Asian Journal of
Scientific Research

Research Article
Modeling the Asymmetric in Conditional Variance

[1,2]Ayodele Abraham Agboluaje, [1]Suzilah Bt Ismail and [3]Chee Yin Yip

[1]School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, Malaysia
[2]Department of Mathematics and Computer Science, Faculty of Natural Sciences, Ibrahim Badamasi Babangida University, Lapai, Nigeria
[3]Department of Economics, Faculty of Business and Finance, Universiti Tuanku Abdul Rahman, Malaysia

Abstract

The purpose of this study is to model the asymmetric in conditional variance of Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) with Combine White Noise (CWN) model to obtain suitable results. Combine white noise has the minimum information criteria and high log likelihood when compare with EGARCH estimation. The determinant of the residual covariance matrix value indicates that CWN estimation is efficient. Combine white noise has minimum information criteria and high log likelihood value that signify suitable estimation. Combine white noise has a minimum forecast errors which indicates forecast accuracy. Combine white noise estimation results have proved more efficient when compared with EGARCH model estimation.

Key words: Combine white noise, determinant residual covariance, log likelihood, minimum forecast errors, minimum information criteria

Corresponding Author: Ayodele Abraham Agboluaje, School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, Malaysia

Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

In high frequency data time series, the conditional variance (volatility) in empirical analysis is asymmetric, as the large negative shocks are traced by the raise in volatility than the large positive shocks[1,2,3]. It is detected in the most high frequency data that in conditional variance, the negative shocks have more effect than the positive shocks[1,4,5,3].

Engle[6] uses lagged disturbances to capture the time varying conditional variance, but cannot capture the large lag length. To capture the dynamic behaviour of conditional variance, it requires a high order of ARCH.

Bollerslev[7] overcomes the weaknesses by generalized ARCH model for the extension of the lag length, but cannot effectively tackle the thick tails in the high frequency data distribution of the time series. Bollerslev[8] employs student's t-distribution to take care of thick tails challenges.

Engle[6] and Bollerslev[7] establish linear ARCH and GARCH models for conditional variance with interest on the magnitude of returns, but ignore the information on the direction of returns, whereas, volatility affects the direction of return[9,10,1]. Vivian and Wohar[11] and Ewing and Malik[12] reveal that the GARCH models exhibit excess kurtosis and volatility persistence which is GARCH weakness.

This conveys the GARCH family. A reaction to news is a shock which is the volatility. The observation of news time can increase the expected volatility mechanism, like economic announcements which may not be a shock[13].

Integrated GARCH model is parallel to ARIMA (0, 1, 1) model as the definition of an ACF of squared observations, if the observations are stationary in first difference, then the model is recognized as IGARCH. Exponential GARCH overcomes the problem of conditional variance persistence measurement[10].

Threshold GARCH (TGARCH) and exponential GARCH (EGARCH) capture the asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in various ways. Leverage is a particular case of asymmetry. To deal with asymmetry, positivity restriction will be imposed on the parameters of the EGARCH model[9,1,2,4,5,3].

It is a known fact, that positive shocks may have less impact on volatility than the negative shocks of the same magnitudes. As both the positive and negative shocks are assigned equal degree of importance in simple GARCH model which cannot take care of leverage effect[9,1,4,5]. Nelson[9] proposes the EGARCH to overcome the leverage effect, but it can only capture the asymmetric volatility. While a negative shock will add more volatility, as the coefficient of the conditional variance will be negative. The positivity restriction positioned on each conditional variance follows the simple GARCH specification and the conditional variance without restriction necessitates the conditional volatility to be negative. Modeling leverage effect is not possible in EGARCH as the general statistical properties to estimate the EGARCH parameters to model the leverage effect are not available[4,5].

The purpose of this study is to uplift the weaknesses of asymmetry in conditional variance in EGARCH model with Combine White Noise (CWN) model to have a reliable estimation for accurate forecasting and to improve the economy.

