Asian Journal of Scientific Research

ISSN 1992-1454

OPEN ACCESS Asian Journal of Scientific Research

ISSN 1992-1454
DOI: 10.3923/ajsr.2018.240.255

Research Article
Maintainability and Capability of the Multi-Server System in Repairing Defective Machine Configurations with Different Probability Distributions

I.M. Hanafy, Mohammed Mohammed El-Genidy and Esraa Ahmed Hebeshy

Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said, Egypt

Abstract
Background and Objective: The efficiency study of the defective machine repairing system is an imperative field in the industrial engineering because it depends on the elapsed time of repairing defective machines in multi-server system configurations. This study seeks to introduce an accurate statistical method for different cases of the multi-server system in order to obtain important statistics such as the failure rate, reliability, availability, maintainability, capability and efficiency of the repairing machine system. **Methodology:** The moment's method and its corresponding software implementation are applied to estimate the parameters of the exponential, Weibull, Gamma and normal distributions. Moreover, conduct a simulation study to generate a dataset of the time of servers in the general case. Finally, Anderson-Darling test is performed to fit datasets with the aforementioned distributions. **Results:** The equations of failure rate, reliability, maintainability, unreliability, availability, nonlinear regression are derived for each stage of the repairing time of the system under consideration and also for the entire system. Thus, in this study, evaluate the capability and efficiency of the system. **Conclusion:** The statistical method, used in this study, shows that it is possible for researchers to find all characteristics of each server in the multi-repairing systems as well as the entire system. As a result, the safety of designing multi-server system will be increased and the economic losses will be reduced.

Key words: Multi-server system, moment's method, Anderson-Darling test, maintainability, reliability, availability, efficiency, capability

Received: August 21, 2017 Accepted: November 16, 2017 Published: March 15, 2018

Citation: I.M. Hanafy, Mohammed Mohammed El-Genidy and Esraa Ahmed Hebeshy, 2018. Maintainability and capability of the multi-server system in repairing defective machine configurations with different probability distributions. Asian J. Sci. Res., 11: 240-255.

Corresponding Author: Mohammed Mohammed El-Genidy, Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said, Egypt Tel: +201124774443/+20663657602 Fax: +20663657601

Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

From the beginning of the industrial revolution until today, the concept of maintenance widely differs such that it becomes a necessity in modern industry. In the past, it entails that a machine will need to fix when it breaks down, otherwise, no maintenance will be required. In other words, in that period, maintenance was simply defined as a way of fixing one's stomach if it is upset. Yet, the concept drastically changes now. The need of maintenance becomes urgent when failure is of large defects in either the design, the process or even a particular part of the machine. Still, human factor errors are also significant, especially in all that is related to the operation conduct.

This is why some researchers attempted to apply the reliability, availability and maintainability (RAM) analysis to process industries by providing a case study of a natural gas processing plant, departing from the reliability simulation until reaching new ideas about complex industrial systems[1]. Presenting a direct proof of the convexity of the long run average cost function, some other researchers sought to resolve the optimization process problem of the industrial runtime[2]. Besides, a review of the maximal queue size with the standard normal distribution of arrival times was carried out in a named study[3] while a different research work adopted basic modeling approaches to check the failure and maintenance data of the repairable systems[4]. The scope of the repairable systems was also investigated to prove that a finite Weibull mixture, with only positive component weights, could be used as an underlying distribution of the time to first failure (TTFF) of the generalized renewal process (GRP) model but only on the condition of the unknown parameters, estimated in the study[5].

In a similar vein, an original maintenance decision was suggested to select the optimal last production stoppage, convenient to operate a maintenance action on a component, in accordance with its degradation conditions. That stoppage was optimal for the criteria of maintainability and reliability, which were the two key elements, bringing together and performing a maintenance action[6]. Later on, some researchers presented an overview of two maintenance techniques, widely discussed in the literature, namely time-based maintenance (TBM) and condition-based maintenance (CBM). They explained how the TBM and CBM techniques would support maintenance decision making[7] and how the act of detecting induction machines at an early stage would prevent breakdowns and costly maintenance[8].

To upgrade productivity, researchers believe that it is essential to improve the performance of the manufacturing systems. A relevant study showed that the required production output was a result of high equipment availability, which was influenced by equipment reliability and maintainability[9]. Another study introduced a technique, based on reliability and maintainability parameters, for effective running of life cycle costing in design and warranty of repairable systems. To hit the goal, the significant life cycle stages of repairable systems were specified and a generalized model of life cycle cost analysis was first proposed[10]. Still, a different study attempted a special repair assumption, called partially perfect repair, assigned for repairable systems with dependent component failures, where only the failed component was repaired to be as good as the new. For this reason, a parametric reliability model was suggested to hold the statistical dependency among different component failures, in which the joint distribution of the latent component failure time was created by copula functions[11].

In the present study, the parameters of Weibull, exponential, Gamma and normal distributions are estimated by the moment's method and the Anderson-Darling goodness-of-fit test to obtain the optimum values of the unknown parameters. As a result, availability, capability and efficiency of the multi-server system are derived. Similarly, maintainability, reliability and failure rate of each server in the system and of the entire system are estimated. Nonlinear regression equations are also introduced to predict the probability values of the elapsed time of each server in the system.

MATERIALS AND METHODS

In this study, the multi-server machine consists of four stages. The first and the fourth are connected in a series while the second and the third are interrelated in a parallel way so as to repair defective machines such that the repairing process includes two parts: (a) An electrical part and (b) A mechanic part (Fig. 1a-d).

The first service is a dismantling process, where a continuous random variable X_1 on the interval $[0, x_1]$ represents the time of the first service while the continuous random variables X_2 on the interval $[0, x_2]$ and X_3 on the interval $[0, x_3]$ stand for the time of the repairing stages, namely the electrical repairing and the mechanical repairing, respectively. Then, the continuous random variable X_4 on the interval $[0, x_4]$ is to label the compile-time of the machine after repairing.

Fig. 1(a-d): Phases of the multi-server system, (a) Dissociation of the machine, (b) Repairing of machine (Electrical), (c) Repairing of machine (Mechanic) and (d) Installation of machine parts

The mutually independent continuous random variables X_1, X_2, X_3 and X_4 have a Weibull distribution with the different scale parameters and shape parameters (λ_1, k_1), (λ_2, k_2), (λ_3, k_3) and (λ_4, k_4), respectively. Therefore, the total time X of the time repairing of the system can be rendered as:

$$X = X_1+\max\{X_2, X_3\}+X_4 = X_1+M+X_4 \quad (1)$$

where, M is a random variable such that $M = \{X_2, X_3\}$.

Figure 1 (a-d) shows the four different stages of the machine repair in the multi-server system, starting from the dissociation stage of the machine, then the electrical and the mechanic repairing stages, ending in the stage of the machine installation.

On the other hand, the probability density functions of the continuous random variables X_1, X_2, X_3 and X_4 can be written us:

$$f(X_1) = \frac{k_1}{\lambda_1}\left(\frac{X_1}{\lambda_1}\right)^{k_1-1} e^{-\left(\frac{X_1}{\lambda_1}\right)^{k_1}}, \; 0 < X_1 < x_1 \quad (2)$$

$$f(X_2) = \frac{k_2}{\lambda_2}\left(\frac{X_2}{\lambda_2}\right)^{k_2-1} e^{-\left(\frac{X_2}{\lambda_2}\right)^{K_2}}, \; 0 < X_2 < x_2 \quad (3)$$

$$f(X_3) = \frac{k_3}{\lambda_3}\left(\frac{X_3}{\lambda_3}\right)^{k_3-1} e^{-\left(\frac{X_3}{\lambda_3}\right)^{K_3}}, \; 0 < X_3 < x_3 \quad (4)$$

and:

$$f(X_4) = \frac{k_4}{\lambda_4}\left(\frac{X_4}{\lambda_4}\right)^{k_4-1} e^{-\left(\frac{X_4}{\lambda_4}\right)^{K_4}}, \; 0 < X_4 < x_4 \quad (5)$$

where, λ, $k > 0$ such that x_1, x_2, x_3 and x_4 are the values of X_1, X_2, X_3 and X_4, respectively. Therefore, the joint distribution density function F_x (x) is obtained as:

$$F_X(x) = p\,(X \le x) = p((X_1 + M + X_4) \le x) \quad (6)$$

where, $x = x_1+m+x_4$ and $m = \max\{x_2, x_3\}$.

