Furthermore, the effectiveness of various parameters such as beam geometry and material type on dynamic magnification factor is studied. This study shows that maximum dynamic magnification factor only depends on boundary condition. This value is obtained at a critical to moving load velocity ratio that is independent of material and beam geometry and only depends on boundary condition. The Maximum dynamic magnification factor for composite and isotropic material in each boundary condition is slightly different.

Key word: Moving force, dynamic magnification factor, straight beam

INTRODUCTION

Transport engineering structures such as bridges are subjected to loads that vary in both time and space (moving forces), in the form of vehicular traffic, which cause them to vibrate. A moving vehicle on a bridge causes deflections and stresses that are generally greater than those caused by the same vehicular loads applied statically. This is due to dynamic interaction between the bridge and the vehicle.

Given a particular beam and the magnitude of the traveling force, the following characteristics may be of interest: maximum response of the beam for a given velocity of the force, maximum response over all possible velocities and the velocity at which it is developed, dependence of forced response and the amplitude of free vibration on velocity and the like.

There is not much work done on the study of impact of the material, beam dimensions and boundary conditions on dynamic behaviour of straight beams under the action of moving loads.

Fryba (1999) reported, analytical solutions for simple problems of simply supported and continuous beams with uniform cross-section.

Pesterev *et al*. (2003), Gbadeyan and Oni (1995), Renard and Taazount (2002) and Ichikawa *et al*. (2000) performed dynamic analysis of beams due with uniform cross-section to moving masses by means of various analytical or numerical methods. Zheng *et al*. (1998) studied dynamic response of a non-uniform continuous Euler-Bernoulli beam with Hamilton's principle. Zhu and Law (2001) used generalized orthogonal function approach to determine the responses of a bridge deck under the action of moving loads. Kadivar and Mohebpour (1997, 1998) investigated forced vibration of composite laminated beams under action of moving loads. He only considered a simply supported isotropic and composite beam. Abu-Hilal and Mohsen (2000) studied vibrational behavior of isotropic single span beams with general boundary conditions. Although Abu-Hilal studied boundary condition effects on dynamic behaviour, but he did not investigated these effects on dynamic magnification factor.

In these studies, simply supported is the major feature of boundary conditions that is considered and the effect of other boundary conditions is not investigated.

Corresponding Author: Kambiz Bakhshandeh, Air Naval Research Center, MUT University, Postal Code 7194915685, Shiraz, Iran Tel: +98-711-6254388 Fax: +98-711-6254388

The objective of this study is to demonstrate the effect of boundary conditions in dynamic magnification factor. Dynamic magnification factor is defined as the ratio of dynamic deflection to static deflection of a beam. This investigation is performed in various boundary conditions with different cross-sections and material properties.

FINITE ELEMENTS MODEL (FEM)

Dynamic behavior of a straight beam subjected to a vertical constant force P traversing the beam with a constant velocity C (Fig. 1) is studied in three-dimensional element by use of finite element software Msc/Nastran.

In isotropic materials such as steel, the beam model is discritized with 20 node solid elements, while for composite material the 20 node continuum composite element is applied (Anonymous, 2004). Each node has three degrees of freedom. Various mesh density in each beam is used in each beam to achieve a good mesh density and therefore improving the accuracy of results. Densifying is iteratively repeated until the difference between consecutive results becomes less than 0.5%. The number of nodes in each model is shown in Table 1.

Finite element models are consisted of 245722 nodes for isotropic beam with cross-section C to 345682 nodes for isotropic beam with cross section A. Various cross-sections are shown in Fig. 2. For transient analysis the Newmark (Anonymous, 2004) method is used to carry out integration with respect to time where, all parameters for t_n are known, t_{n+1} can be calculated using the following equation of motion

$$M\ddot{q}_{n+1} + B\dot{q}_{n+1} + Kq_{n+1} = F^{ext}_{n+1}, \tag{1}$$

Where, M is the mass matrix, B the damping matrix, K the stiffness matrix, F^{ext}_{n+1} the external load vector, $\ddot{q}_{n+1}$ the acceleration at t_{n+1}, q_{n+1} the velocity at t_{n+1} and q_{n+1} is the displacement at t_{n+1}.

