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## **Discrete Kirchhoff Elements for the Reinforced Concrete Beams Modeling Comparison Between the Elasto-Plastic and Damage Behavior Models**

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**Abstract:** The study treats the plane modeling of the reinforced concrete beams, by using the discrete Kirchhoff triangular finite elements available in Castem 2000. The simulation takes into account the nonlinear behavior of the concrete material such as elasto-plastic of Drucker-Prager and Mazars damage models. The obtained results of the beam in 3 points flexural case are in good agreement with the references values. The comparisons illustrate the specifications, the advantage and also the richness of each validated model.

**Key words:** Behavior, nonlinearity, damage, finite element, steel-concrete adherence

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### **INTRODUCTION**

The reinforced concrete structures are widely used in many types of engineering practice. In the majority of cases these structures are analyzed by using simplified rules based on the experimental studies. The use of the digital computers and the development of the finite element method have proved their capacity in the analysis of such complex structures.

The major issue in finite elements modeling of the reinforced concrete structures is the characterization of the material properties of the components that form this composite; for concrete: A lot of models are established: Drucker-Prager, Mazars, Laborde and others ones, all these models aim on the one hand to a reliable representation of the concrete behavior, on other hand to be exploited in a large context and able to be integrated in analysis codes.

Other problems of the reinforced concrete structures modeling are related to the rheology of the steel-concrete interface and to the geometrical description of the reinforcements in the concrete volume.

Several researchers (Khalil and Charif, 2004; Krätzig and Pölling, 2004) treated the problems of modeling of the reinforced concrete structures by the finite element method.

### **BEHAVIOR MODELS OF THE REINFORCED CONCRETE MATERIAL COMPONENTS**

Modeling the composite concrete-reinforcement means the modeling of the concrete and the reinforcement's behavior.

#### **Concrete Model**

Two different models of the concrete behavior are applied; the elasto-plastic and damage model.

#### **Elasto-Plastic Model**

This approach considers the concrete as hardening elasto-plastic material, it's applicable in monotonous or cyclic loading cases. The Drucker-Prager criterion was used by numerous investigators (Cordebois and Sidoroff, 1982).

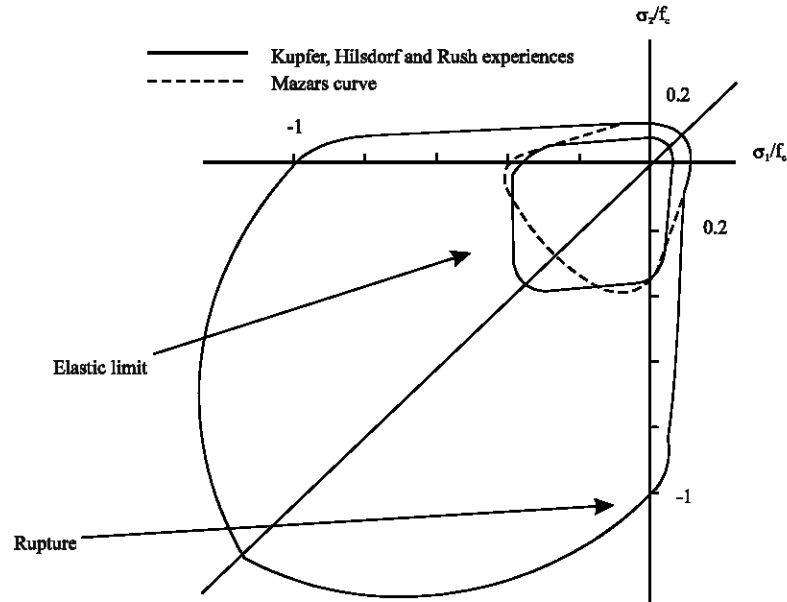


Fig. 1: Jacky Mazars criterion

### Damage Model

This type of model is suitable for the matter deterioration modeling which precedes the rupture. Many models of damage associated with other nonlinear phenomena are elaborated (Gajer and Peter, 1991). Among of these ones; the Mazars model with a scalar variable is the most used model.

