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Markov Chain Model for Predicting Pitting Corrosion Damage in Offshore Pipeline

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ABSTRACT

Pitting is a type of corrosion which the growth prediction is governed by the uncertainties that can cause inaccuracy in the outcome. This paper studies the fitness of Markov Chain model in representing the pitting growth behaviour. This work focuses on the prediction of pitting corrosion damage in offshore pipeline based on Markov Chain model. Data of pitting corrosion was obtained from repeated In-line Inspection (ILI) survey. Historical data of corrosion pit was projected and compared with actual data from later inspection. Based on the results, Markov Chain model has over predicted the growth of corrosion pit with larger dispersion. The predicted distribution of corrosion pit is highly governed by extreme data as compared to distribution of actual data. The discrepancies in prediction result are due to the assumption made on the exponential factor of power law model whereby the factor was set to v = 1 so that the corrosion progress can behaves linearly. Regardless the inaccuracy of prediction, the Markov Chain model is found promising in predicting corrosion progress in pipelines. If more information in regards to environment and pipeline operation can be made available, the overall projection can be improved.

Key words: Corrosion, pipeline, Markov chain

INTRODUCTION

Corrosion can cause serious damage due to volume loss process especially to metal-based element (Shaw and Kelly, 2006; Yahaya *et al.*, 2011). Of all type of corrosion defects found on deteriorating structure, pitting has been acknowledged as the most threatening mechanism that can cause severe leakage in liquid containment structure such as pressure vessels and pipelines owing to its fast growth rate in penetrating the wall thickness. (Velazquez *et al.*, 2009). As opposed to conventional modelling work through laboratory testing, instead of analysing data to produce model, a model based on theoretical mathematical equation is chosen as a hypothetical model to represent dynamic of internal pitting corrosion growth in the pipeline.

The model is based upon Markov Chain model used to incorporate the pure birth nature element of future distribution of the corrosion pit which is independent with its previous state of defect distribution. This theory enable prediction of future growth of corrosion pit without significant influence by erroneous distribution of corrosion pit measured via ILI in the past. This is due to memoryless characteristic of Markov Chain which allows the ILI data to be fully utilised regardless the age of the data. Memoryless property as the state of the system at future time t_{n+1}

is decided by the system state at the current time t_n and does not depend on the state at earlier time instants $t_1, \ldots, t_{n-1}$ is one of the most important characteristics of Markov Chain (Bose, 2002).

Markov process: Markov process can represent a number of real system evolutions such as corrosion. The probabilistic Markov model of equations describes the transition from one state to another whereby the past govern the present state but not the future (Zio, 2009). Researchers have been using Markon Chain model to simulate the time-dependent growth of corrosion defect using different principles. In this paper, the Markov process used in the modelling is a continuous-time non homogenous linear growth type of process defined as following:

$$\frac{dp_{i,j}(t)}{dt} = \begin{cases} \lambda_{j-1}(t)p_{i,j-1}(t) - \lambda_j(t)p_{i,j}(t) j \geq i+1 \\ -\lambda_i(t)p_{i,i}(t) \end{cases} \tag{1}$$

It has been found that the behaviour of the pitting corrosion can be represented by Markov process (Cox and Miller, 1965). Listed below is the criterion that has been considered prior to the classification of the above type of Markov process:

- **Continuous time:** The process of pitting growth is continuous in time
- **Non-homogeneous:** The growth behaviour of pitting corrosion does not depend on the condition of the pitting begin, so the transition probability which represent the growth of pitting corrosion does not depend on the initial distribution of the pitting corrosion
- **Linear:** The corrosion rate is assumed to be linear

The description on Markov model utilised in this work can be found in detail somewhere else (Caleyo *et al*., 2009).