On the other hand, the mutually independent continuous random variables X_1, X_2, X_3 and X_4 have a Weibull distribution and the joint probability distribution is to function:

$$\begin{bmatrix} F_X(x) = p(X_1 \le x_1).p(M \le m).p(X_4 \le x_4) \\ \quad = \int_0^{x_1} f(X_1)dX_1 \,.\, \int_0^{m} f(M)dM \,.\, \int_0^{x_4} f(X_4)dX_4 \end{bmatrix} \quad (7)$$

Let M = max{X_2, X_3} = X_2, then:

$$\begin{bmatrix} F_X(x) = \frac{k_1}{\lambda_1}\int_0^{x_1}\left(\frac{X_1}{\lambda_1}\right)^{k_1-1} e^{-\left(\frac{X_1}{\lambda_1}\right)^{k_1}} dX_1.\frac{k_2}{\lambda_2}\int_0^{x_2}\left(\frac{X_2}{\lambda_2}\right)^{k_2-1} e^{-\left(\frac{X_2}{\lambda_2}\right)^{k_2}} dX_2. \\ \frac{k_4}{\lambda_4}\int_0^{x_4}\left(\frac{X_4}{\lambda_4}\right)^{k_4-1} e^{-\left(\frac{X_4}{\lambda_4}\right)^{k_4}} dX_4 \end{bmatrix} \quad (8)$$

Reliability: It is the probability that a system will be successful in the interval from time 0 to time x and where no failures or repairs are allowed:

$$R(x) = p\,(X>x), \ \ x \ge 0 \quad (9)$$

where, X is a random variable, denoting failure time and the reliability is the product of different reliabilities terms as follow:

$$R = R_x \cdot R_y \cdot R_z \quad (10)$$

Unreliability F(x): It is a measure of failure, defined as the probability that the system will fail per time x:

$$F(x) = p\,(X \le x) = 1\text{-}R\,(x),\ x \ge 0 \quad (11)$$

The unreliability F(x) is thus the failure distribution function. If the time to failure random variable X has a density function f(x), then:

$$F(x) = \int_0^x f(X)dX \quad (12)$$

$$R(x) = \int_x^\infty f(y)dy \quad (13)$$

$$f(x) = \frac{-d}{dx}[R(t)] \quad (14)$$

Failure rate: It is the rate, at which failures occur in certain time intervals [t_1, t_2], often expressed as:

$$\frac{R(t_1) - R(t_2)}{(t_2 - t_1)\,R(t_1)} \quad (15)$$

Hazard function: It is the limit of the failure rate as the interval approaches zero. If the interval is redefined as [t, t+Δt], then:

$$\begin{bmatrix} h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{R(t).\Delta t} = \frac{1}{R(t)} \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{\Delta t} \\ \quad = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} \end{bmatrix} \quad (16)$$

Maintainability: It is the probability that the failed system will be back in service by time t. If the repair time T has a repair time density function (t), then:

$$V(t) = p(T \le t) = \int_0^t g(s)\,ds \quad (17)$$

where, T denotes the random variable of the time to repair or the total down time.

Availability: It is the probability that the system is successful at time t, used for repairable systems or as a measure that allows a system to repair when failure occurs. Mathematically, it is defined as:

$$\begin{bmatrix} A(t) = \frac{\text{System up time}}{\text{System up time} + \text{System down time}} \\ \quad = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \end{bmatrix} \quad (18)$$

where, MTTF is the system mean time to failure and MTTR is the system mean time to repair or the system mean down time. In this regard, it can be rewritten as:

$$\text{MTBF} = \text{MTTF} + \text{MTTR}$$

where, MTBF is the system mean time between failures. This implies that the system has failed and it has been repaired. Besides, availability may be the product of many different terms such as:

$$A = A_x \,.\, A_y \,.\, A_z \quad (19)$$

For non-repairable systems, availability A(t) equals reliability R(t). In repairable systems, the availability A(t) will be equal to or greater than R(t).

Utilization is the ratio of time, spent on productive efforts, in comparison with the total time consumed[1]:

$$\text{Utilization } = \frac{\text{Average of system up time}}{\text{Average of system lost time} + \text{Average of system up time}} \tag{20}$$

Capability: It deals with the productive output, compared with inherent productive output, which is a measure of how well the production activity is performed, compared with the datum this way:

$$\text{Capability} = \text{Utilization} \times \text{Efficiency} \tag{21}$$

where the efficiency measures the productive work output versus the work input.

Method of moments: The procedure of applying the method of moments can be described in the following steps:

- Calculate the moments of population:

$$\overline{M_j} = T(x^j) = \int x^j f(x,\theta)\, dx,\ (j = 1, 2, 3, \ldots, r) \text{ with parameter } \theta \tag{22}$$

- Calculate the moments of the sample:

$$\overline{m_j} = \frac{\sum x^j}{n},\ (j = 1, 2, 3, \ldots, r) \tag{23}$$

where, n is the size of the sample.

- Equalize the moments of population and the moments of the sample $(\overline{M_j} = \overline{m_j})$ to obtain a value of θ (the estimate of the parameter θ)

Hence, the moment generating function is defined as:

$$M(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx \tag{24}$$

Let:

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k},\ x \geq 0 \tag{25}$$

If k>0 is the shape parameter and λ>0 is the scale parameter of the Weibull distribution, then:

$$M(t) = \int_0^{\infty} e^{tx} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} dx$$

Let:

$$y = \left(\frac{x}{\lambda}\right)^k \to y^{\frac{1}{k}} = \frac{x}{\lambda} \to x = \lambda y^{\frac{1}{k}} \to dx = \lambda \frac{1}{k} y^{\frac{1}{k}-1} dy$$

then:

$$\begin{aligned} M(t) &= \int_0^{\infty} e^{t\lambda y^{\frac{1}{k}}} \frac{k}{\lambda} y^{1-\frac{1}{k}} e^{-y} \frac{\lambda}{k} y^{\frac{1}{k}-1} dy = \int_0^{\infty} e^{t\lambda y^{\frac{1}{k}}} e^{-y} dy \\ &= \int_0^{\infty} e^{-y} dy + \frac{t\lambda}{1!}\int_0^{\infty} y^{\frac{1}{k}} e^{-y} dy + \frac{t^2\lambda^2}{2!}\int_0^{\infty} y^{\frac{2}{k}} e^{-y} dy + \frac{t^3\lambda^3}{3!}\int_0^{\infty} y^{\frac{3}{k}} e^{-y} dy + \cdots \\ &= 1 + \frac{t\lambda}{1!}\Gamma\left(1+\frac{1}{k}\right) + \frac{t^2\lambda^2}{2!}\Gamma\left(1+\frac{2}{k}\right) + \frac{t^3\lambda^3}{3!}\Gamma\left(1+\frac{3}{k}\right) + \cdots \\ &= \sum_{n=0}^{\infty} \frac{t^n\lambda^n}{n!}\Gamma\left(1+\frac{n}{k}\right) \end{aligned} \tag{26}$$

Similarly, the first moment around zero will be:

$$\text{Mean} = E(x) = M'(0) = \lambda\Gamma\left(1+\frac{1}{k}\right) \tag{27}$$

and the second moment around zero will be represented by the following formula:

$$E(x^2) = M''(0) = \lambda^2\, \Gamma\left(1+\frac{2}{k}\right) \tag{28}$$

Finally, it will read:

$$\begin{aligned} \text{Variance } &= E(x^2) - (E(x))^2 = M''(0) - (M'(0))^2 \\ &= \lambda^2\left[\Gamma\left(1+\frac{2}{K}\right) - \left(\Gamma\left(1+\frac{1}{K}\right)\right)^2\right] \end{aligned} \tag{29}$$

The parameters of Weibull, exponential, Gamma and normal distributions are evaluated by the moment's method, software and Anderson-Darling goodness-of-fit test to find the optimum estimations.