In this model the material is considered elastic, damageable and isotropic (Khalflah and Charif, 2004), the total strains are also supposed elastic.

Figure 1 represents the damage low in the  $(\sigma_1, \sigma_2)$  plan which is described by the variable  $D$  such as:

$$\begin{aligned} D &= 0 \text{ for uncracked material} \\ D &= 1 \text{ for fractured material} \end{aligned}$$

The surfaces damage modes induce the anisotropy behavior, which result in a dissymmetry between the tensile and the compression behaviors. The taking into account of this phenomenon in the Mazars model intervenes by the scalar variable on two levels.

In the damage threshold, Mazars use the equivalent deformation concept which translates the local extension state :

$$\bar{\epsilon} = \sqrt{\sum_i \langle \epsilon_i \rangle_+^2} \quad (1)$$

With  $\epsilon_i$  is the principal strain in the  $i$  direction

$$\langle \epsilon_i \rangle_+ = \epsilon_i \quad \text{if} \quad \epsilon_i > 0 \quad \text{and} \quad \langle \epsilon_i \rangle_+ = 0 \quad \text{if} \quad \epsilon_i < 0$$

Then the damage threshold is expressed by:

$$f(\epsilon, H) = \bar{\epsilon} - K(D) = 0 \quad (2)$$

where,  $K(D)$  is relates to the surface threshold size in strain space. For a given damage; if  $D = 0$  then,  $K(D) = K_0$  is the initial threshold.

In the unidirectional tensile according to the  $i$  loading direction.  $\bar{\epsilon} = \epsilon_i$  in the unidirectional compression according to the same loading direction  $\bar{\epsilon} = -\nu\epsilon_i\sqrt{2}$ . For the initial threshold damage  $K_0 = 10^{-4}$  become in the tensile case  $\epsilon_i = K_0$  or the compression case  $\epsilon_i = -\frac{K_0}{\nu\sqrt{2}} = -3.5K_0$ .

The two damages coupling; beyond the threshold, the damage evolution are stronger in the tensile than the compression case. In this last case the dilations are due to the Poisson effect. The model proposes distinct forms for the evolution laws:

$$\text{In tensile} \quad D_t = F_t(A_t, B_t, K_0)$$

$$\text{In compression} \quad D_c = F_c(A_c, B_c, K_0)$$

$$\text{For an unspecified loading} \quad D = \alpha_t D_t + \alpha_c D_c; \quad 0 < \alpha_t, \alpha_c < 1$$

With  $\alpha_t$  and  $\alpha_c$  are respectively depending to the tensile tensors  $\langle \sigma \rangle_+$  and the compression tensors  $\langle \sigma \rangle_-$ , ( $\sigma = \langle \sigma \rangle_+ + \langle \sigma \rangle_-$ ), respectively definite with the positive and the negative eigenvalues these coefficients are;

$$\text{if } \sigma = \langle \sigma \rangle_- \text{ then } \alpha_t = 0 \text{ and } \alpha_c = 1$$

$$\text{if } \sigma = \langle \sigma \rangle_+ \text{ then } \alpha_t = 1 \text{ and } \alpha_c = 0$$

The identification of the evolution laws of this model is carried from the bending and the compression tests preferred then the tensile testes. In addition to  $E_0$  and  $\nu$  five parameters are to determinate (Mazars, 1982).

### Steel Models

Unlike concrete, a single stress-strain relation is sufficient to define the material properties of steel needed in the analysis of the reinforced concrete structures.

The linearly elastic-perfectly plastic (Fig. 2) model had been successfully used in many studies of the reinforced concrete structures (Ngo and Scordelis, 1976; Vebo and Ghali, 1977; Bashur and Darwin, 1978; Khalflah and Charif, 2004; Krätzig and Pölling, 2004; Lemaitre and Chaboche, 1988).