Model development

Non-homogeneous Markov process: In order to develop the prediction model using Markov process as aforementioned, the Markov conditional probability of transition from the m-th state to the n-th state (n = m) in the interval (t_0, t) needs to be created first based on the following equation:

$$\rho_{m,n}(t_0, t) = \binom{n-1}{n-m} e - \{\rho(t) - \rho(t_0)\} m \left(1 - e - \{\rho(t) - \rho(t_o)\}\right)^{n-m} \tag{2}$$

Where:

$$\rho(t) = \int_0^t \lambda(t')dt' \tag{3}$$

In words, Eq. 2 means that the pitting corrosion damage increase in an interval of length t-t_0 follows a negative binomial distribution NegBin(r, ρ) with parameters r = m and $\rho = \rho_s = e^{-\{\rho(t)-\rho(t0)\}}$. From Eq. 2 it is now possible to estimate the probability distribution function f(v) of the damage rate v associated with the damage process over the interval of length $\Delta t = t - t_0$, when the damage depth is at the m-th state:

$$f(v;\, m,\, t_0,\, t) = \rho m(t_0)\rho m,\, m+v\Delta t\ (t_0,\, t)\Delta t \quad (4)$$

Now from Eq. 4, the pitting rate probability distribution can be derived using:

$$f(v;m,t_0,t)=\sum_{m=1}^{N} f(v;m,t_0,t) \quad (5)$$

With the initial pit depth $\rho_m(t_0)$ created, the next step is to determine the value of t_s so as to make the distribution able to evolve using the probability of transition.

Probability parameter: This section will describe the derivation of probability parameter which is part of the transition probability. The detail description and derivation can be found in Parzen (1999). Basically the probability parameter $\rho_s = e^{-\{\rho(t)-\rho(t0)\}}$ in Eq. 2 can be expressed as:

$$\rho_s=\left(\frac{t_0-t_{sd}}{t-t_{sd}}\right)^v \geq t_0 \geq t_{sd} \quad (6)$$

First of all the data is assumed to follow a power law model for pit growth. Say that the corrosion damage in the form of pit depth at any point in time t can be represented by a discrete random variable D(t) with P{D(t) = I} = pi(t), i = 1, 2... N. The pipe wall thickness has been divided in total of N discrete states.

If the initial damage state at $t = t_i$ is n_i, so the linear growth Markov process stochastic mean is:

$$M(t) = E\,[D(t)] \quad (7)$$

$$M\,(t) = n_i e^{\rho}(t-t_i) \quad (8)$$

The state of the system when $t = t_i$ which is $n_i = 1$ since the system evolves from depth = 0 at t, so that it start from n = 1 in the Markov Chain. The deterministic mean pit depth at time t is assumed represented by the power law model for the pit growth process and can be expressed as:

$$D\,(t) = \kappa(t-t_{sd})^v \quad (9)$$

Where:
κ = Pitting proportionality
υ = Exponent parameter
t_{sd} = Starting time of pitting corrosion damage

The transition probability function is found when the stochastic mean is assumed to be equal to deterministic mean as described by Caleyo *et al.* (2009):

$$M\,(t) = D(t) \quad (10)$$

The determination of function $\rho(t)$ rely upon the above assumption, $n_i = 1$ and $t_i = 0$, substitute into Eq. 10:

$$n_i e^{\rho}(t-t_i) = \kappa(t-t_{sd})v \tag{11}$$

$$e^{\rho(t)} = \kappa(t-t_{sd})^{v} \tag{12}$$

If both sides are ln function, then they will become:

$$\rho(t) = \ln\ \kappa(t-t_{sd})^{v} \tag{13}$$

From Eq. 2 of probability parameter:

$$\rho_s = e^{-}\{\rho(t)-\rho(t_0)\} \tag{14}$$

Substitute Eq. 13 into Eq. 14:

$$\rho_s = e-\left\{[\ln \kappa(t-t_{sd})^{v}]-\left[\ln \kappa\left(t_0-t_{sd}\right)^{v}\right]\right\} \tag{15}$$

Now probability parameter has been derived and become similar to Eq. 6. If the transition probability function $\rho_m(t_0, t)$ is known, then the pit depth distribution at any future moment in time can be estimated using:

$$\rho_n(t) = \sum^{n} m = 1\ \rho m(t_0)\rho m, n(t_0, t) \tag{16}$$

It is of importance that the diameter of the metal loss should be equal to or less than twice the pipe wall thickness.