The estimates of q_{n+1}, $\dot{q}_{n+1}$ and $\ddot{q}_{n+1}$ are given by

$$\begin{aligned} q_{n+1} &= q^*_n + \beta\ddot{q}_{n+1}\Delta t^2, \\ \dot{q}_{n+1} &= \dot{q}^*_n + \gamma\ddot{q}_{n+1}\Delta t, \\ \ddot{q}_{n+1} &= M^{*^{-1}} F^{residual}_{n+1} \end{aligned} \tag{2}$$

Where, Δt is the time step and γ and β are constants. The q^*_n, $\dot{q}^*_n$, Matrix M* and vector $F^{residual}_{n+1}$ are calculated by these equations.

$$\begin{aligned} F^{residual}_{n+1} &= F^{ext}_{n+1} - B\dot{q}^*_n - Kq^*_n, \\ M^* &= M + B\gamma\Delta t + K\beta\Delta t^2, \\ \dot{q}^*_n &= \dot{q}_n + (1-\gamma)\ddot{q}_n\Delta t, \\ q^*_n &= q_n + \dot{q}_n\Delta t + \frac{(1-2\beta)\ddot{q}_n\Delta t^2}{2}, \end{aligned} \tag{3}$$

The analyses stages are illustrated in the flow chart are shown in Fig. 3.

Validation of Finite Element Model

For this purpose, a uniform simply supported steel beam with below specifications analysis with FE and analytical method (Saranjam *et al*., 2006) and the results are compared.

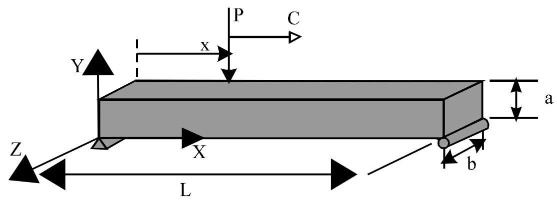

Fig. 1: Simply supported beam subjected to a moving load

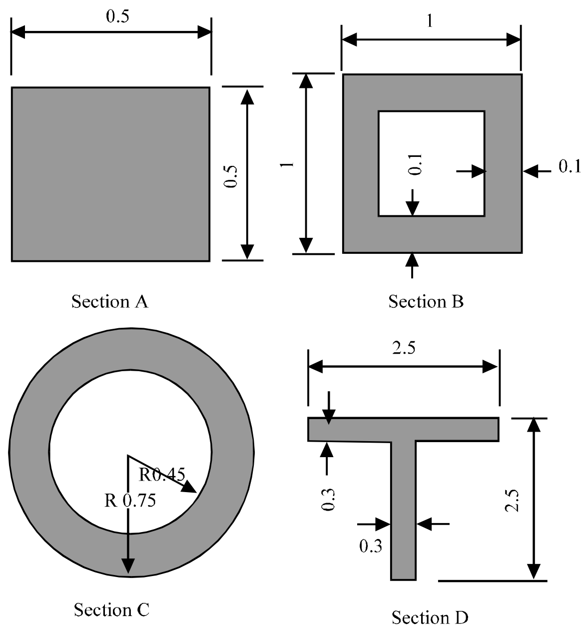

Fig. 2: Cross-sections dimensions (cm)

Table 1: No. of nodes and elements in various beams

Beam types	Nodes	Elements
Isotropic beam-cross-section A	345682	80001
Isotropic beam-cross-section B	292203	36001
Isotropic beam-cross-section C	249059	41355
Isotropic beam-cross-section D	245722	42689
Composite beam	345682	80001

Table 2: Dynamic magnification factor for moving load on simply supported steel beam

		Dynamic magnification factor D_{mf}		
T_f/T	Velocity C (m sec^{-1})	FEM	Analytic	Error e (%)
0.125	1.47	1.030	1.059	2.73
0.25	2.94	1.157	1.572	1.03
0.75	8.82	1.556	1.572	1.02
1.00	11.76	1.691	1.705	0.82
1.25	14.70	1.714	1.731	1.00
1.50	17.65	1.687	1.701	0.82
2.00	23.53	1.533	1.548	0.98