RESULTS AND DISCUSSION

In this study, the multi-server system of repairing defective machines is presented, whereby servers are connected in a serial structure, a parallel structure or a combination of both in the same system. Compared to previous studies, where only reliability analysis is of two unit cold standby repairable system, where a k-out-of-n system is introduced[12,13], the present study manages to obtain the reliability, maintainability and failure rate of each server and of the entire system at the same time. Moreover, the probabilistic analysis of a serial-parallel repairable system, stability analysis of n-unit serial-parallel system and a study of the reliability and the failure rate resulted from comparing BIT and non-BIT maintenance cycles are all considered in some significant research papers[14-16]. Unlike what has been done in a previous study, where only the availability modeling of the repairable systems was considered[17], here we derive the availability, capability and efficiency of the multi-server system. We also apply the Weibull, exponential, Gamma and normal distributions to the multi-server system, where Anderson-Darling goodness-of-fit test is operated to detect the optimum estimations of their significant parameters. On the contrary, the previous studies[18,19] satisfied themselves with the multi-queue system of the repairing time for the defective machines under the normal distribution and the estimation of the system's performance under Pareto distribution.

Simulation study: The Mathematica software (Mathematica 4, version number 4.0.1.0, Wolfram Research, Inc. 100 Trade Center Drive Champaign, IL 61820-7237 USA) is performed to generate the maximum values of X_1, X_2 and X_4, using the following command Random [Real, 10]:

x_1 = 3.2742003794474304
x_2 = 7.0414845931166390
x_4 = 9.3756707581605900

where, $x_1 < x_2 < x_4$.

Then, generate the sample of size 10 for each of the values of X_1, X_2 and X_4 by the command:

Table [Random [Real, {0, 3.2742003794474304}], {10}]

x_1 = 0.111636, 2.64264, 0.948226, 2.80813, 0.31546, 1.08563, 2.18926, 1.80423, 0.303384, 1.29565

Table [Random [Real, {3.2742003794474304, 7.041484593116639 }], {10}]

x_2 = 6.55715, 5.17854, 4.89542, 4.26829, 4.82385, 5.85769, 6.82149, 4.57258, 5.14641, 5.90915

Table [Random [Real, {7.041484593116639, 9.37567075816059 }], {10}]

x_4 = 7.46072, 7.08169, 8.89207, 8.81981, 7.38114, 7.53193, 8.21608, 9.15208, 7.15624, 9.09217

Estimating the parameters: To estimate the scale parameter λ and the shape parameter k of the distribution, list the above dataset of the generating values of random variables X_1, X_2 and X_4 in Table 1 such that $x_1 = x$, $y = x_2 - x_1$ and $z = x_2 - x_4$.

If x represents the time of the machine dissociation, then y is the time of the machine repairing and z is the time of the installation of the machine parts:

$$\text{Mean} = \lambda_1 \Gamma\left(1+\frac{1}{k_1}\right) = 1.350425 \tag{30}$$

$$\text{Variance} = \lambda_1^2\left[\Gamma\left(1+\frac{2}{k_1}\right) - \left(\Gamma\left(1+\frac{1}{k_1}\right)\right)^2\right] = 0.960137 \tag{31}$$

The Eq. 30 and 31 can be solved by the Mathematica software on the computer with this command:

$$\text{Find root}\left[\left(\frac{1.350425}{\text{Gamma}\left[1+\frac{1}{k_1}\right]}\right)^2\left(\text{Gamma}\left[1+\frac{2}{k_1}\right] - \text{Gamma}\left[1+\frac{1}{k_1}\right]^2\right) == 0.960137, \{k_1, 0.2\}\right]$$

Then, $k_1 = 1.39617$, $\lambda_1 = 1.48103$:

$$\text{Mean} = \lambda_2 \Gamma\left(1+\frac{1}{k_2}\right) = 4.052632 \tag{32}$$

$$\text{Variance} = \lambda_2^2\left[\Gamma\left(1+\frac{2}{k_2}\right) - \left(\Gamma\left(1+\frac{1}{k_2}\right)\right)^2\right] = 2.045366 \tag{33}$$

Likewise, the Eq. 32 and 33 can be solved by the running Mathematica software on the computer with the command:

Table 1: Time values of the four stages of the multi-server system

x	y	z
0.111636	6.445514	0.90357
2.642640	2.535900	1.90315
0.948226	3.947194	3.99665
2.808130	1.460160	4.55152
0.315460	4.508390	2.55729
1.085630	4.772060	1.67424
2.189260	4.632230	1.39459
1.804230	2.768350	4.57950
0.303384	4.843026	2.00983
1.295650	4.613500	3.18302

$$\text{Find root}\left[\left(\frac{4.052632}{\text{Gamma}\left[1+\frac{1}{k_2}\right]}\right)^2\left(\text{Gamma}\left[1+\frac{2}{k_2}\right]-\text{Gamma}\left[1+\frac{1}{k_2}\right]^2\right)==2.045366,\{k_2,\ 0.3\}\right]$$

Then, $k_2 = 3.10018$, $\lambda_2 = 4.53159$:

$$\text{Mean} = \lambda_4\Gamma\left(1+\frac{1}{k_4}\right) = 2.675336 \quad (34)$$

$$\text{Variance} = \lambda_4^2\left[\Gamma\left(1+\frac{2}{k_4}\right)-\left(\Gamma\left(1+\frac{1}{k_4}\right)\right)^2\right] = 1.776031 \quad (35)$$

The Eq. 34 and 35 can be solved by the running Mathematica software on the computer with the command:

$$\text{Find root}\left[\left(\frac{2.675336}{\text{Gamma}\left[1+\frac{1}{k_4}\right]}\right)^2\left(\text{Gamma}\left[1+\frac{2}{k_4}\right]-\text{Gamma}\left[1+\frac{1}{k_4}\right]^2\right)==1.776031,\{k_4,\ 0.3\}\right]$$

Then, $k_4 = 2.11011$, $\lambda_4 = 3.0207$.
The values of $F_{X,Y,Z}(x, y, z)$ are computed as:

$F_{X,Y,Z}$ (0.111636, 6.445514, 0.90357):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{0.111636}\left(\frac{X_1}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_1}{1.48103}\right)^{1.39617}} dX_1\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{6.445514}\left(\frac{Y_1}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_1}{4.53159}\right)^{3.10018}} dY_1\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{0.90357}\left(\frac{Z_1}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_1}{3.0207}\right)^{2.11011}} dZ_1\right) = 0.00191007\end{array}\right] \quad (36)$$

$F_{X,Y,Z}$ (2.64264, 2.53591, 1.90315):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{2.64264}\left(\frac{X_2}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_2}{1.48103}\right)^{1.39617}} dX_2\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{2.5359}\left(\frac{Y_2}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_2}{4.53159}\right)^{3.10018}} dY_2\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{1.90315}\left(\frac{Z_2}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_2}{3.0207}\right)^{2.11011}} dZ_2\right) = 0.0428165\end{array}\right] \quad (37)$$

$F_{X,Y,Z}$ (0.948226, 3.947194, 3.99665):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{0.948226}\left(\frac{X_3}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_3}{1.48103}\right)^{1.39617}} dX_3\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{3.97194}\left(\frac{Y_3}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_3}{4.53159}\right)^{3.10018}} dY_3\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{3.99665}\left(\frac{Z_3}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_3}{3.0207}\right)^{2.11011}} dZ_3\right) = 0.166163\end{array}\right] \quad (38)$$

$F_{X,Y,Z}$ (2.80813, 1.46016, 4.55152):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{2.80812}\left(\frac{X_4}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_4}{1.48103}\right)^{1.39617}} dX_4\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{1.46016}\left(\frac{Y_4}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_4}{4.53159}\right)^{3.10018}} dY_4\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{4.55152}\left(\frac{Z_4}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_4}{3.0207}\right)^{2.11011}} dZ_4\right) = 0.0243689\end{array}\right] \quad (39)$$

$F_{X,Y,Z}$ (0.31546, 4.50839, 2.55729):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{0.31546}\left(\frac{X_5}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_5}{1.48103}\right)^{1.39617}} dX_5\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{4.50839}\left(\frac{Y_5}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_5}{4.53159}\right)^{3.10018}} dY_5\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{2.55729}\left(\frac{Z_5}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_5}{3.0207}\right)^{2.11011}} dZ_5\right) = 0.0344938\end{array}\right] \quad (40)$$

$F_{X,Y,Z}$ (1.08563, 4.77206, 1.67424):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{1.08563}\left(\frac{X_6}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_6}{1.48103}\right)^{1.39617}} dX_6\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{4.77206}\left(\frac{Y_6}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_6}{4.53159}\right)^{3.10018}} dY_6\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{1.67424}\left(\frac{Z_6}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_6}{3.0207}\right)^{2.11011}} dZ_6\right) = 0.082427\end{array}\right] \quad (41)$$

