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# Analytical Solution for Simply Supported and Multilayered Magneto-Thermo-Electro-Elastic Plates

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## ABSTRACT

An analytical solution for simply supported and multilayered magneto-thermo-electro-elastic plates is presented in this study. The fundamental theory was derived based on the generic first-order transversely shearable deformation shell theory involving Codazzi-Gauss geometrical discretion, in which this fundamental equation and its boundary conditions were strenuously derived using Hamilton's principle. Then the developed theory was applied to plate of rectangular plane-form, at which the Navier' solution procedure for the response was derived and its mode shapes were evaluated in the simply supported boundary condition. Moreover, the theory is intended for a wide range of common smart materials. Thus, among the entire primary variable the center deflection was selected for validation and verification purpose and studied for four different laminations schemes. Whereas, the result has shown a close agreement with those of higher order shear deformation theory that obtained from literature.

**Key words:** Laminated composite plate, smart composite, piezoelectric/magnetostrictive, structronics, free vibration, Navier' solution

## INTRODUCTION

Magneto-thermo-electro-elastic (MTEE) concept is a synergistic integration of smart, adaptive or responsive materials that contains the main structure and the distributed functional materials (e.g., piezoelectric, piezomagnetic, electrostrictive, magnetostrictive and alike materials). Which refer to a class of structures that had the capability of simultaneously sensing/actuating; mechanical, electrical, magnetic and even thermal effects, as well as simultaneously generating a control forces to eliminate the undesirable effects or to enhance the desirable one. Whereas, structronics are largely improving the working performance and lifetime of devices that construct from it (Bassiouny, 2006; Badri and Al-Kayiem, 2011a-c). Several accurate solutions of MTEE plate have been presented using 3-D and 2-D theories or the discrete layer approaches. The exact closed-form solutions for multilayered piezoelectric-magnetic and purely elastic plates have been proved for special cases of Pan's analysis. Heyliger and Pan (2004) demonstrated the free vibration analysis of the simply supported and multilayered MEE plates under cylindrical bending. Then, studied two cases of the MEE plates subjected to static fields, one under cylindrical bending and the other of completely traction-free under surface potentials. Following up the previous Stroh formulation. Pan and Han (2005) presented the 3-D solutions of multilayered and FG MEE plates.

Wang *et al.* (2003) proposed a modified state vector approach to obtain 3-D solutions for MEE laminates, based on the mixed formulation of solid mechanics. By an asymptotic approach, Tsai *et al.* (2008) studied 3-D static and dynamic behavior of doubly curved functionally graded MEE shells under the mechanical load, electric displacement and magnetic flux by consideration the edge boundary conditions as full simple supports. In comparison with the recent development of 3D solutions of smart plate, we found that the literature dealing with theoretical work in smart composites plate concerning coupled field phenomena in general and in MTEE in particular, is rather scarce, especially for shear deformation studies. In addition, the distribution of sensors and actuators in the plate structure are not well understood.

In this study, a theory of laminated composite MTEE plates based on the First-order Transversely Shearable (FSDT) model will be developed. New issues elicited by the structural lamination, such as the distributions of center deflection over the thickness of plate are addressed. The results supplied herein are expected to provide a foundation for the investigation of the interactive effects among the thermal, magnetic, electric and elastic fields in thin-walled structures and of the possibility to apply the MTEE adapting.

## THEORY OF VARIATIONAL PRINCIPLE

The energy functional are important for their use in approximate methods as well as deriving a consistent set of equations of motion coupled with free charge equation and the boundary conditions (Reddy, 1984; Bao, 1996; Tzou *et al.*, 2004; Badri and Al-Kayiem, 2012a-c). In summary, the total energy of a shell element is defined as:

$$\delta\int_{t_0}^{t_1}[k-p]\,dt=0 \tag{1}$$

where, p is total potential energy:

$$p=\iiint_v[Q(s_i,\varepsilon_j,g_l,t)+(Tt)]dV-\iint_{\Omega 0}\left(t(s_i,\varepsilon_j,g_l)+W(s_i,\varepsilon_j,g_l)\right) \tag{2}$$