Exponential parameter: The exponential parameter, v represent the exponent of the power law describing the pit depth evolution with time. The exponential parameter does not denote the character of the pitting corrosion defect growth either linear or non-linear. Therefore the correct determination of the value of v is very important to fit the Markov model to the data.

Caleyo *et al.* (2009) carried out predictive model to estimate the value of v for different soil types as they studied parameters for soil corrosion which is external corrosion. This paper only considers internal corrosion inside a pipeline. Since additional information in regards to environment and pipeline operation are lacking it was assumed that v = 1. On the other hand, the value of v can be estimated based on equation stated below if the parameters in the equation are available:

$$v = \frac{\ln\left(\frac{d_2}{d_1}\right)}{\ln\left(\frac{t_2-t_{sd}}{t_1-t_{sd}}\right)} \tag{17}$$

Data analysis

Data preparation: The metal loss data used in this study was obtained from two repeated ILI survey on offshore pipeline. The ILI was conducted in year 1992 and 1995. Data from year 1992 was used to predict the future distribution of corrosion pit in year 1995 for comparison purposes. In order to fit the data into Markov Chain model, the initial pit depth distribution $\rho_m(t_0)$ was developed beforehand. The process began with discretising the wall thickness of the pipeline into M states. The pit depth fall under each state then counted and normalised using the total number of pits, N (in this case, N = 1000). From that, the frequency of each depth can be obtained, that is the value of $\rho_m(t_0)$ for each m; t_0 being the time of the ILI.

For this case study, the wall thickness was divided into ten equal numbers of states with 2.22 mm interval between each state. The pit depth data were then distributed according to the, respective states resulting in an initial probability; ρ_m. Table 1 shows the tabulation of states with the pit depth data that fall into the corresponding states. According to the table, the state distribution of wall thinning process is governed by 2nd state (shallow depth of defect) with 35.5%, reflecting the initial distribution of corrosion depth. The pattern of defect distribution according to the states will change as corrosion progress in time.

The data was thoroughly analysed mainly to determine the important deterioration parameters such as defect dimension and corrosion growth value. Each of the parameter can be represented by mean and variation values. The variation of data shows how the intensity and dispersion of uncertainties exists within inspection data. Fig. 1 illustrates the steps of probabilistic analysis.

Table 1: Pipe wall thickness divided into different states to reflect the evolution of defect distribution until the defect reach the full penetration at 10th state

No. of states	Maximum pit depth (mm)	No. of data	Initial probability, p_m
1	2.22	236	0.236
2	4.44	353	0.353
3	6.66	269	0.269
4	8.88	104	0.104
5	11.10	30	0.030
6	13.32	7	0.007
7	15.54	1	0.001
8	17.76	0	0.000
9	19.98	0	0.000
10	22.20	0	0.000