$F_{X,Y,Z}$ (2.18926, 4.63223, 1.39459):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{2.18926}\left(\frac{X_7}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_7}{1.48103}\right)^{1.39617}} dX_7\right). \\ \left(\frac{3.10018}{4.53159}\int_0^{4.63223}\left(\frac{Y_7}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_7}{4.53159}\right)^{3.10018}} dY_7\right). \\ \left(\frac{2.11011}{3.0207}\int_0^{1.39459}\left(\frac{Z_7}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_7}{3.0207}\right)^{2.11011}} dZ_7\right) = 0.0960329\end{array}\right] \quad (42)$$

$F_{X,Y,Z}$ (1.80423, 2.76835, 4.5795):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{1.80423}\left(\frac{X_8}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_8}{1.48103}\right)^{1.39617}} dX_8\right).\\ \left(\frac{3.10018}{4.53159}\int_0^{2.76835}\left(\frac{Y_8}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_8}{4.53159}\right)^{3.10018}} dY_8\right).\\ \left(\frac{2.11011}{3.0207}\int_0^{4.5795}\left(\frac{Z_8}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_8}{3.0207}\right)^{2.11011}} dZ_8\right) = 0.129946\end{array}\right] \tag{43}$$

$F_{X,Y,Z}$ (0.303384, 4.843026, 2.00983):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{0.303384}\left(\frac{X_9}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_9}{1.48103}\right)^{1.39617}} dX_9\right).\\ \left(\frac{3.10018}{4.53159}\int_0^{4.843026}\left(\frac{Y_9}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_9}{4.53159}\right)^{3.10018}} dY_9\right).\\ \left(\frac{2.11011}{3.0207}\int_0^{2.00983}\left(\frac{Z_9}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_9}{3.0207}\right)^{2.11011}} dZ_9\right) = 0.0252759\end{array}\right] \tag{44}$$

$F_{X,Y,Z}$ (1.29565, 4.6135, 3.18302):

$$\left[\begin{array}{l}\left(\frac{1.39617}{1.48103}\int_0^{1.29565}\left(\frac{X_{10}}{1.48103}\right)^{1.39617-1} e^{-\left(\frac{X_{10}}{1.48103}\right)^{1.39617}} dX_{10}\right).\\ \left(\frac{3.10018}{4.53159}\int_0^{4.6135}\left(\frac{Y_{10}}{4.53159}\right)^{3.10018-1} e^{-\left(\frac{Y_{10}}{4.53159}\right)^{3.10018}} dY_{10}\right).\\ \left(\frac{2.11011}{3.0207}\int_0^{3.18302}\left(\frac{Z_{10}}{3.0207}\right)^{2.11011-1} e^{-\left(\frac{Z_{10}}{3.0207}\right)^{2.11011}} dZ_{10}\right) = 0.247483\end{array}\right] \tag{45}$$

$$F(x) = 1 - e^{-\left(x/\lambda_1\right)^{k1}} = 1 - e^{-\left(x/1.48103\right)^{1.39617}} \tag{46}$$

$$F(y) = 1 - e^{-\left(y/\lambda_2\right)^{k2}} = 1 - e^{-\left(y/4.53159\right)^{3.10018}} \tag{47}$$

$$F(z) = 1 - e^{-\left(z/\lambda_3\right)^{k3}} = 1 - e^{-\left(z/3.0207\right)^{2.11011}} \tag{48}$$

$$R(x) = e^{-\left(x/\lambda_1\right)^{k1}} = e^{-\left(x/1.48103\right)^{1.39617}} \tag{49}$$

$$R(y) = e^{-\left(y/\lambda_2\right)^{k2}} = e^{-\left(y/4.53159\right)^{3.10018}} \tag{50}$$

$$R(z) = e^{-\left(z/\lambda_3\right)^{k3}} = e^{-\left(z/3.0207\right)^{2.11011}} \tag{51}$$

As a result, the characteristics of the multi-server system in Fig. 1(a-d) at the different stages are summarized in Table 2-6 and Fig. 2-7.

Table 2: Values of the probability density function, cumulative distribution function, reliability and failure rate of the time x at the first stage of the multi-server system

f(x)	F(x)	R(x)	h(x)
0.329470	0.026704	0.973296	0.338510
0.125681	0.894009	0.105991	1.185770
0.461982	0.415251	0.584749	0.790052
0.105549	0.913104	0.086896	1.214655
0.455166	0.109014	0.890986	0.510857
0.435960	0.476992	0.523008	0.833563
0.195946	0.821960	0.178040	1.100573
0.273044	0.732148	0.267852	1.019384
0.450934	0.103542	0.896458	0.503017
0.389975	0.563815	0.436185	0.894059

Table 3: Values of the probability distribution function, cumulative distribution function, reliability and failure rate of the time at the second and third stages of the multi-server system

f(y)	F(y)	R(y)	h(y)
0.072759	0.949253	0.050747	1.433765
0.171330	0.152398	0.847602	0.202135
0.266770	0.478887	0.521113	0.511924
0.061545	0.029424	0.970576	0.063411
0.252939	0.626267	0.373733	0.676791
0.235774	0.690828	0.309172	0.762598
0.245625	0.657152	0.342848	0.716425
0.195610	0.195074	0.804926	0.243016
0.230191	0.707364	0.292636	0.786612
0.246820	0.652541	0.347459	0.710357

Table 4: Values of the probability density function f(z), cumulative distribution function F(z), reliability R(z) and failure rate h(z) of the time of the fourth stage z of the multi-server system

f(z)	F(z)	R(z)	h(z)
0.169167	0.075352	0.924648	0.182953
0.286833	0.314261	0.685739	0.418283
0.156716	0.835586	0.164414	0.953179
0.102402	0.907005	0.092995	1.101161
0.287275	0.505243	0.494757	0.580639
0.272061	0.250143	0.749857	0.362817
0.243534	0.177788	0.822212	0.296194
0.099963	0.909836	0.090164	1.108679
0.291032	0.345100	0.654900	0.444392
0.242338	0.672667	0.327333	0.740341

Table 5: Values of the joint probability distribution function, joint cumulative distribution function, reliability and failure rate of the total time of the multi-server system

f(x, y, z)	F(x, y, z)	R(x, y, z)	h(x, y, z)
0.004055	0.001910	0.045670	0.088795
0.006176	0.042817	0.061606	0.100256
0.019314	0.166163	0.050100	0.385510
0.000665	0.024369	0.007843	0.084814
0.033074	0.034494	0.164750	0.200752
0.027965	0.082427	0.121251	0.230633
0.011721	0.096033	0.050188	0.233542
0.005339	0.129946	0.019439	0.274649
0.030209	0.025276	0.171804	0.175837
0.023326	0.247483	0.049609	0.470191

Figure 2 presents the relation between the dissociation's time of the machine "x" and the cumulative distribution function "F(x)" at the first stage of the multi-server system.

Figure 3 provides the relation between the dissociation's time of the machine "x" and the reliability function "R(x)" at the first stage of the multi-server system.

Figure 4 presents the relation between the repairing time of the machine "y" and the cumulative distribution function "F(y)" at the second and third stages of the multi-server system. Figure 5 provides the relation between the repairing time of the machine "y" and the reliability function "R(y)" at the second and third stages of the multi-server system.

Table 6: Values of the lost time of the four stages of the multi-server system

Lost time x	Lost time y	Lost time z
0.0487635	0.876075	0.159620
0.0222204	0.386766	0.216278
0.0487365	0.472721	0.096398
0.0087198	0.270785	0.158316
0.0372085	0.359596	0.170749
0.0126956	0.374318	0.252179
0.0290181	0.998537	0.107770
0.0279260	0.151431	0.079838
0.0375909	0.522491	0.285872
0.0295097	0.380102	0.137919

Figure 6 presents the relation between the installation's time of the machine "z" and the cumulative distribution function "F(z)" at the fourth stage of the multi-server system.

Figure 7 provides the relation between the installation's time of the machine "z" and the reliability function "R(z)" at the fourth stage of the multi-server system.