Table 3: Material properties of aluminum

Properties	Magnitudes
E	70 (Gpa)
υ	0.3
ρ	2700 (kg m^{-3})

The beam has length L = 100 cm, width b = 0.5 cm, thickness α = 0.5 cm (Fig. 1), modulus of elasticity $E = 2.1\times10^{11}$Pa, mass density ρ = 7800 kg m^{-3}, Poisson ratio υ= 0.3, moment of inertia I = 0.00520833 cm^4 and fundamental period T_f = 0.085005 s (Meirovitch, 1967).

All degrees of freedom in Z direction are constrained in order to calculate the modes which are in X-Y plane and contribute to dynamic response of the beam in Y direction. The beam is simply supported and can be analyzed employing the following conditions (Fryba, 1999):

- Initial conditions:

$$W(x,0) = \dot{W}(x,0) = 0, \tag{4}$$

- Boundary conditions:

$$\begin{aligned} W(0,t) = W(L,t) = 0, \\ M(0,t) = M(L,t) = 0, \end{aligned} \tag{5}$$

Where:
W (x,t) = Beam deflection in at point x and time t
M (x,t) = The bending moment at point x and time t

To validate the finite elements model, the results of dynamic response of a simply supported isotropic beam subjected to a P = 10.0KN (Fig. 1) moving force is compared with analytical results. The error e is defined as

$$e = \frac{D_{mf\,exact} - D_{mf\,FEM}}{D_{mf\,exact}} \times 100 \tag{6}$$

Where, $D_{mf\,exact}$ and $D_{mf\,FEM}$ are dynamic magnification factors calculated with exact formula and finite elements method, respectively.

The dynamic magnification factor for a simply supported beam is calculated by dividing dynamic deflection by static deflection of beam center.

Table 2 shows calculated dynamic magnification factors, for involved beam. The quantity of T is the time required for the moving load to travel the beam span. Closeness of the results of the agreement between the finite elements method and theoretical solution is acceptable (Table 2).

Parametric Study

In previous part the finite elements model is validated for a uniform simply supported steel beam subjected to a single moving force with a constant velocity, thus for evaluating boundary conditions effect on dynamic magnification factor, the finite elements method can be used.

In this part four boundary conditions, Piuned-fixed, Fixed-pinned, Fixed-fixed and Pinned-pinned is investigated.