MATERIALS AND METHODS

Consider the autoregression model by Eq. 1:

$$y_t = \varphi y_{t-1} + \varepsilon_t \tag{1}$$

Permit the stochastic approach of a real-valued time to be ε_t and the complete information through t time is I. The GARCH model is by Eq. 2 and 3:

$$\varepsilon_t | I_{t-1} \sim N(0, h_t) \tag{2}$$

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} = \omega + A(L)\varepsilon_t^2 + B(L)h_t \tag{3}$$

The EGARCH specification is in Eq. 4:

$$\log h_t = \alpha + \beta |z_{t-1}| + \delta z_{t-1} + \gamma \log h_{t-1}, |\gamma| < 1 \tag{4}$$

where, $z_t = \varepsilon_t / \sqrt{h_t}$ is the standardized shocks, $z_t \sim iid(0, \alpha), |\gamma| < 1$ is when there is stability. The impact is asymmetric if $\delta \neq 0$, although, there is existence of leverage if $\delta < 0$ and $\delta < \beta < -\delta$ while, both β and δ must be positive which the variances of two stochastic processes are, then, modeling leverage effect is not possible[4,5].

The unequal variances (heteroscedastic errors) behaviors in the process of estimation being exhibited by GARCH models can be simplified into combine white noise models. The standardized residuals of GARCH errors which are unequal variances are decomposed into equal variances (white noise)

in series to deal with the heteroscedasticity. The regression model is employed to transform each equal variances series to model.

Moving average process is employed for the estimation of these white noise series which is called combine white noise as following Eq. 5-7:

$$Y_1 = \varepsilon_{1t} + \theta_{11}\varepsilon_{1,t-1} + \theta_{12}\varepsilon_{1,t-2} + ...\theta_{jq}\varepsilon_{j,t-q}$$
$$Y_2 = \varepsilon_{2t} + \Phi_{21}\varepsilon_{2,t-1} + \Phi_{22}\varepsilon_{2,t-2} + ...\Phi_{jq}\varepsilon_{j,t-q}$$
$$\vdots$$
$$Yj = \varepsilon_{jt} + \varphi_{j1}\varepsilon_{j,t-1} + \varphi_{j2}\varepsilon_{j,t-2} + ...\varphi_{jq}\varepsilon_{j,t-q} \tag{5}$$
$$Y_{jt} = \sum_{j=1}^{q}\theta_j\varepsilon_{j,t-q} + \sum_{j=1}^{q}\Phi_j\varepsilon_{j,t-q} + ...\sum_{j=1}^{q}\varphi_j\varepsilon_{j,t-q}$$

$$\begin{aligned} &= A(L)\varepsilon_t + B(L)\varepsilon_t + ... \\ &= \varepsilon_t[A(L) + B(L) + ...] \end{aligned} \tag{6}$$

$$\begin{aligned} &= Q\varepsilon_t \\ &= U_t \end{aligned} \tag{7}$$

This Eq. 8 can be written as:

$$Y_t = U_t, \left(U_t \sim N(0,\sigma_c^2)\right) \tag{8}$$

where, $A(L)+B(L)+\cdots = Q$ which are the matrix polynomial, U_t is the error term of combine white noise model and σ_c^2 is the combination of equal variances.

The combine variances of the combine white noise is in Eq. 9:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + ... \tag{9}$$

Considering the best two variances in the best two models produced by the Bayesian model averaging output. The combine variance follows in Eq. 10:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{10}$$

The variance of errors, σ_c^2 in the combine white noise can be written in Eq. 11:

$$\sigma_c^2 = W^2\sigma_1^2 + (1-W)^2\sigma_2^2 + 2\rho W\sigma_1(1-W)\sigma_2 \tag{11}$$

where, the balanced weight specified for the model is W. The least of σ_c^2 appearing, when the equation is differentiated with respect to W and equate to zero, obtaining from Eq. 12:

$$W = \frac{\sigma_c^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{12}$$

where, ρ is the correlation; intra-class correlation coefficient is used for a reliable.

RESULTS

Materials and empirical analysis: The data of U.K. Real Gross Domestic Product (GDP) quarterly data from 1960-2014 is obtained from the DataStream of Universiti Utara Malaysia library. The behavior of data in time plot is non-stationary with an upward trend.