where, Q ($s_i$, $\varepsilon_j$, $g_l$, t), t ($s_i$, $\varepsilon_j$, $g_l$) and W ($s_i$, $\varepsilon_j$, $g_l$) are the thermodynamic potential "Gibbs free energy", tractions and the work done by body force, electrical and magnetic charge, respectively. Moreover, the kinetic energy is:

$$k=\frac{1}{2}\iiint_v[\dot{u}^2+\dot{v}^2+\dot{w}^2]dV \tag{3}$$

Substituting Eq. 2 and 3 into Eq. 1 yields:

$$\begin{aligned}&\int_{t_0}^{t_1}\frac{1}{2}\iiint_v[\dot{u}^2+\dot{v}^2+\dot{w}^2]dv\,dt-\left\{\int_{t_0}^{t_1}\iiint_v[\delta Q(s_i,\varepsilon_j,g_l,t)\right.\\&\left.+(t\delta\tau)]dV\,dt-\int_{t_0}^{t_1}\iint_{\Omega_0}\left(\delta t(s_i,\varepsilon_j,g_l)+\delta W(s_i,\varepsilon_j,g_l)\right)dAdt\right\}=0\end{aligned}$$

The kinetic energy of the shell can be expressed as:

$$k = \frac{1}{2}\iint_{\Omega_0}\int_{-h/2}^{h/2}\left(\begin{matrix}[\dot{u}_0^2+\dot{v}_0^2+\dot{w}_0^2]\\ +\varsigma^2[\dot{\psi}_\alpha^2+\dot{\psi}_\beta^2]+2\varsigma[\dot{u}_0^2\dot{\psi}_\alpha^2+\dot{v}_0^2\dot{\psi}_\beta^2]\end{matrix}\right)\times\left(1+\frac{\varsigma}{R_\alpha}\right)\left(1+\frac{\varsigma}{R_\beta}\right)ABd\varsigma dA \quad (4)$$

It is known that, for quasi-static infinitesimal reversible processes, the linear thermodynamic potential energy Q of a system subject to mechanical, electric, magnetic and thermal influences from its surroundings, can be approximated by:

$$Q(s_i, \varepsilon_j, g_l, t) \cong \frac{1}{2}(Q(s_{ij}\,\varepsilon_{kl} - \varepsilon_n\,\xi_n - g_q x_q - t\tau)$$

where, $s_{ij}$, $\varepsilon_k$, $g_l$ and t are the dependent variables of Q, while $\varepsilon_{ij}$, $\xi_k$, $x_l$ and $\tau$ are the natural independent variables. In order to obtain the thermodynamic potential for which these variables are natural, is performed by Perez-Fernandez *et al.* (2009), that is:

$$\begin{aligned}2Q = {} & \varsigma_{ijkl}^{\varepsilon,g,t}\,\varepsilon_{ij}\,\varepsilon_{kl} - \epsilon_{mn}^{s,g,t}\,\xi_m\xi_n - \mu_{pq}^{s,\varepsilon,t}\,\chi_p\chi_q - \theta^{s,\varepsilon,g}\,\tau^2 - 2Q_{mkl}^{g,t}\,\varepsilon_{kl}\,\xi_m - 2k_{pkl}^{\varepsilon,t}\,\varepsilon_{kl}\,\chi_p \\ & - 2\lambda_{kl}^{\varepsilon,g}\,\varepsilon_{kl}\,\tau - 2\eta_{pn}^{s,t}\,\xi_n\chi_p - 2\rho_n^{s,g}\,\xi_n\tau - 2\gamma_q^{s,\varepsilon}\,\chi_q\,\tau\end{aligned}$$

where, Q is commonly known as Gibbs free energy, the superscripts indicate that the magnitudes must be kept constant when measuring them in the laboratory frame. The constitutive relations can be expressed formally by differentiation of Q corresponding to each dependent variable as:

$$\left.\begin{aligned} s_{ij} &= \left(\frac{\partial Q}{\partial \varepsilon_{kl}}\right) = \varsigma_{ijkl}^{\varepsilon,g,t}\,\varepsilon_{ij} - Q_{mkl}^{g,t}\,\xi_m - k_{pkl}^{\varepsilon,t}\,\chi_p - \lambda_{kl}^{\varepsilon,g}\tau \\ \varepsilon_k &= \left(\frac{-\partial Q}{\partial \xi_n}\right) = Q_{ijn}^{g,t}\,\varepsilon_{ij} + \epsilon_{mn}^{s,g,t}\,\xi_m - \eta_{pn}^{s,t}\,\chi_p - \rho_n^{s,g}\tau \end{aligned}\right\} \quad (5a)$$

$$\left.\begin{aligned} g_l &= \left(\frac{-\partial Q}{\partial \chi_q}\right) = k_{ijq}^{\varepsilon,t}\,\varepsilon_{ij} - \eta_{mq}^{s,t}\,\xi_m + \mu_{pq}^{s,\varepsilon,t}\,\chi_p - \gamma_q^{\varepsilon,g}\tau \\ t &= \left(\frac{-\partial Q}{\partial \tau}\right) = \lambda_{ij}^{\varepsilon,g}\,\varepsilon_{ij} + \rho_m^{s,g}\,\xi_m + \gamma_p^{s,\varepsilon}\,\chi_p + \theta^{s,\varepsilon,g}\tau \end{aligned}\right\} \quad (5b)$$

Then the total thermodynamic potential is given by:

$$\delta Q = \frac{\partial Q}{\partial \varepsilon}.\delta\varepsilon - \frac{\partial Q}{\partial \xi} - \frac{\partial Q}{\partial \chi}.\delta\chi - \frac{\partial Q}{\partial \tau}.\delta\tau \quad (6)$$

While the tractions are:

$$t(s_i, \varepsilon_j, g_l) = (\tilde{s}_{nn}\,\delta u_n + \tilde{s}_{nt}\delta v_t + \tilde{s}_{n\varsigma}\delta w_r) + (\tilde{\varepsilon}_{nn}\delta\varphi + \tilde{\varepsilon}_{nt}\delta\varphi) + (\tilde{g}_{nn}\delta\vartheta + \tilde{g}_{nt}\delta\vartheta) \quad (7)$$

Moreover, the external work is:

$$W(s_i, \varepsilon_j, g_l) = (f_\alpha^s u_o + f_\beta^s v_o + f_n^s w_o + c_\alpha^s \psi_\alpha + c_\beta^s \psi_\beta - f^\varepsilon \varphi_o + c^\varepsilon \varphi_1 - f^g \vartheta_o + c^g \vartheta_1) \tag{8}$$

where, $f_\alpha^s$, $f_\beta^s$ and $f_n^s$ are the distributed forces in $\alpha$, $\beta$ and $\zeta$ directions, respectively and $c_\alpha^s$ and $c_\beta^s$ are the distributed couples about the middle surface of the shell. In addition $f^\varepsilon$, $c^\varepsilon$, $f^g$ and $c^g$ are the distributed forces and couples due to electrical and magnetic charge. Substituting Eq. 6-8 in Eq. 2 and equating the resulted equation with Eq. 1, yields after expanding the terms:

$$\begin{aligned}
&\int_{t_0}^{t_1}\iint_{\Omega_0}\delta\begin{pmatrix}\frac{\bar{I}_1}{2}[\dot{u}_0^2+\dot{v}_0^2+\dot{w}_0^2]\\ \frac{\bar{I}_3}{2}[\psi_\alpha^2+\psi_\beta^2]\\ \bar{I}_2[\dot{u}_0^2\psi_\alpha^2+\dot{v}_0^2\psi_\beta^2]\end{pmatrix}ABdAdt-\left\{\int_{t_0}^{t_1}\iiint_{V}\begin{pmatrix}[\varsigma_{ij}\varepsilon-Q_{ij}\xi-k_{ij}\chi-\lambda_i\tau]\delta\varepsilon\\ -[Q_{ij}\varepsilon-\varepsilon_{ij}\xi+\eta_{ij}\chi-\rho_i\tau]\delta\xi\\ -[k_{ij}\varepsilon+\eta_{ij}\xi+\eta_{ij}\chi+\rho i\tau]\delta\chi\\ \cancel{-[\lambda_{\bar{\tau}}\varepsilon+\rho_{\bar{\tau}}\xi+\gamma_{\bar{\tau}}\chi+\theta\tau]\delta\tau}\\ \cancel{+[\lambda_{\bar{\tau}}\varepsilon+\rho_{\bar{\tau}}\xi+\lambda_{\bar{\tau}}\chi+\theta\tau]\delta\tau}\end{pmatrix}dVdt\right.\\
&\left.-\int_{t_0}^{t_1}\iint_{\Omega_0}\begin{pmatrix}\begin{Bmatrix}(s_{nn}(\delta u_{on}+\zeta\delta\psi_{\alpha n})+\ddot{s}_{nt}(\delta v_{ot}+\zeta\delta\psi_{\beta t})+\ddot{s}_{n\zeta}\delta w_r\\ -\left(\ddot{\varepsilon}_{nn}(\delta\varphi_0+\zeta\delta\varphi_1)\right)-\left(g_{nn}(\delta\theta_0+\zeta\delta\theta_1)\right)\end{Bmatrix}\\ -\left(\ddot{\varepsilon}_{nt}(\delta\varphi_0+\zeta\delta\varphi_1)\right)-\left(g_{nt}(\delta\theta_0+\zeta\delta\theta_1)\right)\\ +\left\{[f_\alpha^\varepsilon u_0+f_\alpha^\varepsilon v_0+f_n^\varepsilon w_0+c_\alpha^\varepsilon\psi_\alpha+c_\beta^\varepsilon\psi_\beta]\right\}\\ -[c^\varepsilon\varphi_0+c^\varepsilon\varphi_1]-[c^g\vartheta_0+c^g\vartheta_1]\end{pmatrix}ABsAdt\right\}=0
\end{aligned} \tag{9}$$

Not that, the temperature $\tau$ is a known function of position. Thus, temperature field enter the formulation only through constitutive equations. While, $I_1$, $I_2$ and $I_3$ are, the inertia terms and they define as:

$$\bar{I}_j = \left[I_j + I_{j+1}\left(\frac{1}{R_\alpha}+\frac{1}{R_\beta}\right)+\frac{I_j+2}{R_\alpha R_\beta}\right] \text{for } j=1,2,3 \text{ where } [I_1, I_2, I_3, I_4, I_5]$$
$$= \sum_{k=1}^{N}\int_{h_{k-1}}^{h_k} I^k(1, \zeta, \zeta^2, \zeta^3, \zeta^4, \zeta^5)d\zeta$$

where, $I^k$ is the mass density of the kth layer of the shell per unit mid-surface area. While the energy expressions described above are used to derive the equations of motion. Note that, the kinetic relations (i.e., the force and moment resultants per unit length at the boundary $\Omega$) are obtained by integrating the stresses over the plate thickness as in Eq. 10. Or we can rewrite Eq. 10 in term of constitutive relations Eq. 5a and b directly as that expressed below in Eq. 11. Thus, the constitutive terms in Eq. 9 could be replaced by the kinetic relations Eq. 11 for a reason of casting the equation of motion to be dependent of forces and moment resultant as well as to reduce the volume integral to double integral. Through, a recast of Eq. 9 to put in the famihar form, the governing equations of motion and the equation charge equilibrium for first-order shearable deformation case can be derived based on the fundamental Lemma of calculus of variations. By integrating the displacement gradients by parts to reheve the virtual displacements and setting its coefficients to zero individually:

$$\begin{Bmatrix} N_\alpha^s & M_\alpha^s \\ N_\beta^s & M_\beta^s \\ Q_\alpha^s & P_\alpha^s \\ Q_\beta^s & P_\beta^s \\ N_{\alpha\beta}^s & M_{\alpha\beta}^s \\ N_\alpha^\varepsilon & M_\alpha^\varepsilon \\ N_\beta^\varepsilon & M_\beta^\varepsilon \\ N_\alpha^g & M_\alpha^g \\ N_\beta^g & M_\beta^g \end{Bmatrix} = \int_{-h/2}^{h/2} (1,\zeta) \begin{Bmatrix} s_\alpha \\ s_\beta \\ s_{\beta\zeta} \\ s_{\alpha\zeta} \\ s_{\alpha\beta} \\ \varepsilon_\alpha \\ \varepsilon_\beta \\ g_\alpha \\ g_\beta \end{Bmatrix} d\zeta \tag{10}$$