The reinforcement steels are modeled with one-dimensional truss, which are connected to concrete elements (Fig. 3) (Gajer and Peter, 1991; Krätzig and Pölling, 2004).

### Steel-Concrete Adherence

A perfect bond between steel and concrete is considered; then theirs deformations are identical and the stiffness of the composite element is the superposition of the two components stiffness (Khalflah and Charif, 2004; Krätzig and Pölling, 2004).

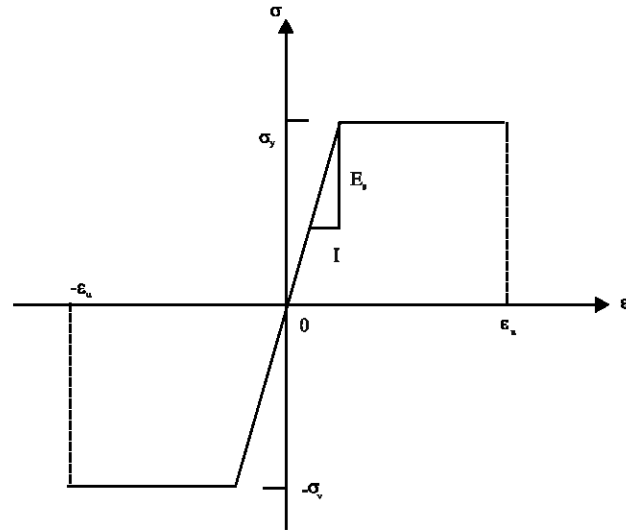


Fig. 2: Steel stress-strain relation

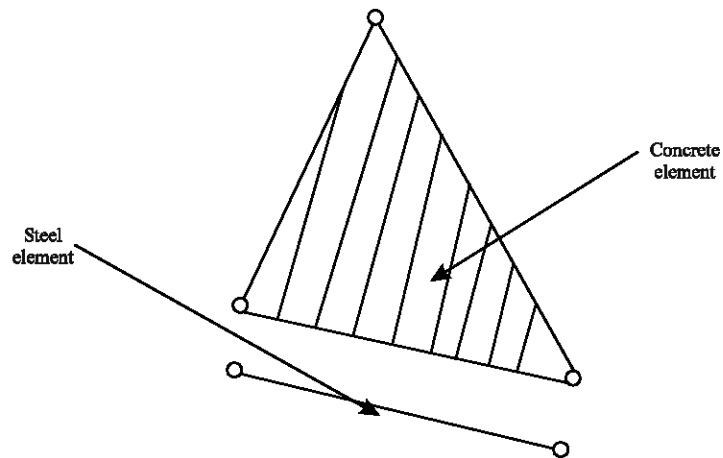


Fig. 3: Discrete steel representation

Table 1: The finite elements used in of the reinforcements and concrete modeling

Designation	Description	df/node
BARR	Two nodes truss element	$U_x U_y U_z$
DKT	Three nodes triangular element with Kirchhoff's discrete hypothesis	$U_x U_y U_z$ $R_x R_y R_z$

### FINITE ELEMENTS USED

The finite elements used in the modeling of the reinforcements and of the concrete are respectively BARR and the DKT elements available in the Castem 2000 code library (Table 1).

### VALIDATION OF THE NUMERICAL MODEL

The simulation relates to a simply supported beam with a concentrate load at its mid-span, the geometry and the materials properties are respectively represented in Fig. 4 and Table 2.

#### Elasto-Plastic Model

In order to study the effect of finite element mesh size on the structural response of the beam, three different mesh configurations were investigated (378, 504 and 840 total elements) (Fig. 5). A very satisfactory agreement with the experimental behavior (Mazars, 1982) is established by 840 elements model (800 concrete elements and 40 elements for steel).

The global curve of the load-deflection evolution (Fig. 6) agrees with the experimental study (Mazars, 1982), however the response exhibit more concure with increasing grid refinement.

The differences between the simulation results and the experimental values are shown in Table 3.