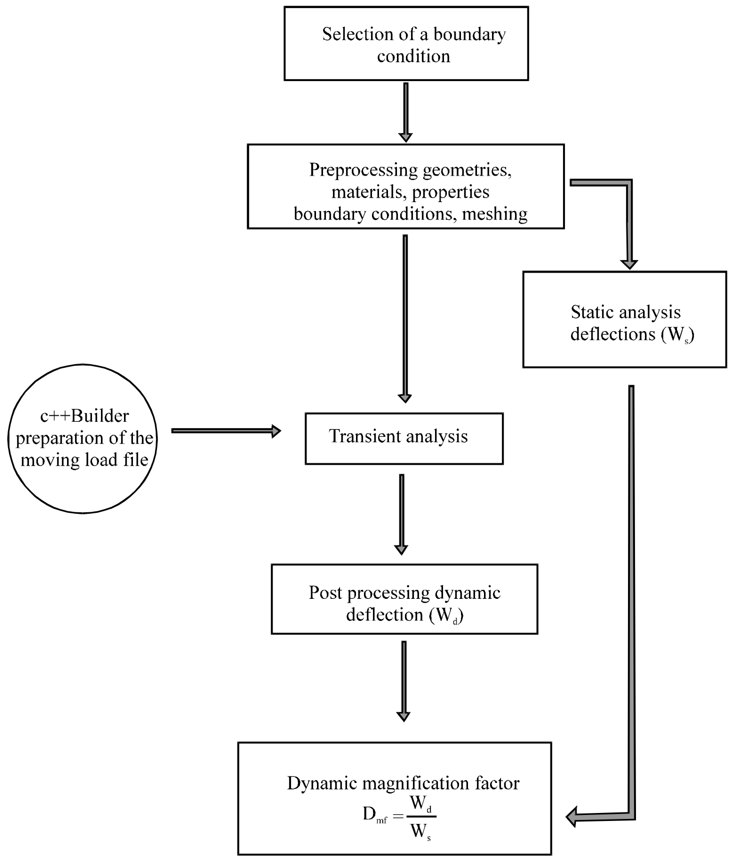

Fig. 3: A Flowchart of Analysis Scheme

Dynamic magnification factor for each boundary condition is calculated in the location of maximum static deflection. For a simply supported beam, this location is in the mid-point of the beam span but for other boundary condition, this location is changed.

For investigation of beam geometry (moment of inertia), four cross-sections are considered. The beam length is similar to simply supported beam that is described earlier (Fig. 2).

In each boundary condition, steel, aluminum and composite material is used. Cross-section for composite beam is the section A in Fig. 3. Mechanical property of aluminum is shown (Table 3)

The composite material is consisted of 20 layers with equal thickness of 1 mm and angle sequence $(0/90)_{10s}$. Property of each lamina shown in (Table 4).

The principle directions 1 and 2 are in X and Y directions, respectively (Fig. 1).

RESULTS AND DISCUSSION

As mentioned in previous section, dynamic magnification factor is calculated by dividing the maximum dynamic deflection by maximum static deflection of beam. In simply supported beam, the maximum dynamic and static deflections are in the mid-point of the beam span, but in other boundary conditions, this location may be changed. In Table 5 the location and magnitude of maximum static deflection is shown for involved boundary conditions.

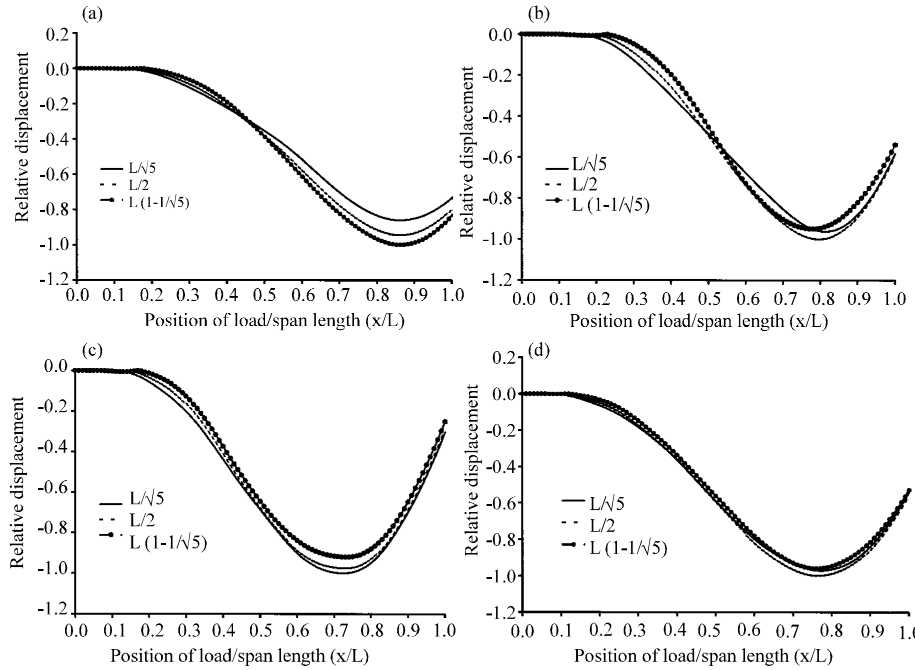

Fig. 4: Comparison of relative displacement in various boundary conditions

Table 4: Material properties of Lamina

Properties	Magnitudes
E_{11}	140 (Gpa)
$E_{22} = E_{33}$	10 (Gpa)
$G_{12} = G_{13}$	5.2 (Gpa)
G_{23}	3.9 (Gpa)
$\upsilon_{12} = \upsilon_{13}$	0.3
υ_{23}	0.5
ρ	1400 (kg m^{-3})
Ply thickness	1 (mm)