## EQUATIONS OF MOTION

In order to solve the equation of motion, we introduce the following assumptions to cast the equation of motion in thick (or shear deformation) plate theories. Where the deepness (or shallowness) of the shell, is One criterion used in developing plate equations. Thus, shell is referred to as a plate, when it has zero curvature or infinity radius of curvature (i.e., the term $1+\iota/R_1$: where, $R_1$ is either of the curvature parameter $R_\alpha$, $R_\beta$, or $R_{\alpha\beta}$ (Qatu, 2004; Badri and Al-Kayiem, 2011a, b). If it is represented by the plane coordinate systems for the case of rectangular orthotropy, this leads to constant Lame´ parameters (i.e., A, B = 1). In addition, the radii of curvature are assumed to be very large compared to the in-plane displacements (i.e., $u_i/R_i = 0$, where $I = \alpha$, $\boldsymbol{\beta}$ and $\alpha$, $\boldsymbol{\beta}$ and $\boldsymbol{\alpha\beta}$, $u_0$ or $v_0$).

Hence:

$$\begin{bmatrix} \begin{Bmatrix} N_{ky}^s & M_{ky}^\varepsilon \\ Q_{ky}^\varepsilon & P_{ky}^\varepsilon \end{Bmatrix} \\ \begin{Bmatrix} N_{ny}^\varepsilon & M_{ny}^\varepsilon \end{Bmatrix} \\ \begin{Bmatrix} N_{qy}^\varepsilon & M_{qy}^g \end{Bmatrix} \end{bmatrix} = \int_{-h/2}^{h/2} (1,\zeta) \begin{pmatrix} [\varsigma_{ijkl}^{\varepsilon,g,t}\, \varepsilon_{ij} - Q_{mkl}^{g,t}\, \xi_m - k_{pkl}^{\varepsilon,t}\chi_p - \lambda_{kl}^{\varepsilon,g}\,\tau] \\ [Q_{ijn}^{g,t}\, \varepsilon_{ij} - \epsilon_{mn}^{s,g,t}\, \xi_m - \eta_{pn}^{s,t}\chi_p - \rho_n^{s,g}\,\tau] \\ [k_{ijq}^{\varepsilon,t}\, \varepsilon_{ij} - \eta_{mq}^{s,t}\xi_m - \mu_{pq}^{s,\varepsilon,t}\chi_p - \gamma_q^{s,\varepsilon}\,\tau] \end{pmatrix} \left(1+\frac{\zeta}{R_x}\right) d\zeta \begin{cases} \text{For } (x,y) \\ \quad =(\alpha,\beta) \\ \text{and } x \neq y \end{cases} \tag{11}$$

The procedure outlined above, is valid irrespective of using the Navier' solution. The Navier'-type solution can be applied to obtain exact solution as $(k_{ij}+\lambda^2 M_{ij})\ \{\Delta\} = \{F\}$, which is an eigenvalue problem. For nontrivial solution, the determinant of the matrix in the parenthesis is set to zero. Then the configuration of $k_{ij}$ terms for SS-1, cross-ply and rectangular plane form is given by Badri and Al-Kayiem (2012b).