Table 2: Material properties of concrete and steel

Concrete	Young's modulus	34000 MPa
	Poisson's ratio	0.20
	Compressive strength	32.7 MPa
	Tensile strenght	3.27 MPa
Reinforcement	Young's modulus	210000 MPa
	Elastic limit	348 MPa

Table 3: Comparison between numerical and experimental results

Load (kN)	Reference (Mazars, 1982)	Model	Differences
Cracking	11.1	11.40	2.70
Collapse	30.0	29.20	2.66

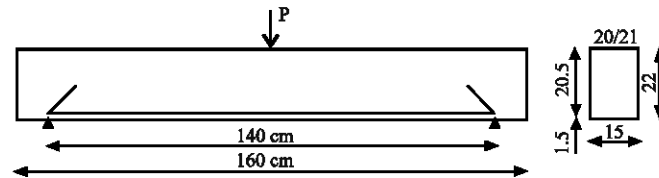


Fig. 4: Geometry and beam reinforcement

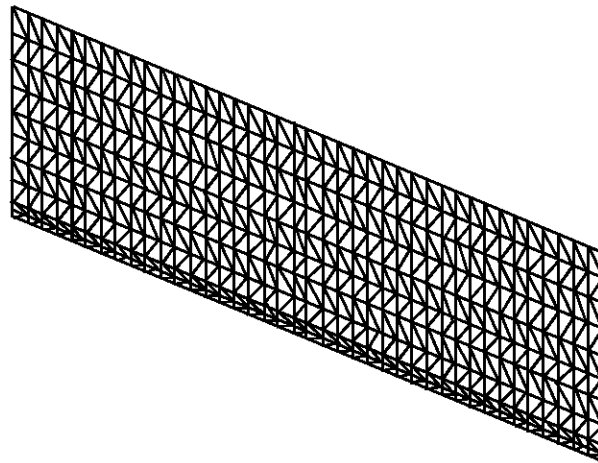


Fig. 5: A used mesh size example

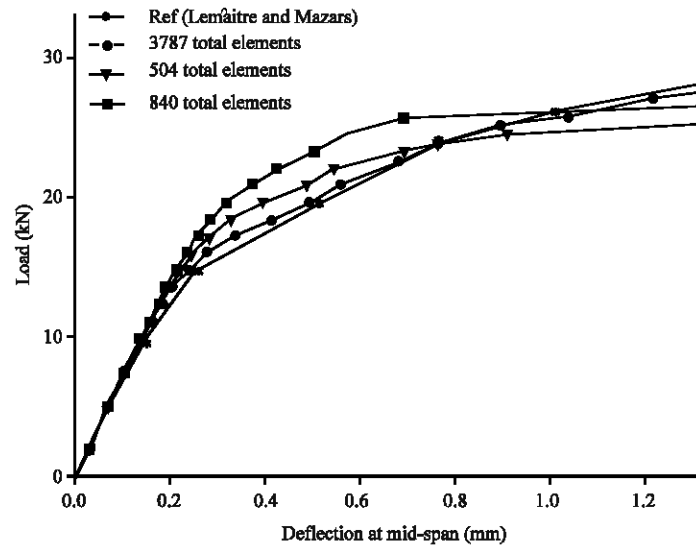


Fig. 6: Load-deflection curve for the elasto-plastic model

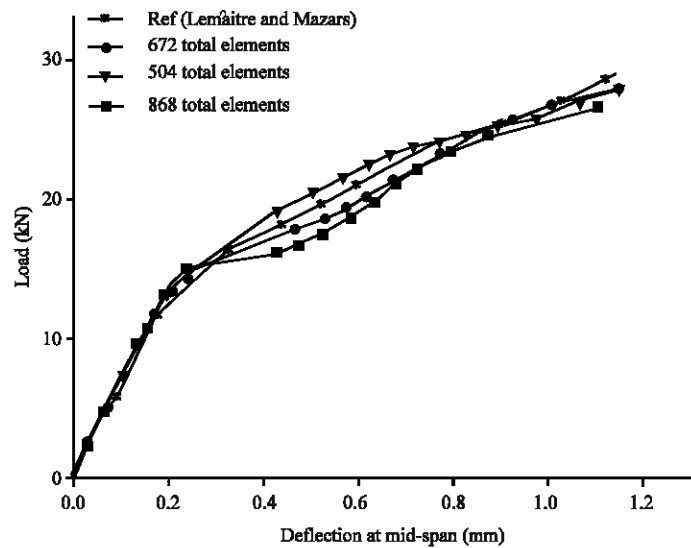


Fig. 7: Load-deflection curve for the damage model

#### Damage Model

The validation of the Mazars damage model presented on Fig. 7 underlines the capacity of the model to simulate the various phases of the total behavior of the reinforced concrete beam.