Estimation of the scale parameter λ and the shape parameter k of the lost time of the three different stages will be as follows:

$$\lambda_1 = \frac{0.0302389}{\Gamma\left[1+\frac{1}{k_1}\right]} \tag{52}$$

$$\text{Variance} = \left(\frac{0.0302389}{\Gamma\left[1+\frac{1}{k_1}\right]}\right)^2 \left(\Gamma\left[1+\frac{2}{k_1}\right] - \Gamma\left[1+\frac{1}{k_1}\right]^2\right) = 0.00018116 \tag{53}$$

Solving Eq. 52 and 53 is as follows:

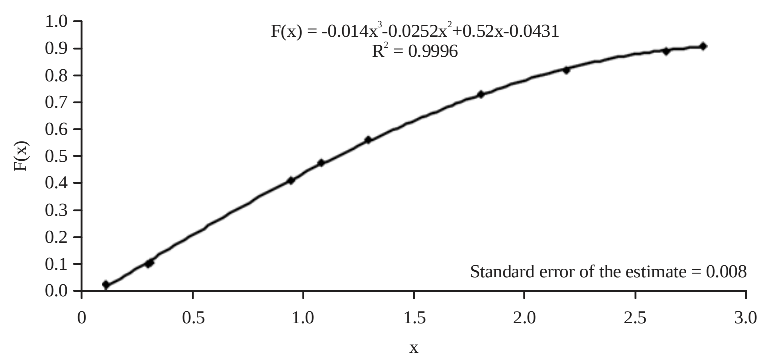

Fig. 2: Cumulative distribution function F(x) at the first stage

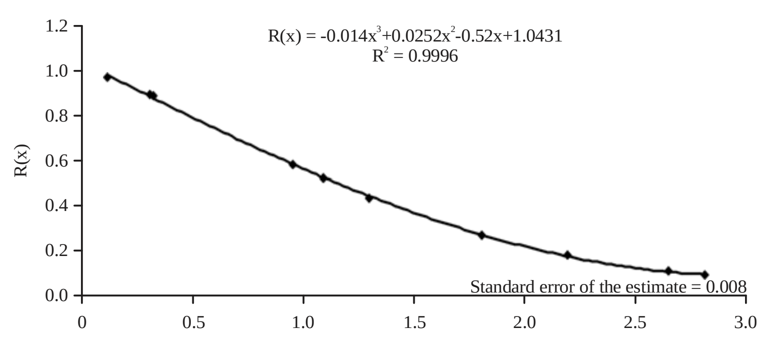

Fig. 3: Reliability R(x) at the first stage

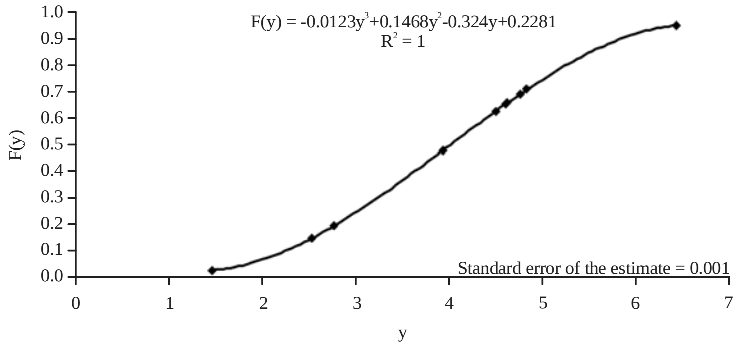

Fig. 4: Cumulative distribution function F(y) at the second and third stages

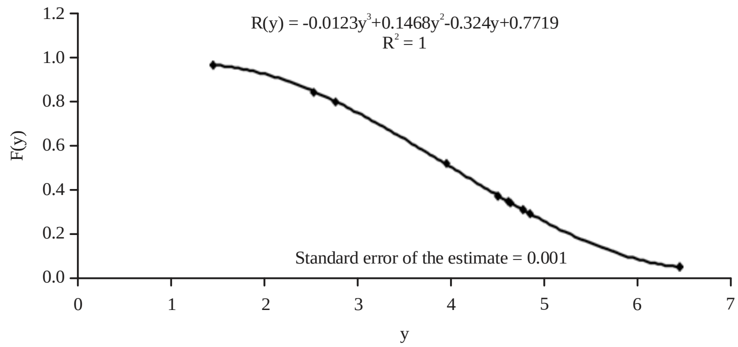

Fig. 5: Reliability R(y) at the second and third stages

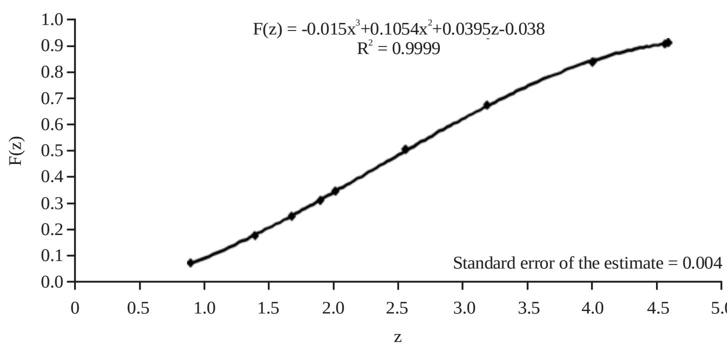

Fig. 6: Cumulative distribution function F(z) at the fourth stage

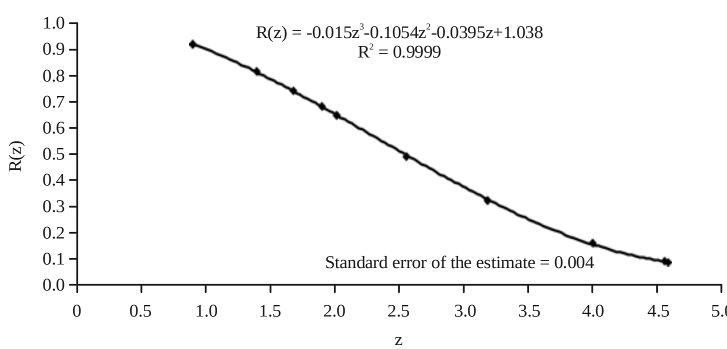

Fig. 7: Reliability R(z) at the fourth stage

$$\text{Find root}\left[\left(\frac{0.0302389}{\text{Gamma}\left[1+\frac{1}{k_1}\right]}\right)^2\left(\text{Gamma}\left[1+\frac{2}{k_1}\right]-\text{Gamma}\left[1+\frac{1}{k_1}\right]^2\right)==0.00018116,\{k_1,\ 0.3\}\right]$$

Then: $k_1 = 2.39114$, $\lambda_1 = 0.0341134$:

$$\lambda_2 = \frac{0.4792822}{\Gamma\left[1+\frac{1}{k_2}\right]} \quad (54)$$

$$\text{Variance} = \left(\frac{0.4792822}{\Gamma\left[1+\frac{1}{k_2}\right]}\right)^2\left(\Gamma\left[1+\frac{2}{k_2}\right]-\Gamma\left[1+\frac{1}{k_2}\right]^2\right)=0.06929732 \quad (55)$$

Similarly, from Eq. 54, 55, 56 and 57, we obtain:

$$\text{Find root}\left[\left(\frac{0.4792822}{\text{Gamma}\left[1+\frac{1}{k_2}\right]}\right)^2\left(\text{Gamma}\left[1+\frac{2}{k_2}\right]-\text{Gamma}\left[1+\frac{1}{k_2}\right]^2\right)==0.06929732,\{k_2,\ 0.3\}\right]$$

Then: $k_2 = 1.89313$, $\lambda_2 = 0.540056$:

$$\lambda_4 = \frac{0.14508239}{\Gamma\left[1+\frac{1}{k_4}\right]} \quad (56)$$

$$\text{Variance} = \left(\frac{0.14508239}{\Gamma\left[1+\frac{1}{k_4}\right]}\right)^2\left(\Gamma\left[1+\frac{2}{k_4}\right]-\Gamma\left[1+\frac{1}{k_4}\right]^2\right)=0.00675924 \quad (57)$$

$$\text{Find root}\left[\left(\frac{0.14508239}{\text{Gamma}\left[1+\frac{1}{k_4}\right]}\right)^2\left(\text{Gamma}\left[1+\frac{2}{k_4}\right]-\text{Gamma}\left[1+\frac{1}{k_4}\right]^2\right)==0.00675924,\{k_4,\ 0.3\}\right]$$