Table 5: Location of maximum static deflection for various boundary conditions (Abu-Hilal and Mohsen, 2000)

	Pinned-piuned	Fixed-fixed	Piuned-fixed	Fixed-pinned
X_{max}	L/2	L/2	$L/\sqrt{5}$	$L(1-1/\sqrt{5})$
Maximum displacement	$PL^3/48EI$	$PL^3/192EI$	$PL^3/48\sqrt{5}EI$	$PL^3/48\sqrt{5}EI$

Relative displacement of the points that are specified in this table is compared in Fig. 4. Relative displacement is defined as dynamic deflection divided by maximum static deflection that is specified in Table 5. As shown in this figure, for each boundary condition the location of maximum deflection is the location of static deflection. Thus, dynamic and static deflection for involved boundary condition are calculated on the points that are specified in Table 5.

As it can be seen from Fig. 5-7 the dynamic magnification factor in various beams with different materials, boundary conditions and cross-sections (moment of inertia) that are subjected to a moving constant load can be divided into two regions: undercritical and overcritical. In the undercritical region, the dynamic magnification factor increases with the increasing velocity of the moving load, but in the overcritical region dynamic magnification factor decreases when the moving load velocity increases. The quantity of C_f in these figures is the velocity of which the time for traveling span is equal to the fundamental period of the beam.

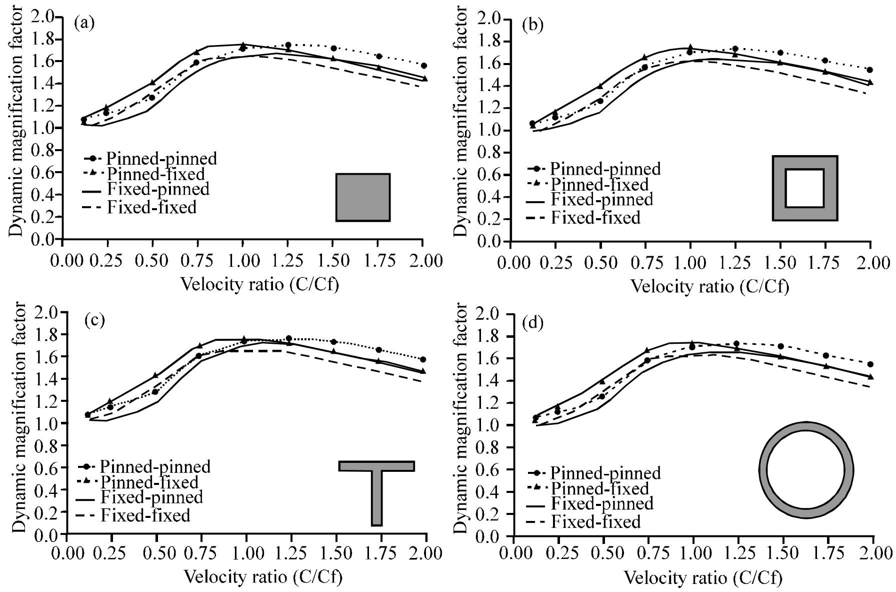

Fig. 5: Dynamic magnification factor in steel beam with different cross-sections and boundary conditions