The data is transformed in returns series to observe the volatility clustering, long tail skewness and excess kurtosis which are the characteristics of heteroscedasticity. The graph demonstrated unequal variances that showed the existence of volatility.

In histogram normality test: Jarque-Bera test was highly significant with probability value of 0.000000 which was an indication of non-normality. There were right tail skewness value of 0.375454, excess kurtosis value of 7.014953 and standard deviation value of 0.966867 which was less than one. These signified that the distribution was not normal.

The heteroscedasticity ARCH LM tests for the effect of heteroscedasticity in the data series indicated that the F-Statistic and Obs*R-squared were not significant with probability values of 0.8064 and 0.8053. These indicated the presence of ARCH effect in the data.

Table 1 showed that the AIC, BIC and HQ have minimum information criteria with log-likelihood that were used to select the appropriate model between ARCH and GARCH models. Exponential generalized autoregressive conditional heteroscedasticity model was choosing because it has minimum values of AIC, BIC and HQ with high log-likelihood values.

Combine White Noise (CWN) has the minimum information criteria with high log likelihood value. Combine white noise model gave the best results with minimum information criteria and high log likelihood when compared with GARCH model estimation. The estimation of GARCH model and Combine White Noise (CWN) model with their forecasting values were in Table 2.

In GARCH modeling, the leverage is not possible because any restriction imposed will be positivity restriction which has no leverage effect[4,5]. Although, the data used in this study has no leverage effect, it was asymmetric which GARCH cannot

Table 1: ARCH, GARCH and Combine White Noise (CWN) models

Parameters	α	β	δ	γ	AIC	BIC	HQ	LL
ARCH	0.334938 (0.0003)	0.424743 (0.0000)			2.68436	2.74646	2.70944	-288.60
EGARCH	0.291288 (0.0000)	0.218189 (0.0106)	0.09329 (0.1228)	0.98997 (0.0000)	2.35147	2.37644	2.46014	-249.31
CWN					-0.4444	-3.3515		383.158

α: Coefficient of the mean equation, β and δ: Coefficients of the variance equations, γ: Coefficient of the log of variance equation, ARCH: Autoregressive conditional heteroskedasticity, GARCH: Generalized autoregressive conditional heteroskedasticity, EGARCH: Exponential generalized autoregressive conditional heteroskedasticity, In the parentheses are the Probability Values (PV), AIC: Akaike information criteria, BIC: Bayesian information criteria, CWN: Combine white noise

Table 2: Summary of GARCH and Combine White Noise (CWN) models estimation and forecasting evaluation

Parameters	CWN	GARCH
Estimation residual diagnostic		
Stability test (Lag structure)	Stable	Stable
Correlogram (Square) residual	Covariance stationary	Stationary
Portmanteau tests	No autocorrelation	No autocorrelation
Histogram-normality tests	Not normal	Not normal
ARCH test	No ARCH effect	No ARCH effect
Dynamic forecast evaluation		
RMSE	0.167297	0.653369
MAE	0.040005	0.408789
MAPE	1.427953	169.7009
Residual diagnostics		
Correlogram (Square) residual	Stationary	Stationary
Histogram-normality tests	Not normal	Almost normal
Serial correlation LM tests	No serial correlation	No serial correlation
Heteroscedasticity test	No ARCH effect	No ARCH effect
Stability diagnostic		
Ramsey reset tests	Stable	Stable
Determinant residual covariance	0.000104	

RMSE: Root mean squared error, MAE: Mean obsolute error, MAPE: Mean absolute percentage error, CWN: Combine white noise, ARCH: Autoregressive conditional heteroskedasticity, GARCH: Generalized autoregressive conditional heteroskedasticity

model effectively as CWN did. No proposition has removed heteroscedasticity completely[14-16].

To avoid the above challenges, the standardized residuals graph of the GARCH model (GARCH errors) with unequal variances and zero mean are decomposed into equal variances series (white noise series). There were some graphs of equal variances (white noise series) with mean zero being obtained from graph of GARCH errors. These white noise series were fit into regression model to make each a model.