## ILLUSTRATED EXAMPLE

In the present examine, laminated composite square plate (a/b-1) with both the upper and lower surfaces embedded smart materials is considered. The plate structures considered here are made of Terfonal-D smart composite material. The material properties are given in several papers and books like (Reddy, 2004; Badri and Al-Kayiem, 2011c) and it will not repeat here. The adhesive used to bond the structural layers or smart-material layers are neglected in the analysis. The laminated composite structures are composed of N layers and all the layers are assumed to be of the same thickness. The side-to-thickness ratios stack range (a/h = 10 to a/h = 100) are considered to represent the thick and thin laminated composites. Four different laminations schemes (i.e., symmetric cross-ply, symmetric angle-ply, symmetric general angle-ply and asymmetric

Table 1: Static analysis of nondimensionalized center deflection as $\bar{w}=10^2\times w_0/h$ and load parameter $P^s=P_0^s\alpha^4/E_2h^4$ of laminated composite plate (a/b = 1, CFRP and Terfonal-D, 10-layer and SS-1)

| Load parameter | | Symmetric cross-ply (M,90,0,90,0)s Presented | HSDT | Symmetric angle-ply (M,45,-45,45,-45)s Presented | HSDT | Symmetric general angle-ply (M,45,-45,0,90)s Presented | HSDT | Asymmetric general angle-ply (M,45,-45, 15,-15,0,90,30,-30, M) Presented | HSDT |
|---|---|---|---|---|---|---|---|---|---|
| Thick (a/h = 10) | 1 | 0.0089 | 0.00869 | 0.0053 | 0.00684 | 0.0067 | 0.00697 | 0.0070 | 0.00706 |
| | 3 | 0.0268 | 0.02606 | 0.0160 | 0.02053 | 0.0200 | 0.02090 | 0.0209 | 0.02115 |
| | 5 | 0.0446 | 0.04341 | 0.0266 | 0.03420 | 0.0333 | 0.03482 | 0.0348 | 0.03521 |
| | 10 | 0.0892 | 0.08666 | 0.0532 | 0.06827 | 0.0666 | 0.06952 | 0.0696 | 0.07018 |
| | 15 | 0.1338 | 0.12958 | 0.0798 | 0.10208 | 0.0999 | 0.10401 | 0.1044 | 0.10480 |
| | 30 | 0.2677 | 0.25519 | 0.1596 | 0.20102 | 0.1999 | 0.20531 | 0.2088 | 0.20594 |
| | 45 | 0.4015 | 0.37402 | 0.2394 | 0.29469 | 0.2998 | 0.30186 | 0.3132 | 0.30188 |
| | 60 | 0.5353 | 0.48504 | 0.3193 | 0.38232 | 0.3997 | 0.39279 | 0.4176 | 0.39204 |
| | 80 | 0.7138 | 0.62070 | 0.4257 | 0.49002 | 0.5330 | 0.50523 | 0.5568 | 0.50360 |
| | 100 | 0.8922 | 0.74426 | 0.5321 | 0.58856 | 0.6662 | 0.60858 | 0.6960 | 0.60638 |
| Thin (a/h = 100) | 10 | 0.0864 | 0.07518 | 0.0525 | 0.05660 | 0.0659 | 0.05799 | 0.0689 | 0.05880 |
| | 20 | 0.1727 | 0.14969 | 0.1051 | 0.11278 | 0.1319 | 0.11560 | 0.1378 | 0.11687 |
| | 30 | 0.2591 | 0.22294 | 0.1576 | 0.16816 | 0.1978 | 0.17252 | 0.2067 | 0.17395 |
| | 40 | 0.3454 | 0.29445 | 0.2101 | 0.22243 | 0.2638 | 0.22843 | 0.2756 | 0.22978 |
| | 50 | 0.4318 | 0.36386 | 0.2627 | 0.27537 | 0.3297 | 0.28313 | 0.3445 | 0.28423 |
| | 60 | 0.5182 | 0.43098 | 0.3152 | 0.32683 | 0.3957 | 0.33646 | 0.4134 | 0.33719 |
| | 70 | 0.6045 | 0.49572 | 0.3678 | 0.37674 | 0.4616 | 0.38834 | 0.4823 | 0.38863 |
| | 80 | 0.6909 | 0.55807 | 0.4203 | 0.42508 | 0.5276 | 0.43871 | 0.5512 | 0.43854 |
| | 90 | 0.7772 | 0.61811 | 0.4728 | 0.47188 | 0.5935 | 0.48759 | 0.6201 | 0.48696 |
| | 100 | 0.8636 | 0.67592 | 0.5254 | 0.51719 | 0.6595 | 0.53501 | 0.6890 | 0.53394 |