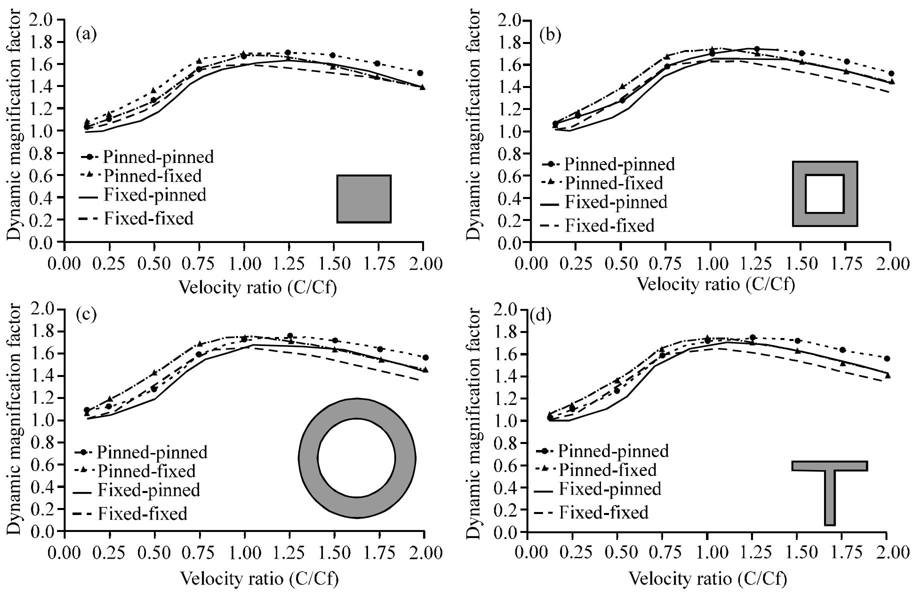

Fig. 6: Dynamic magnification factor in aluminum beam with different cross-sections and boundary conditions

According to these figures in various beams with different boundary conditions, material types and cross-sections, maximum dynamic magnification factors in Pinned-pinned and Piuned-fixed conditions are the same and higher than the others. The Fixed-fixed condition has a lower maximum dynamic magnification factor.

The important results of this investigation are that maximum dynamic magnification factors for Pinned-pinned and Fixed-pinned conditions in various materials and cross-sections are developed at $C_c/C_f = 1.25$ but for Fixed-fixed and Pinned-fixed conditions this ratio will be 1 (C_c is the critical velocity in which the maximum dynamic magnification factor is developed). Thus in uniform straight beams C_c/C_f ratio is independent of material type and beam geometry and ouly depends on boundary conditions.

As shown in Fig. 5-7, in spite of the fact that, total weight of composite and aluminum beams are about 5 and 3 times less than the total weight of steel beam, respectively; maximum dynamic magnification factor is approximately close to the value of that the same bonndary conditions. Thus as in C_c/ C_f ratio, maximum dynamic magnification factor for a uniform straight beam ouly depends on boundary conditions.

In Fig. 8-10, the variation of dynamic deflection of the beam (W_d) divided by the static deflection (W_s) is drawn with respect to the position of the moving load for steel and composite beams. Note that, the curves obtained from aluminum beams are similar to that of steel beams thus for avoidance of chaos, behaviours of steel beam only, shown in Fig. 8.

The velocity of moving load is the critical velocity for each bonndary condition. It can be seen that maximum dynamic deflections of steel and aluminum beams are 1.73 times greater than static deflection and belong to Pinned-fixed and Pinned-pinned conditions (Fig. 8-10). This maximum is developed when the moving load passes from the location of maximum static deflection and has a delay with respect to the position of the moving load. In other bonndary conditions, this delay increases and

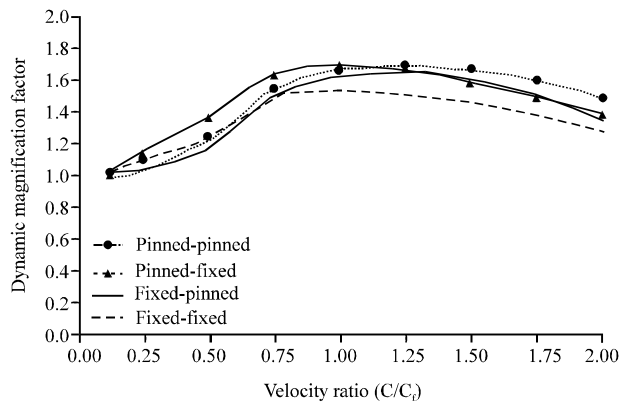

Fig. 7: Dynamic magnification factor in composite beam with different boundary conditions