The first best models out of the five best models from the output of Bayesian model averaging (BMA) produced two best models[17]. Fit linear regression with autoregressive errors to confirm the BMA output with 220 the number of observation, with zero mean and variance one[18]. Therefore, the best two models were the white noise models.

Table 3 indicated that an independent samples test revealed that the variability in the distribution of the two data sets was significantly different value which was less than the p-value 0.05. Thus, the two models have unequal variances[19-22].

Table 2 showed that Combine White Noise (CWN) emerged as a better model for estimation and forecasting when compared with the convention EGARCH model which modeled the asymmetric in GARCH family model.

DISCUSSION

Mutunga *et al.*[23] demonstrated that the first order asymmetric EGARCH model outperformed the first order Glosten-Jagannathan-Runkle GARCH model in forecasting volatility with lower mean square error and mean absolute error. But, CWN model outperformed the first order asymmetric EGARCH model with minimum information criteria and minimum forecast errors.

Bekaert *et al.*[24] improved on the conventional asymmetric volatility models by setting in the conditional shock distribution with time varying heteroscedasticity, skewness and kurtosis called Bad Environment-Good Environment (BEGE) model. Bad environment good environment model incorporated in standard asymmetric GARCH model outperformed the standard asymmetric GARCH model. Decomposition of standardized residual errors of the standard asymmetric GARCH model into white noise series were modeled to give combine white model. Combine white noise model results out performed the standard asymmetric GARCH model results.

Chuffart[25] argued that logistic smooth transition GARCH and Markov-Switching GARCH models were employed to confirm that Bayesian Information Criteria (BIC) can lead to wrong specification. Combine white noise model employs Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and log likelihood for specification of the model to obtain the right model specification.

Chang *et al.*[26] developed ample conditions of strict stationary and ergodicity for three nonlinear models of Self-Exciting Threshold Auto-Regressive (SETAR)-GARCH process, the multiple-regime logistic transition auto-regressive (STAR) model with GARCH errors and Exponential STAR-GARCH model. They considered the STAR-GARCH models estimation results to be vital in financial econometrics. The

Table 3: Levene's test for equal variances

Levene's test for equality of variances	t-test for equality of means							95% confidence interval of the difference	
	F	Significant	t	df	Sig. (2-tailed)	Mean difference	SD	Lower	Upper
Independent samples test									
B equal variances assumed	5.504	0.019	1.133	438	0.258	0.01545	0.01364	-0.01135	0.04226
Equal variances not assumed			1.133	255.502	0.258	0.01545	0.01364	-0.01140	0.04231

df: Degree of freedom, SD: Standard deviation

development of CWN model was from GARCH family errors. Combine white noise was tested using different countries data set with outstanding performance when compared with family GARCH model (EGARCH) which[23] considered to be suitable.

McAleer[4] described the asymmetry and leverage to be indistinguishable and that leverage is asymmetry. The challenge was that there were no statistical properties for the estimation of this leverage effect. The estimation was only possible through positivity restriction of the parameters, which was not an estimate for the leverage effect. Combine white noise model estimated with available statistical properties of maximum likelihood estimation to obtain efficient estimation and proved better than the estimation of the existing models.

McAleer and Hafner[5] introduced one line derivation of EGARCH to model the asymmetric leverage effect, but in this process, stationarity and invertibility conditions were not determined. This made it impossible to model the leverage effect. Combine white noise model stationarity and invertibility were possible.

Therefore, from these discussions, CWN model is suitable for efficient estimation.

CONCLUSION

Exponential GARCH models have been able to model the asymmetric but cannot model leverage effect. The CWN estimation proved more efficient in modeling asymmetric.

The standardized residual EGARCH errors were decomposed into Combine White Noise (CWN). The estimation of combine white noise model passed stability condition, serial correlation, the ARCH effect tests, but failed normality test.

The minimum information criteria and high log likelihood values revealed that CWN model yielded better results when compared with the conventional EGARCH model. Equally, the minimum forecast errors showed that CWN has better results. The determinant of the residual of covariance matrix value indicated that CWN was efficient.