The discretization with 504 total elements presents a cracking load of 13.4 kN and a rupture under a load equal to 29.9 kN. These values are almost identical to the experimental data (Mazars, 1982).

Figure 8 shows the beam response with the two behavior models; the cracking phase is well represented by the damage model, while that before the rupture is correctly simulated with the elasto-plastic one.

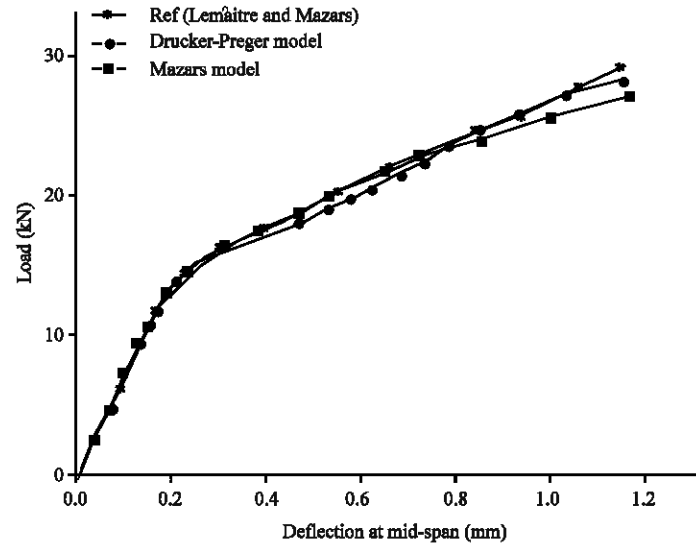


Fig. 8: Load-deflection curve for the elasto-plastic and the damage model

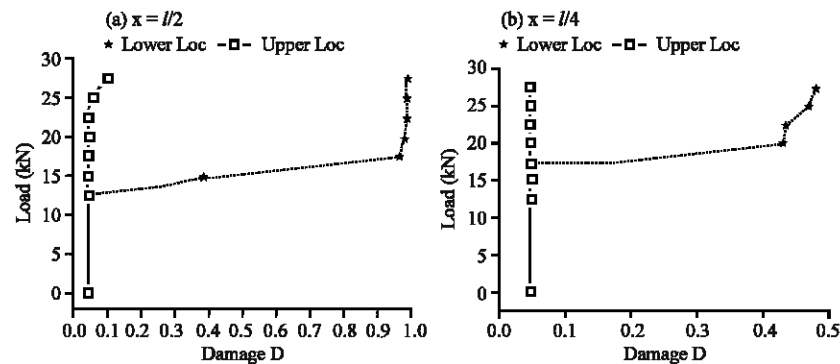


Fig. 9: Damage evolution in the beam

This illustrates the wealth of the information obtained on the beam behavior. In term of degrees of freedom; the Mazars model response is obtained by 1464 and 2440 DDL for the elasto-plastic model.

It should be noted that the validation with the damage law requires more time for analysis than the elasto-plastic model.