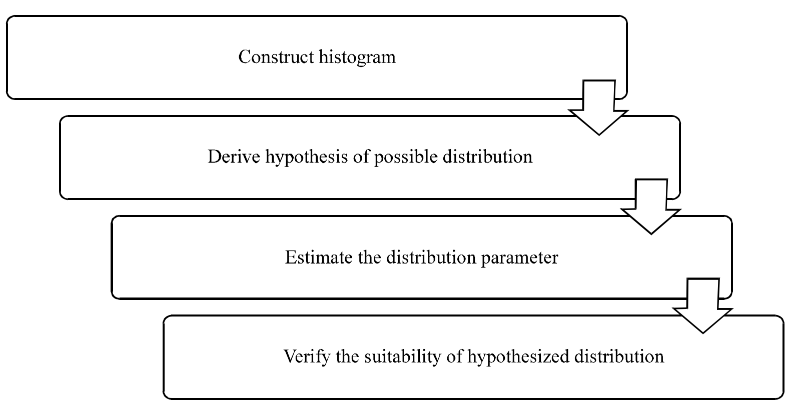

Fig. 1: Steps in probabilistic analysis

A histogram of corrosion data was constructed from frequency table so as to determine the most appropriate distribution to present the corresponding data. The shape examined from the histogram puts the types of distribution into view. Histogram itself is not sufficient to describe the behaviour of the data since the information from histogram is limited to frequency and rough shape of distribution. Moreover, histogram only represents the sample out of the population. Therefore it is very crucial to optimise the information from sample so that a conclusion can be derived about population. To do so, a probabilistic distribution must be developed from the corresponding histogram. The data was assumed to follow a Weibull pattern of distribution. A graphical plot, one of the goodness-of-fit test, was utilised to fit the data into the best distribution. The linear fitting of data indicate how well the chosen hypothesised distribution can represent the data. Figure 2 and 3 display the probability plot of the actual inspection data in year 1995 (later inspection) and predicted distribution based on Markov model.

Table 2 list the Weibull parameters of both distributions to indicate the shape and orientation of the distributions. The β and θ represent the sharpness and the variation of the data, respectively represented by Weibull distribution. Both distributions have shape parameter, β>1 indicating the

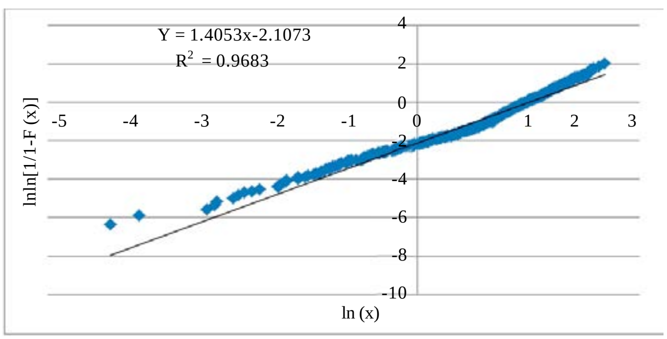

Fig. 2: Goodness-of-fit test of Weibull probability plot for actual data in (1995), The R^2 value indicates how well the data fit to the straight line, hence prove the hypothesis of Weibull distribution

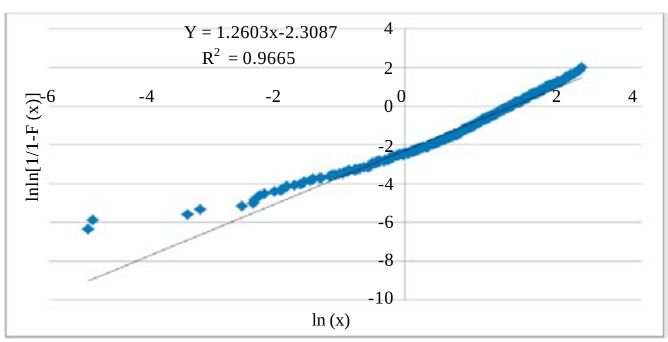

Fig. 3: Goodness-of-fit test of Weibull probability plot of predicted data in (1995) based on markov chain model, The R^2 value indicates how well the data fit to the straight line, hence prove the hypothesis of Weibull distribution

Table 2: Weibull parameters of actual and predicted data

No. of states	Beta (β)	Theta (θ)
Actual	1.4053	4.4796
Markov model	1.2603	6.2455

existence of peak zone as opposed to exponential shape. However, the predicted distribution has greater value of scale parameter, θ. Hence, less peak shape due to greater deviation of the data from its central tendency.