Then: $k_4 = 1.82877$, $\lambda_4 = 0.163263$:

Then, the maintainability V(x), V(y) and V(z) are derived as follows (Table 7):

$$V(x) = \int_0^{\text{lost time }(x)} \frac{2.39114}{0.0341134}\left(\frac{x}{0.0341134}\right)^{2.39114-1} e^{-\left(\frac{x}{0.0341134}\right)^{2.39114}} dx \quad (58)$$

$$V(y) = \int_0^{\text{lost time }(y)} \frac{1.89313}{0.540056}\left(\frac{y}{0.540056}\right)^{1.89313-1} e^{-\left(\frac{y}{0.540056}\right)^{1.89313}} dy \quad (59)$$

$$V(z) = \int_0^{\text{lost time }(z)} \frac{1.82877}{0.163263}\left(\frac{z}{0.163263}\right)^{1.82877-1} e^{-\left(\frac{z}{0.163263}\right)^{1.82877}} dz \quad (60)$$

Table 7: Values of the maintainability of the four stages of the multi-server system

V(x)	V(y)	V(z)
0.904613	0.917826	0.616943
0.301475	0.412285	0.812202
0.904316	0.540292	0.317193
0.037597	0.237119	0.611431
0.707941	0.370639	0.662248
0.089801	0.393223	0.890811
0.492986	0.959290	0.373654
0.461886	0.086131	0.236847
0.716703	0.609107	0.938305
0.506906	0.402087	0.520276

Table 8: Values of the uptime of the four stages of the multi-server system

Uptime x	Uptime y	Uptime z
0.0628725	5.569439	0.743950
2.6204196	2.149134	1.686872
0.8994895	3.474473	3.900252
2.7994102	1.189375	4.393204
0.2782515	4.148794	2.386541
1.0729344	4.397742	1.422061
2.1602419	3.633693	1.286820
1.7763040	2.616919	4.499662
0.2657931	4.320535	1.723958
1.2661403	4.233398	3.045101

Table 9: Values of the availability of the four stages of the multi-server system

A(x)	A(y)	A(z)
0.563191981	0.864079886	0.823345175
0.991591590	0.847483734	0.886357880
0.948602443	0.880238721	0.975880250
0.996894795	0.814551145	0.965216895
0.882050022	0.920238489	0.933230490
0.988305776	0.921560500	0.849377031
0.986745247	0.784437085	0.922722807
0.984521929	0.945299185	0.982566241
0.876094652	0.892114765	0.857763094
0.977224019	0.917610924	0.956670395

If uptime = Total time-Lost time, then evaluating uptimes of x, y and z are as indicated in Table 8.

Therefore, the availability of the three stages x, y and z is computed in Table 9.

Thus, the maintainability and availability of the entire system are derived in Table 10.

Then, some characteristics of the system are derived as follows:

Utilization = 91.63%
Efficiency = 77.55%
Capability = 71.06%

Anderson-darling (A-D) test for goodness-of-fit: The Anderson-Darling test is used to check whether a sample of data comes from a population with a specific distribution and

Table 10: Values of the maintainability and availability of the total time of the multi-server system

Lost time	Uptime	Maintainability	Availability
1.0844585	6.3762615	0.512233788	0.400675053
0.6252644	6.4564256	0.100951527	0.744857708
0.6178557	8.2742143	0.154978819	0.814856692
0.4378208	8.3819892	0.005450870	0.783777157
0.5675535	6.8135865	0.173767613	0.757499810
0.6391926	6.8927374	0.031456051	0.773598641
1.1353251	7.0807549	0.176707157	0.714223961
0.2591949	8.8928851	0.009422359	0.914442740
0.8459539	6.3102861	0.409615935	0.670407884
0.5475307	8.5446393	0.106042817	0.857857283

Table 11: Anderson-Darling test of Weibull distribution for the time of the four stages of the multi-server system

		x	y	z
Significance level α	Critical values of c_α	A* = 0.42037789	A* = 0.506210237	A* = 0.27550427
0.25	0.474	Accept	Reject	Accept
0.10	0.637	Accept	Accept	Accept
0.05	0.757	Accept	Accept	Accept
0.025	0.877	Accept	Accept	Accept
0.01	1.038	Accept	Accept	Accept

Table 12: Anderson-darling test of Weibull distribution for the lost time of the four stages of the multi-server system

		x	y	z
Significance level α	Critical values of c_α	A* = 0.294094825	A* = 0.523951251	A* = 0.89174911
0.25	0.474	Accept	Reject	Reject
0.10	0.637	Accept	Accept	Reject
0.05	0.757	Accept	Accept	Reject
0.025	0.877	Accept	Accept	Reject
0.01	1.038	Accept	Accept	Accept

whether it makes use of the specific distribution in calculating critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution.

The Anderson-darling test is defined as:

- **H_0:** The data follows the Weibull distribution f(x) in Eq. 25
- **H_a:** The data does not follow the Weibull distribution f(x) in Eq. 25

Test statistic: The Anderson-Darling test statistic is defined as:

$$A^2 = -\sum_{i=1}^{n}[(2i-1)\{\ln F_X(x_i) + \ln[1 - F_X(x_{n+1-i})]\}/n] - n \quad (61)$$

The adjusted A-D statistic of Weibull distribution is given by:

$$A^* = A^2\left(1 + \frac{0.2}{\sqrt{n}}\right) \quad (62)$$

The values of the Anderson-Darling statistic for Weibull distribution of the three stages x, y, z and their lost times are listed in Table 11 and 12.

Figure 8 provides an algorithm, summarizing the statistical method used in this study to derive the failure rate, cumulative distribution function and reliability.

Let T_1 be a lifetime of the series system (Minimum lifetime) and T_n be a lifetime of the parallel system (Maximum lifetime). The values of the dataset of size 50 for t_1, t_2 and t_3 are generated by the command: Table [Random[Real, {0, 0.25 }], {50}] (for the corresponding results, Table 13).

Similarly, the values of the dataset of size 50 for $F(t_1)$, $F(t_2)$ and $F(t_3)$ are generated by the command: Table[Random[Real, {0.02, 1 }], {50}] (Table 14).

The SPSS 16.0 (IBM SPSS software, New York, USA) is performed to obtain the values of the parameters of the nonlinear regression equations $F(t_1)$, shown in Table 15.

The adjust R squared has the same meaning of a. R^2.

Similarly, the values of the parameters of $F(t_2)$ and $F(t_3)$ are derived in Table 16 and 17.

The adjusted A-D statistic is given by for exponential distribution:

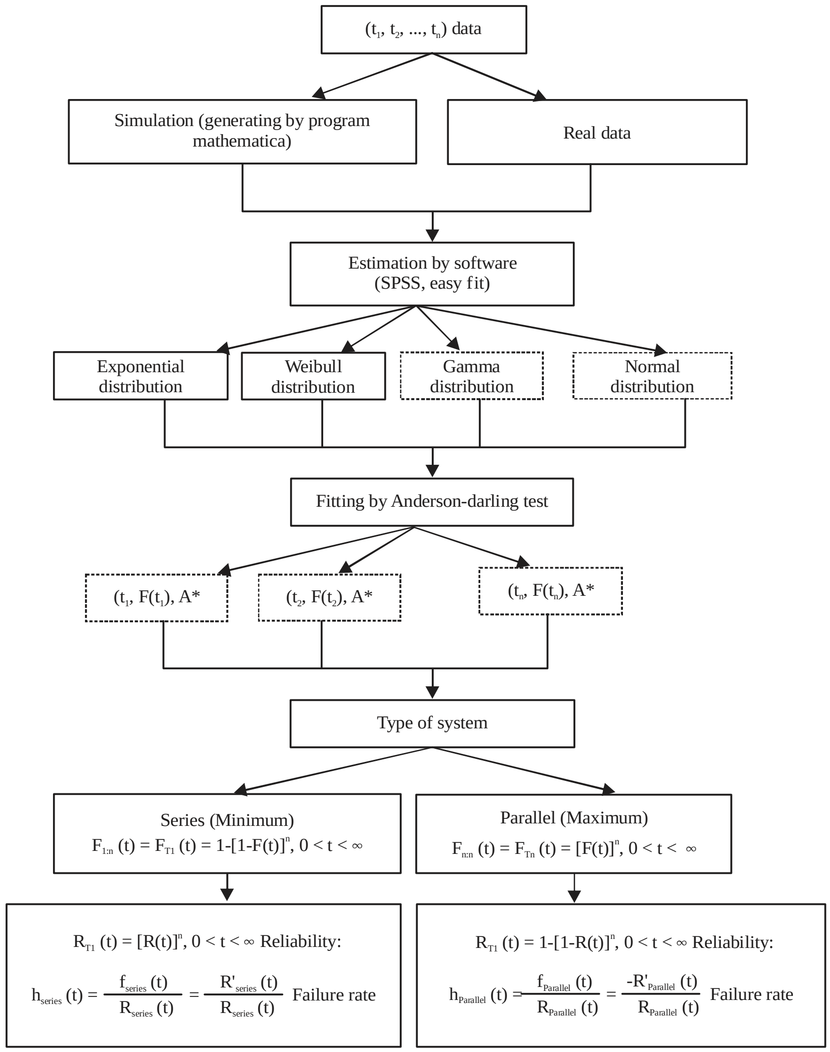

Fig. 8: Algorithm of the reliability and the failure rate

$$A^* = A^2\left(1+\frac{0.6}{\sqrt{n}}\right) \quad (63)$$

for Gamma distribution: The critical value of c_a and the adjusted A-D statistic depends on the value of the parameter k as follows.

If k =1 then:

$$A^* = A^2\left(1+\frac{0.6}{n}\right) \quad (64)$$

If $\geq$ 2then:

$$A^* = A^2 + \left(\frac{0.2+\frac{0.3}{k}}{n}\right) \quad (65)$$

For normal distribution:

$$A^* = A^2\left(1+\frac{0.75}{n}+\frac{2.25}{n^2}\right) \quad (66)$$

Thus, the following results are obtained as follows:

Result 1: The time t_1 has the Gamma distribution with the shape parameter k = 0.972 and the scale parameter b = 0.111. Then, the cumulative distribution function CDF F (t_1) of the time t_1/min is defined as:

$$F(t_1) = 8.32996\int_0^{t_1} T^{-0.028}\, e^{\frac{-T}{0.111}}\, dT \quad (67)$$

Result 2: The time t_2 has normal distribution with μ (Mean) = 7.398 and σ (Standard Deviation) = 4.559.

Table 13: Values of the times t_1, t_2 and t_3 per hour and minute of the multi-server system

t_1 h	t_2 h	t_3 h	t_1 min	t_2 min	t_3 min	t_1 h	t_2 h	t_3 h	t_1 min	t_2 min	t_3 min
0.0224596	0.0077687	0.00359684	1.347576	0.466121	0.215810	0.134370	0.119560	0.143170	8.062200	7.173600	8.590200
0.0228399	0.0169274	0.00719892	1.370394	1.015644	0.431935	0.134840	0.124951	0.143291	8.090400	7.497060	8.597460
0.0231129	0.0233177	0.00924137	1.386774	1.399062	0.554482	0.172937	0.134615	0.143996	10.37622	8.076900	8.639760
0.0240543	0.0303192	0.02400900	1.443258	1.819152	1.440540	0.175588	0.138401	0.149478	10.53528	8.304060	8.968680
0.0242609	0.0386953	0.02488080	1.455654	2.321718	1.492848	0.178409	0.147240	0.168520	10.70454	8.834400	10.11120
0.0257753	0.0399104	0.03060260	1.546518	2.394624	1.836156	0.184292	0.149400	0.169704	11.05752	8.96400	10.18224
0.0263792	0.0452851	0.03129770	1.582752	2.717106	1.877862	0.186501	0.150704	0.185954	11.19006	9.042240	11.15724
0.0311154	0.0478627	0.03842120	1.866924	2.871762	2.305272	0.187486	0.152994	0.186669	11.24916	9.179640	11.20014
0.0350159	0.0480574	0.04686390	2.100954	2.883444	2.811834	0.194704	0.153758	0.193868	11.68224	9.225480	11.63208
0.0360978	0.0545568	0.06487190	2.165868	3.273408	3.892314	0.194992	0.155270	0.197151	11.69952	9.316200	11.82906
0.0378526	0.0578424	0.06967250	2.271156	3.470544	4.180350	0.209340	0.155387	0.200748	12.56040	9.323220	12.04488
0.0440390	0.0599592	0.06971890	2.642340	3.597552	4.183134	0.217164	0.167962	0.208042	13.02984	10.07772	12.48252
0.0503917	0.0668932	0.07012360	3.023502	4.013592	4.207416	0.225339	0.186635	0.216568	13.52034	11.19810	12.99408
0.0555544	0.0734951	0.07909530	3.333264	4.409706	4.745718	0.226157	0.192200	0.218932	13.56942	11.53200	13.13592
0.0601522	0.0778681	0.09117990	3.609132	4.672086	5.470794	0.228146	0.199700	0.219345	13.68876	11.98200	13.16070
0.0640495	0.0867528	0.09455330	3.842970	5.205168	5.673198	0.228491	0.205877	0.226740	13.70946	12.35262	13.60440
0.0687004	0.0870964	0.09548910	4.122024	5.225784	5.729346	0.241363	0.207200	0.228586	14.48178	12.43200	13.71516
0.0704046	0.0902108	0.10032100	4.224276	5.412648	6.019260	0.242939	0.208547	0.228836	14.57634	12.51282	13.73016
0.0722508	0.1018540	0.11052700	4.335048	6.111240	6.631620	0.243645	0.212484	0.230602	14.61870	12.74904	13.83612
0.0774673	0.1019650	0.11412400	4.648038	6.117900	6.847440	0.245806	0.233284	0.238192	14.74836	13.99704	14.29152
0.0937594	0.1036540	0.11841100	5.625564	6.219240	7.104660	0.247838	0.235795	0.238548	14.87028	14.14770	14.31288
0.0944307	0.1114220	0.12283200	5.665842	6.685320	7.369920	0.247882	0.236550	0.240346	14.87292	14.19300	14.42076
0.1007240	0.1134060	0.12389300	6.043440	6.804360	7.433580	0.248852	0.237920	0.240577	14.93112	14.2752	14.43462
0.1014540	0.1139330	0.13076500	6.087240	6.835980	7.845900	0.249270	0.239001	0.240858	14.95620	14.34006	14.45148
0.1088880	0.1187810	0.13534800	6.533280	7.126860	8.120880	0.249712	0.241970	0.242057	14.98272	14.5182	14.52342

Table 14: Cumulative distribution function at the time t_1, t_2 and t_3 per minute of the multi-server system

$F(t_1)$	$F(t_2)$	$F(t_3)$	$F(t_1)$	$F(t_2)$	$F(t_3)$
0.022430	0.068280	0.032157	0.570938	0.461239	0.530176
0.095259	0.086645	0.037750	0.602625	0.553998	0.574275
0.153195	0.088029	0.058409	0.608272	0.563061	0.582787
0.158313	0.088212	0.083806	0.624722	0.571022	0.585828
0.171479	0.130747	0.154167	0.659440	0.617211	0.592476
0.181765	0.151797	0.160495	0.687381	0.655255	0.595610
0.193197	0.164451	0.187419	0.689811	0.665491	0.606192
0.206601	0.167473	0.190494	0.708354	0.670315	0.634019
0.227284	0.169315	0.208294	0.712379	0.727958	0.730254
0.231550	0.174871	0.211593	0.716937	0.738248	0.737495
0.261967	0.211502	0.220973	0.722418	0.745179	0.753247
0.292369	0.247014	0.259186	0.743463	0.747720	0.761421
0.303603	0.257085	0.279576	0.743957	0.761075	0.782814
0.338268	0.268761	0.294662	0.770227	0.785529	0.804621
0.351735	0.279645	0.336034	0.773185	0.806277	0.812759
0.371672	0.280839	0.337383	0.785868	0.821848	0.831661
0.375117	0.316788	0.342935	0.791656	0.831861	0.847559
0.421670	0.333937	0.390249	0.797754	0.855419	0.858901
0.425689	0.337454	0.395807	0.823064	0.872698	0.871353
0.452156	0.357761	0.406972	0.850249	0.882829	0.923984
0.474970	0.371559	0.412566	0.855612	0.906110	0.931916
0.498247	0.372112	0.423913	0.863169	0.925136	0.949666
0.513500	0.425973	0.470937	0.912815	0.946902	0.972959
0.514184	0.431390	0.476371	0.923423	0.978950	0.974318
0.549463	0.435889	0.483093	0.928486	0.981635	0.979126