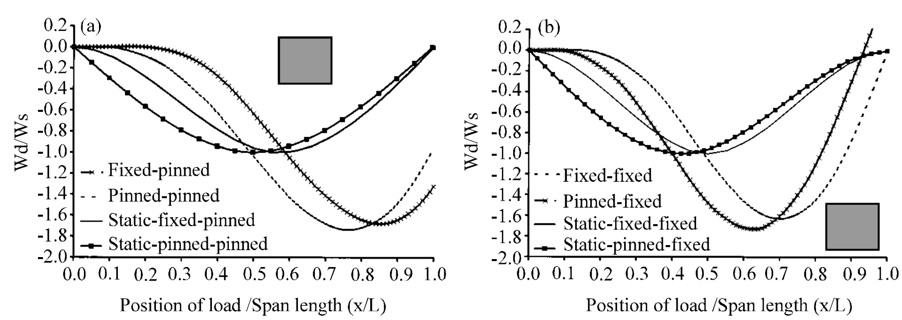

Fig. 8: Comparison between dynamic and static deflection of steel beam with cross-section A in critical velocities

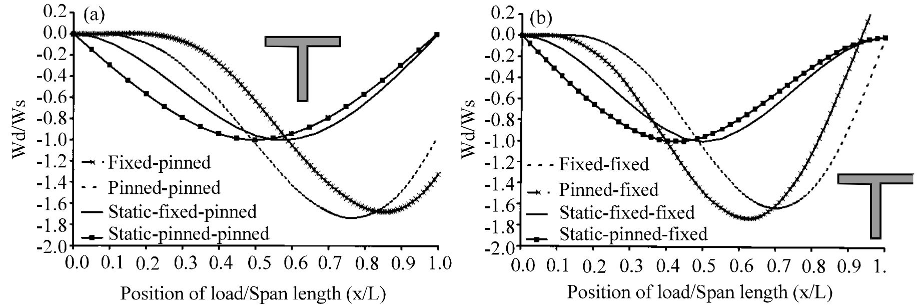

Fig. 9: Comparison of dynamic and static deflection of steel beam with cross-section D in critical velocities

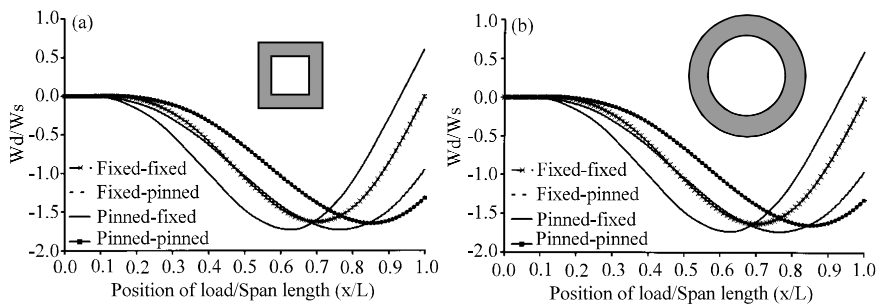

Fig. 10: Comparison of dynamic deflection of steel beam for various boundary conditions in critical velocities

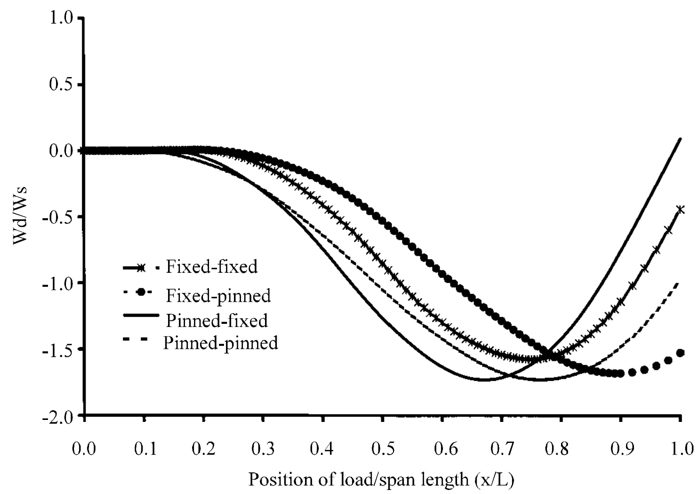

Fig. 11: Comparison of dynamic deflection of composite beam in critical velocities for various boundary conditions

the location of maximum dynamic deflection shift to the right of the beam so that in Fixed-pinned condition maximum dynamic magnification factor approximately reaches the third quarter of the beam span. The higher delay time belongs to Fixed-pinned condition.