Markov Chain modelling: The modelling begins with calculation of the probability parameter, ρ_s. In this case, the pit depth evolution was assumed to follow the linear behaviour with the exponent factor, υ equals to one. Whilst the time of the pipeline started having pit corrosion was as represented by Weibull distribution. Both distributions have shape parameter, β>1 indicating the existence of peak zone as opposed to exponential shape. However, the predicted distribution has greater value of scale parameter, θ. Hence, less peak shape due to greater deviation of the data from its central tendency. estimated as 2.9 years after the installation based on the study done reported by Caleyo *et al.* (2009). With these two parameters in hand, the probability parameter can be computed using Eq. 6. After that, the process continued with calculating the transition probability function, $\rho_{m,n}$ using Eq. 2 and by that pit depth distribution at any future moment in time, ρ_n can be estimated right away.

RESULTS AND DISCUSSION

The results from Markov modelling were compared to the actual data from ILI survey of the same predicted year concerning the mean and standard deviation of each actual and Markov ρ redicted data. The overall results are tabulated in the Table 3.

It can be seen that the mean value from the Markov model is higher than the actual ILI data. The main reason is the assumption that has b een made earlier regarding the time the pitting started to grow, t_{sd} and the exponential parameter, v involved in Markov process. These two parameters must be accurately identified since it can govern the outcome of the prediction using Markov model. If these two parameters can be computed accurately, then the Markov model will be able to produce better result.

Figure 4 presents the visual comparison between the actual distributions from ILI data against Markov model predicted from year 1992 to year 1995. From the figure, Markov model is seen as capable of regenerate the distribution as shown from the shape to further the exposure time of the pipeline to the pitting corrosion. The predicted distribution is dominated by extreme data, hence highly scattered. The extreme effect can be dampened by having v<1, reflecting concave power law behavior (Norhazilan *et al.*, 2012). It represent high rate progress of corrosion in the beginning and getting slower as time progress. The inaccurate prediction of corrosion data as shown in Fig. 4 is highly due to assumption on power law parameter, v = 1. As mentioned previously, the extreme behaviour of predicted data is influenced by the assumed linear growth behaviour. Reduction of v, provided that environmental parameters can become available, may improve the prediction results.

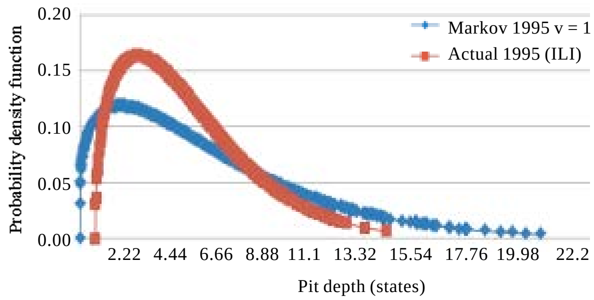

Fig. 4: Comparison of actual and predicted distribution of pit depth in year 1995

Table 3: Comparison of distribution parameters between actual and predicted data in year (1995)

Parameter	Actual 1995	Markov predicted 1995	Percentage
Mean	4.0804	5.8062	42.3
Standard deviation	2.9429	4.6378	57.5

CONCLUSION

A prediction of pit depth distribution using Markov Chain model has been carried out and validated using actual pitting corrosion data. The use of a continuous-time, non-homogenous linear growth (pure birth) Markov process for modelling pitting corrosion is particularly and potentially attractive due to the existence of a closed form solution to the system of Kolmogorov's forward equations that describe this type of Markov process. The use of this solution can avoids a reduction of the number of states for the sake of mathematical simplicity or the requirement of assumptions about the staying time of the pitting damage in a given state.

This study if extended to the better level of accuracy shall provide better understanding on the performance of offshore pipeline under corrosion attack. The finding may also assist pipeline operator to monitor the performance of the pipeline using well-designed pipeline integrity management scheme when the uncertainties in prediction of corrosion growth has been eliminated or reduced by utilisation of the unique characteristic in Markov properties. Furthermore, reliable corrosion model can assist engineers to come out with better cathodic protection system to prolong the integrity of the underground and offshore pipelines.

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