general angle-ply laminates) under SS-1 boundary condition are considered in this study. As a baseline of computer simulation, unless otherwise specified, symmetric cross-ply laminates with (SS-1) boundary condition are mainly used. The HSDT that developed by Lee (2004), are used here in the verifications. The shear correction factor used in FSDT is ($K^2$ = 5/6). Numerical values of nondimensional center deflection as function of the load parameter are tabulated in Table 1 and the effects of two kind of plate thickness are studied. As stated earlier by Tsai *et al.* (2008), that the distribution of displacements through the thickness by kinematics field in classical plate theories may lead to unexpected error.

Consequently, the Higher-order Shears Deformation Theory (HSDT) that allows the transversal displacement w and its corresponding strain $\varepsilon_{\varsigma\varsigma}$, to vary nonlinearly through the cross-thickness, should be more accurate. Thus, a correspondence has been observed between the results of the presented theory with those obtained by Lee (2004) that use an exact model based on a HSDT and satisfactory agreement is found.

Even though, shear deformation theory is relevant in the stress calculations but still not essentially for electric and magnetic potentials as well electric displacement and magnetic induction. Whereas only including of nonlinear constitutive relations of smart materials in the structural analysis could justify the discrepancies found in the predictions with shear deformation theories. A similar conclusion was also reported by Lee (2004).

In the other hand, Fig. 1-3 show the magnetic potential $\vartheta$, electrical potential $\varphi$, center deflections w, angle of twist $\psi_\alpha$ and $\psi_\beta$ the in-plane displacement u and v responses for sandwich plate formed from three smart layers. It is perceived that the elastic deflections, electrical potential

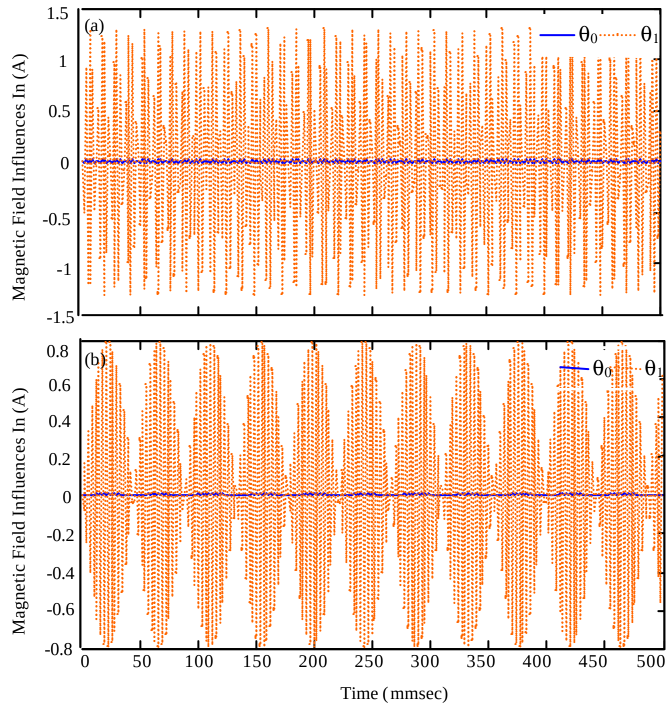


Fig. 1(a-b): The uncontrolled magnetic responses of laminated composite plate of (a/b = 1 and m = n = 5) (a) P/M/P and (b) M/P/M scheme, P: $BaTiO_2$, M: $CoFe_2O_4$