According to Fig. 11 these behaviours are also seen in a composite beam. In a composite beam, the Fixed-pinned condition has maximum delay time and maximum dynamic magnification factor is about 1.74 that is the same as in to steel and aluminum beams.

CONCLUSIONS

Formulations and comparative finite elements results have been presented for a conforming finite elements method that may be used to model a beam under action of several moving loads and this comparison shows that finite elements method has a good accuracy for moving load problem.

In a straight uniform beam, maximum points of dynamic and static deflections are in the same position.

Dynamic magnification factor (W_d/W_s) of a beam can be separated into undercritical and overcritical regions.

Piuned-fixed and Piuned-pinned conditions have maximum dynamic magnification factors and maximum delay time is belong to Fixed-fixed condition in various materials and beam geometries.

Important conclusion of this investigation may be described as that the ratio of C_c/C_f is independent of material types and beam dimensions. This ratio is only depending on boundary conditions.

Another important conclusion is that maximum dynamic magnification factors for a uniform straight beam are ouly dependent on boundary conditions. These values are approximately are the same for both isotropic and composite material in a boundary conditions (about 1.73).

REFERENCES

Abu-Hilal, M. and M. Mohsen, 2000. Vibration of beams with general boundary conditions due to a moving harmonic load. J. Sound Vibrat., 232: 703-171.

Anonymous, 2004. Manual: MSC/NASTRAN for Windows: Quick Start Guide., The MacNeal-Schwendler.

Fryba, L., 1999. Vibration of Solids and Structures under Moving Loads. 3rd Edn. Thomas Telford, London.

Gbadeyan, J.A. and S.T. Oni, 1995. Dynamic behaviour of beams and rectangular plates under moving loads. J. Sound Vibrat., 182: 677-695.

Ichikawa, M., Y. Miyakawa and A. Matsuda, 2000. Vibration analysis of the continuous beam subjected to a moving mass. J. Sound Vibrat., 230: 493-506.

Kadivar, M.H. and S.R. Mohebpour, 1997. Forced vibration of unsymmetric laminated composite beams under the action of moving loads. Comput. Sci. Technol., 58: 1675-1684

Kadivar, M.H. and S.R. Mohebpour, 1998. Finite element dynamic analysis of unsymmetric composite laminated beams with shear effect and rotary inertia under the action of moving loads. Finite Elements in Analysis and Design, 29: 259-273.

Meirovitch, L., 1967. Analytical Methods in Vibrations. Macmillan, New York.

Pesterev, A.V., B. Yang, L.A. Bergman and C.A. Tan, 2003. Revisiting the moving force problem. J. Sound Vibrat., 261: 75-91.

Renard, J. and M. Taazount, 2002. Transient responses of beams and plates subject to traveling load, miscellaneous results. Eur. J. Mechanics A/Solid, 21: 301-322.

Saranjam, B., K. Bakhshandeh and M.H. Kadivar, 2006. Dynamic behaviour of a beam with non-uniform linear varying cross-section under moving load. Strojnicky Casopis J. Mechanical Eng., 57: 45-58.

Zheng, D.Y., Y.K. Chenung, T.K. Au and Y.S. Cheng, 1998. Vibration of multi-span non-uniform beams under moving loads by using modified beam vibration functions. J. Sound Vibrat., 212: 455-467.

Zhu, X.Q. and S.S. Law, 2001. Orthogoual function in moving loads identification on a multi-span bridge. J. Sound Vibrat., 245: 329-345.