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Research Article
Simulation Estimation of Goodness-of-fit Test for Right Skewed Distributions

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Abstract

Objective: This study derives a goodness-of-fit test based on chi-square statistic using simulation and examines the values of the χ^2 test statistic behavior with the level of skewness for two different distributions, namely chi-square and inverse Gaussian. **Methodology:** For this purpose, simulation estimation was conducted to generate random numbers from different skewed distributions. Different sample sizes and skewness values were considered and the corresponding values of the χ^2 test statistic were derived. **Results:** The results show a statistically significant evidence for an inverse relationship between the value of χ^2 test and the level of skewness for all distributions, i.e. the value of χ^2 test statistic decreases as the value of skewness increases and vice versa. The research results also show that the method, estimation by simulation, produces an estimator which is shown to have asymptotic assumed distribution with large sample size. **Conclusion:** These results are relevant to theories in which shape and skewness measure can be used to determine the validity of the assumed right skewed distribution to fit the data well. The results also have practical implications for portfolio managers who are managing funds to optimize risk-adjusted performance and individual investors who prefer positive skewness in rates of return.

Key words: Goodness-of-fit test, chi-square test statistic, simulation, right skewed distributions, skewness measure, shape parameter, χ^2 distribution, inverse Gaussian distribution

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Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

A goodness-of-fit test based on a chi-square statistic for distribution fitting is a vital aspect of statistical hypothesis testing and model validation. Numerous are those articles on goodness-of-fit tests that provide a theoretical basis for studying empirical distribution functions, such as Kolmogorove-Simirnov[1] and Cramer-Von Mises type tests[2-6].

Power[7] and Mayer and Butler[8] address issues related to model validation. They found that validation is a yes or no proposition in the sense that a model does or does not meet the specified validation criteria. These criteria may include requirements for statistical properties goodness-of-fit of the data generated by the model and thus are not necessarily deterministic.

Goncu and Yang[9] have validated the goodness-of-fit results via bootstrapping experiments to the empirical distributions of Chinese index returns. They found that as the time scale of log-returns decrease Normal Inverse Gaussian (NIG) model outperforms the Variance Gamma (VG) model consistently and the difference between the goodness-of-fit statistics increase. They also conclude that for returns at high-frequency at different time scales, the NIG model provides significantly better fit to the empirical returns distributions.

Since, Pearson[10] have investigated the properties of various statistics of skewness, measuring skewness combined with the effects of the role of sample size are becoming more paramount. Several empirical studies have examined the degrees of skewness, power for various samples sizes extracted from different populations and distributions. Tabor[11] has used simulation estimation to investigate different ways to measure skewness. It has ranked eleven different statistics in terms of their power for detecting skewness in samples from populations with varying levels of skewness. It concludes that the students should think creatively far beyond a classroom and see things in an entirely different way about measuring characteristics of distributions.

The purpose of this study is two-fold: First to derive a goodness-of-fit test based on a chi-square statistic using simulation. Second, this study examines the behavior of the values of the χ^2 test statistic with the level of skewness for two different distributions, namely chi-square and inverse Gaussian. To our knowledge, not much empirical study has been exists addressing this relationship. Few studies have derived a goodness-of-fit test based on a chi-square statistic using simulation[12]. This study has supplements the existing literature[13-15] and extends the previous researches of Elobaid[16] who investigated the behavior of the χ^2 test statistic values with the variation of the skewness of Weibull distribution.

MATERIALS AND METHODS

The study uses the following methodology structure. The data from χ^2 and inverse Gaussian distributions were generated via simulation using SAS programs for different sample sizes. The random variable x was generated for χ^2 distribution, which characterized by one parameter that is its degrees of freedom υ and has the probability density function:

$$f(x,\upsilon) = \frac{1}{2^{\frac{\upsilon}{2}}\,\Gamma\frac{\upsilon}{2}}(x)^{\frac{\upsilon}{2}-1}\exp[-(\frac{x}{2})],\ x \geq 0,\ \upsilon > 0 \tag{1}$$

and the cumulative distribution function:

$$F(x,\upsilon) = \frac{1}{\Gamma\frac{\upsilon}{2}}\,y(\frac{\upsilon}{2},\frac{x}{2}) \tag{2}$$

where, γ (s,t) is the lower incomplete gamma function.

Similarly a random variable x was generated for Inverse Gaussian distribution, which has the probability density function:

$$f(x;\mu,\lambda) = (\frac{\lambda}{2\pi x^3})^{1/2}\exp\left[-\lambda(x-\mu)^2/2\mu^2x\right];x \geq 0,\upsilon > 0 \tag{3}$$

where, μ is a measure of location and λ is a reciprocal measure of dispersion. The corresponding cumulative distribution function is given by:

$$F(x;\mu,\lambda) = (\frac{\lambda}{\mu})[1\text{-}(1\text{-}2\mu^2x/\lambda)^{\frac{1}{2}} \tag{4}$$

Figure 1 and 2 show the pdfs for χ^2 and inverse Gaussian distributions, respectively.

For each sample of each distribution, corresponding measure of skewness were calculated. Various degrees of skewness were obtained by changing the degrees of freedom for the χ^2 distribution and by controlling the reciprocal measure of dispersion λ (shape parameter) for the inverse Gausian distribution.

To compute χ^2 test statistics the observations in each sample were classified into intervals. The observed and the expected values were computed. Table 1 and 2 of χ^2 statistic's values versus skewness measures were then constructed for each distribution. Figure 3 show the schematic diagram of the procedures.