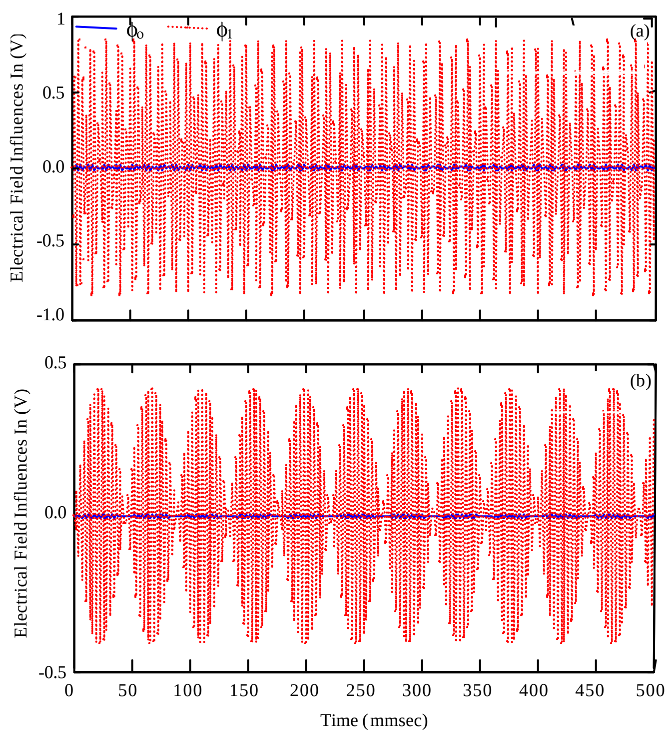


Fig. 2(a-b): The uncontrolled electrical responses of laminated composite plate of (a/b = 1 and m = n = 5) (a) P/M/P and (b) M/P/M scheme, P: $BaTiO_2$, M: $CoFe_2O_4$

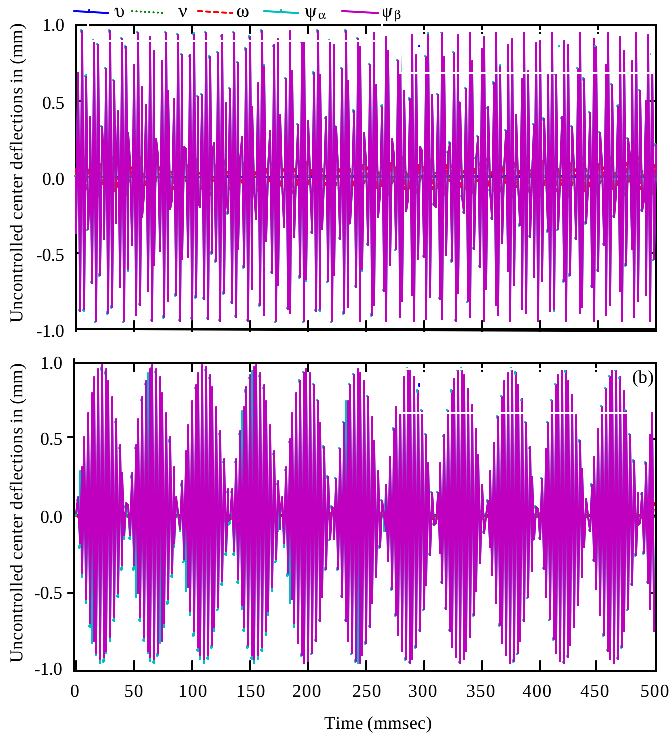


Fig. 3(a-b): The uncontrolled elastic responses of laminated composite plate of (a/b = 1 and m = n = 5) (a) P/M/P and (b) M/P/M scheme, P: $BaTiO_2$, M: $CoFe_2O_4$

and magnetic potential have similar occurrence. It is interesting to note that the sensory responses have simple discriminate behavior against the variation in the plate dimensions.

## CONCLUSION

In this study, a model is developed for static and dynamic analysis of MTEE and multilayered plate structure and/or plate embedded a smart material lamina and influenced by MTEE load. The fundamental theory is derived based on FSDT involving Codazzi-Gauss geometrical discretion. The theory is casted in version of general laminated composite plate of rectangular plane-form, in which the generic forced-solution procedures for the response were derived and its mode shapes were evaluated in simply supported boundary condition. Thus the center deflection was selected among the primary variable for validation and verification purpose. Whereas, result have been shown a close agreement with those of HSDT that obtained by previous researchers. The present results may serve as a reference in developing the MTEE plate theories and to improve the benchmark solutions for judging the existence of imprecise theories and other numerical approaches.

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