Combine white noise was a suitable model based on the reports from the empirical data analysis. For these explanations, CWN is suggested for the modeling of data that exhibits asymmetric conditional variances and leverage effect.

REFERENCES

1. Hentschel, L., 1995. All in the family nesting symmetric and asymmetric garch models. J. Financial Econ., 39: 71-104.
2. Watari, M.A., A.A. Zaidan and B.B. Zaidan, 2013. Securing m-government transmission based on symmetric and asymmetric algorithms: A review. Asian J. Sci. Res., 6: 632-649.
3. Moorthy, R., H.K. Sum and G. Benny, 2015. Power asymmetry and nuclear option in India-Pakistan security relations. Asian J. Sci. Res., 8: 80-94.
4. McAleer, M., 2014. Asymmetry and leverage in conditional volatility models. Econometrics, 2: 145-150.
5. McAleer, M. and C.M. Hafner, 2014. A one line derivation of EGARCH. Econometrics, 2: 92-97.
6. Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50: 987-1007.
7. Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. J. Econ., 31: 307-327.
8. Bollerslev, T., 1987. A conditionally heteroskedastic time series model for speculative prices and rates of return. Rev. Econ. Stat., 69: 542-547.
9. Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59: 347-370.
10. Harvey, A.C., 1993. Time Series Models, 2nd Edn. Hemel Hempstead, Herts, UK..
11. Vivian, A. and M.E. Wohar, 2012. Commodity volatility breaks. J. Int. Financial Mark. Instit. Money, 22: 395-422.
12. Ewing, B.T. and F. Malik, 2013. Volatility transmission between gold and oil futures under structural breaks. Int. Rev. Econ. Finance, 25: 113-121.
13. Engle, R., 2001. GARCH 101: The use of ARCH/GARCH models in applied econometrics. J. Econ. Perspect., 15: 157-168.
14. White, H., 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica, 48: 817-838.

15. Antoine, B. and P. Lavergne, 2014. Conditional moment models under semi-strong identification. J. Econ., 182: 59-69.

16. Uchoa, C.F.A., F. Cribari-Neto and T.A. Menezes, 2014. Testing inference in heteroskedastic fixed effects models. Eur. J. Operat. Res., 235: 660-670.

17. Asatryan, Z. and L.P. Feld, 2015. Revisiting the link between growth and federalism: A Bayesian model averaging approach. J. Compar. Econ., 43: 772-781.

18. Higgins, M.L. and A.K. Bera, 1992. A class of nonlinear ARCH models. Int. Econ. Rev., 33: 137-158.

19. Lim, T.S. and W.Y. Loh, 1996. A comparison of tests of equality of variances. Comput. Stat. Data Anal., 22: 287-301.

20. Boos, D.D. and C. Brownie, 2004. Comparing variances and other measures of dispersion. Stat. Sci., 19: 571-578.

21. Sbaibi, R., Y. Aboussaleh and A.O.T. Ahami, 2014. Sex differences in means and variance of intelligence among middle school children in the rural commune Sidi El Kamel (North-Western Morocco). Asian J. Sci. Res., 7: 376-385.

22. Bast, A., W. Wilcke, F. Graf, P. Luscher and H. Gartner, 2015. A simplified and rapid technique to determine an aggregate stability coefficient in coarse grained soils. Catena, 127: 170-176.

23. Mutunga, T.N., A.S. Islam and L.A.O. Orawo, 2015. Implementation of the estimating functions approach in asset returns volatility forecasting using first order asymmetric GARCH models. Open J. Stat., 5: 455-464.

24. Bekaert, G., E. Engstrom and A. Ermolov, 2015. Bad environments, good environments: A non-gaussian asymmetric volatility model. J. Econ., 186: 258-275.

25. Chuffart, T., 2015. Selection criteria in regime switching conditional volatility models. Econometrics, 3: 289-316.

26. Chang, C.L., Y. Li and M. McAleer, 2015. Volatility spillovers between energy and agricultural markets: A critical appraisal of theory and practice. Instituto Complutense de Analisis Economico.