The examination of the Fig. 9a, b, respectively shows the damage evolution along the depth of the beam, for  $x = l/2$  (mid-span) and  $x = l/4$  can affirm that the damage is more pronounced at the mid-span and at the tended concrete:

At the mid-span, a load of 12.65 kN constitutes a threshold of the damage evolution in the beam bottom; a clear degradation is noted starting from 17.5 kN. However, in the top a weak variation of damage ( $D < 0.1$ ) is starting from 25 kN.

Here,  $x = l/4$  and in the upper location no damage is noted. The load of 17. kN causes the first degradations in lower fibers; the evolution of the parameter D remains under the 0.5 value.

Figure 10 shows the damage zones in X-Y plane, on the right the load equal to 15 kN, the phenomenon emerges in the central zone with 10 cm of length. This length of damage is extended to 30 cm to touch the top location under a load of 27.5 kN.



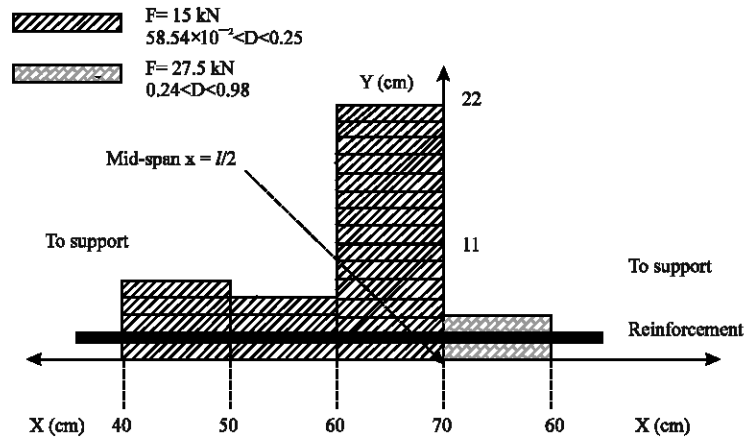


Fig. 10: Damage progression in the beam

## CONCLUSIONS

The study presents a contribution to the reinforced concrete beams modeling using the Castem 2000 finite elements software.

A very good confrontation of the reference results is obtained, that can say that the global response (load-deflection curve) correctly simulates the cracking and the collapse phases with an inaccuracy rate lower than 3%. The built cartography of the damage can predict the scenario of degradation.

The diversity of the obtained results allows on the one hand the choice of the suitable model (elasto-plastic or damage) according to the desired results (cracking load, collapse load and damage cartography), on the other hand it allows an idea about the time reserved to analysis such a structure.

## REFERENCES

- Bashur, F.K. and D. Darwin, 1978. Nonlinear model for reinforced concrete slabs. J. Struct. Div., ASCE, 104: 157-170.
- Cordebois, J.P. and F. Sidoroff, 1982. Anisotropic damage in elasticity and plasticity. J. Mec. Theor. Applied.
- Gajer, G. and F.D. Peter, 1991. Simplified non orthogonal crack model for concrete. J. Struct. Eng., 117: 25-44.
- Khalifallah, S. and A. Charif, 2004. The plan behavior of structures under monotonous loading. Algeria Equipment ENTP, 37: 18-23.
- Krätzig, W.B. and R. Pölling, 2004. An elasto-plastic damage model for reinforced concrete with minimum of material parameters. Comput. Struct., 82: 1201-1215.
- Lemaitre, J. and J.L. Chaboche, 1988. Mechanic of Solid Materials. 2nd Edn., Dunod Edition, Paris, ISBN 2-10-001397-1.
- Mazars, J., 1982. New concepts in the composite concrete behavior modeling, application to the structures analysis. Ann. ITBTP, N461: 02-20.
- Ngo, D. and A.C. Scordelis, 1967. Finite element analysis of reinforced concrete beams. ACI. J., 64: 152-163.
- Vebo, A. and A. Ghali, 1977. Moment-curvature relation of reinforced concrete slabs. J. Struct. Div., ASCE, 103: 515-531.