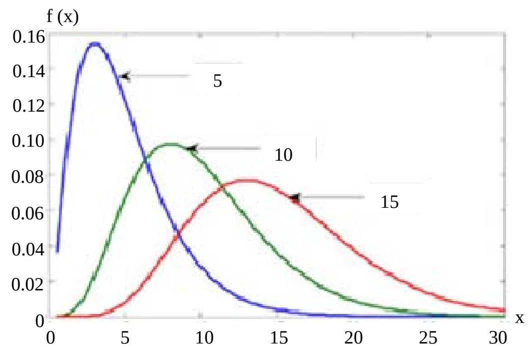

Fig. 1: An example of probability density function of χ^2 distribution using different degrees of freedom

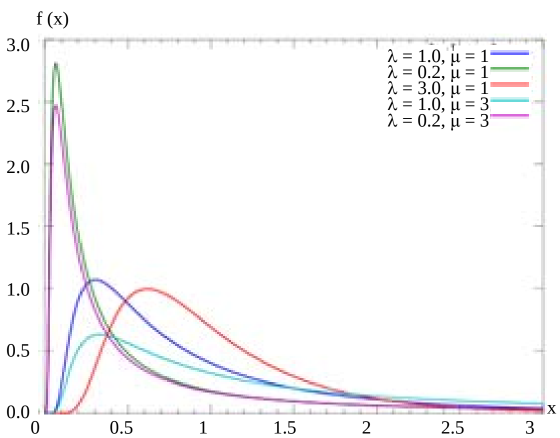

Fig. 2: An example of probability density function of IG distribution using different values of mean μ and shape parameter λ

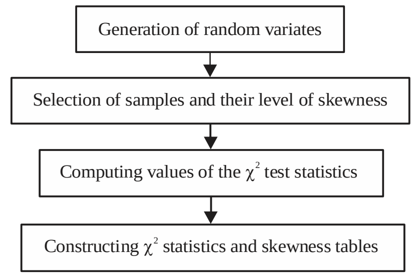

Fig. 3: Schematic representation of the procedures employed to examines the values of the χ^2 test statistic behavior with the level of skewness

Table 1: Behavior of (Sk) and χ^2 test statistics values for χ^2 distribution using three sample sizes and different values of degrees of freedom

Values of df sample size	df = 20	df = 10	df = 5
n = 20			
Sk	0.114	0.874	1.203
χ^2	17.050	7.649	2.818
n = 50			
Sk	0.113	0.669	1.243
χ^2	15.777	9.346	4.014
n = 100			
Sk	0.354	0.957	1.257
χ^2	11.133	8.500	7.735

Table 2: Behavior of (Sk) and χ^2 test statistics for inverse Gaussian distribution using 3 different sample sizes and different values of shape parameter λ

Values of λ sample size	λ = 32	λ = 8	λ = 0.5
n = 20			
Sk	0.374	0.750	2.917
χ^2	94.719	33.373	4.829
n = 50			
Sk	0.850	1.384	3.378
χ^2	186.618	48.714	5.913
n = 100			
Sk	0.474	0.933	2.358
χ^2	331.854	91.109	18.559

RESULTS AND DISCUSSION

The values of χ^2 test statistic and skewness measures (Sk) were tabulated for the distributions selected. The results given in Table 1 and 2 are for χ^2 and inverse Gaussian distributions, respectively.

Table 1 reports the results of skewness (Sk) and χ^2 test statistics for χ^2 distribution using different sample sizes. The results show that the value of χ^2 test statistic decreases as the value of skewness increases. Using significant level of 0.05 the values of χ^2 statistic shows that distribution of the population from which the data sets was originate is the χ^2 distribution indicating that χ^2 test statistic become more significant and sufficient when the values of skewness measure increase[16].

Similar trend of skewness and χ^2 test statistics results for inverse Gaussian distribution has been observed in Table 2. The results show that with small values of the shape parameter (λ = 8 and λ = 0.5) the values of skewness increase while the χ^2 test statistic values decrease. Moreover, increasing the sample size produces an estimator, which is shown to have asymptotic assumed distribution.

These results indicate that for different sample sizes of right skewed distributions the value of χ^2 test statistics is inversely proportional to the values of Sk. This finding has practical implications in many areas such as in finance. It has very important implications for individual as well as institutional investors. It is important for individual investors who prefer positive skewness[17]. It can also help institutional investors to construct portfolios with high skewness[18-20].

CONCLUSION AND FUTURE RECOMMENDATIONS

In this study, we have investigated the behavior of the values of the χ^2 test statistic with the level of skewness for chi-square and inverse Gaussian distributions. A goodness-of-fit test based on a chi-square statistic was also derived using simulation estimation. Using three different sample sizes for each distribution, the degrees of skewness are considered by changing the degrees of freedom for the χ^2 distribution and controlling the shape parameter for inverse Gausian distribution.

The results shows that χ^2 test statistics is affected by the measurement of skewness, that is to say the value of χ^2 test statistic decreases as the value of skewness increases and vice versa. In addition, the study results show that the method, estimation by simulation, produces an estimator which is shown to have asymptotic assumed distribution with large values of skewness for both χ^2 and inverse Gausian distributions.

These findings are relevant to researcher's wishes to study the implications of shape parameters and skewness measure to assess the validity of how well different distributions fit the data. The study can also help investors to examine skewness in stock returns distribution.

Few studies have derived a goodness-of-fit test based on a chi-square statistic using simulation. This study has supplements the existing literature and extends the previous researches by examining the relationship between χ^2 statistics and skewness measures for chi-square and Inverse Gaussian distributions. To the best of our knowledge, not much literature exists addressing this relationship.

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