Table 15: Estimating the parameters of the cumulative distribution function $F(t_1)$, combined with the exponential, Weibull, Gamma and normal distribution

CDF of distributions	Estimate parameters	a. R^2
Exponential	a (Scale) = 0.115	0.972
Weibull	a (Scale) = 8.696, b (Shape) = 0.983	0.972
Gamma	a (Shape) = 0.972, b (Scale) = 0.111	0.972
Normal	μ (Mean) = 7.125, σ (Standard deviation) = 7.422	0.950

Table 16: Estimating the parameters of the cumulative distribution function $F(t_2)$, combined with the exponential, Weibull, Gamma and normal distributions

CDF of distributions	Estimate parameters	a. R^2
Exponential	a (Scale) = 0.108	0.874
Weibull	a (Scale) = 8.782, b (Shape) = 1.795	0.978
Gamma	a (Shape) = 2.615, b (Scale) = 0.326	0.971
Normal	μ (Mean) = 7.398, σ (Standard deviation) = 4.559	0.987

Table 17: Estimating the parameters of the cumulative distribution function $F(t_3)$, combined with the exponential, Weibull, Gamma and normal distributions

CDF of distributions	Estimate parameters	a. R^2
Exponential	a (Scale) = 0.103	0.895
Weibull	a (Scale) = 9.596, b (Shape) = 1.624	0.958
Gamma	a (Shape) = 2.148, b (Scale) = 0.243	0.949
Normal	μ (Mean) = 8.025, σ (Standard deviation) = 5.490	0.977

Then, the cumulative distribution function (CDF) $F(t_2)$ of the time t_2 per min is defined as:

$$F(t_2) = \frac{1}{11.4277}\int_0^{t_2} e^{-\frac{1}{2}\left(\frac{T-7.398}{4.559}\right)^2} dT \quad (68)$$

Result 3: The time t_3 has normal distribution with μ (Mean) = 8.025, σ (Standard Deviation) = 5.490. Then, the cumulative distribution function (CDF) $F(t_3)$ of the time t_3 per min is defined as:

$$F(t_3) = 0.072667\int_0^{t_3} e^{-\frac{1}{2}\left(\frac{T-8.025}{5.490}\right)^2} dT \tag{69}$$

Result 4: The cumulative distribution function (CDF) of the entire lifetime in the case of series system is defined as:

$$\begin{aligned} F_{T_1}(t) &= 1-[1-F(t)]^n = 1-[1-F(t_2)]^{50} \\ &= 1-\left[1-0.0875067\int_0^{t_2} e^{-\frac{1}{2}\left(\frac{T-7.398}{4.559}\right)^2} dT\right]^{50} \end{aligned} \tag{70}$$

where, T_1 is the entire lifetime of the series system, $T_1 = \min\{t_1, t_2, t_3\}$ and t_2 is the minimum time.

Result 5: The reliability of the entire lifetime in the case of series system is defined as:

$$\begin{aligned} R_{Series}(t) &= R_{T_1}(t) = [R(t)]^n = [1-F(t_2)]^{50} \\ &= \left[1-0.0875067\int_0^{t_2} e^{-\frac{1}{2}\left(\frac{T-7.398}{4.559}\right)^2} dT\right]^{50} \end{aligned} \tag{71}$$

Result 6: The failure function of the entire lifetime in the case of the series system is defined as:

$$\begin{aligned} h_{Series}(t) &= \frac{f_{Series}(t)}{R_{Series}(t)} = \frac{n(1-F(t_2))^{n-1} f(t_2)}{R_{Series}(t_2)} \\ &= \frac{(4.37534)e^{-\frac{1}{2}\left(\frac{t_2-7.398}{4.559}\right)^2}}{\left[1-0.0875067\int_0^{t_2} e^{-\frac{1}{2}\left(\frac{T-7.398}{4.559}\right)^2} dT\right]} \end{aligned} \tag{72}$$

Result 7: The cumulative distribution function (CDF) in the case of the parallel system is defined as:

$$F_{T_n}(t) = [F(t)]^n = [F(t_3)]^{50} = \left[0.072667\int_0^{t_3} e^{-\frac{1}{2}\left(\frac{T-8.025}{5.490}\right)^2} dT\right]^{50} \tag{73}$$

where, T_n is the entire lifetime in the case of the parallel system, $T_n = \max\{t_1, t_2, t_3\}$ and t_3 is the maximum time.

Result 8: The reliability in the case of the parallel system is defined as:

$$\begin{aligned} R_{Parallel}(t) &= R_{T_n}(t) = 1-[1\text{-}R(t)]^n = 1-[F(t_3)]^{50} \\ &= 1-\left[0.072667\int_0^{t_3} e^{-\frac{1}{2}\left(\frac{T-8.025}{5.490}\right)^2} dT\right]^{50} \end{aligned} \tag{74}$$

Result 9: The failure function in the case of the parallel system is:

$$h_{Parallel}(t) = \frac{(5.83219\times10^{-56})\, e^{-\frac{1}{2}\left(\frac{t_3-8.025}{5.490}\right)^2}\left[\int_0^{t_3} e^{-\frac{1}{2}\left(\frac{T-8.025}{5.490}\right)^2} dT\right]^{49}}{1-\left[0.072667\int_0^{t_3} e^{-\frac{1}{2}\left(\frac{T-8.025}{5.490}\right)^2} dT\right]^{50}} \tag{75}$$

Thus, the all mentioned results (1-9) can be operated on the computer to obtain the characteristics of the multi-server system in the different cases of server connections with different probability distributions.

CONCLUSION

In the present study, we have managed to figure out the maintainability, availability, capability and efficiency of the multi-server system. Moreover, we can also derive the reliability and failure rate of each server and of the entire system with the application of the nonlinear regression equations, predicting the probability of the elapsed time of each server of the system. Hopefully, researchers will be able to economically deal with the different systems of repairing defective machines at minimal costs in this proposed way.

SIGNIFICANCE STATEMENTS

This study shows that the statistical method, used here, gives accurate results of the multi-server system of repairing defective machines, the point that is highly beneficial to the industrial workshop applications. It also helps to uncover the critical areas of the time lost in repairing defective machines and the cost spent on adding unnecessary new servers to an operating system. Now, researchers can work out a new statistical method of multi-server systems and, possibly, of other new distributions.

REFERENCES

1. Shaikh, A. and A. Mettas, 2010. Application of Reliability, Availability and Maintainability Simulation to Process Industries: A Case Study. In: Simulation Methods for Reliability and Availability of Complex Systems, Faulin, J., A.A. Juan, S. Martorell and J.E. Ramirez-Marquez (Eds.). Springer, London, ISBN: 978-1-84882-212-2, pp: 173-197.

2. Cheng, F.T., K.K. Chen, S.W. Chiu and M.H. Hwang, 2011. On improving the optimization process of a manufacturing run time problem with defective and machine breakdown. Int. J. Phys. Sci., 14: 3564-3566.
3. El Genidy, M.M., 2011. Maximal queue size with standard normal distribution for arrival times. Asian J. Math. Stat., 4: 161-167.
4. Lindqvist, B.H., 2006. On the statistical modeling and analysis of repairable systems. Stat. Sci., 21: 532-551.
5. Veber, B., M. Nagode and M. Fajdiga, 2008. Generalized renewal process for repairable systems based on finite Weibull mixture. Reliab. Eng. Syst. Saf., 93: 1461-1472.
6. Edouard, T., L. Eric and I. Benoit, 2008. Maintenance decision making tool reaching a compromise between maintainability and reliability performances. IFAC Proc. Volumes, 41: 907-912.
7. Ahmad, R. and S. Kamaruddin, 2012. An overview of time-based and condition-based maintenance in industrial application. Comput. Ind. Eng., 63: 135-149.
8. El Bouchikhi, E.H., V. Choqueuse and M.E.H. Benbouzid, 2013. Current frequency spectral subtraction and its contribution to induction machine's bearings condition monitoring. IEEE Trans. Energy Convers., 28: 135-144.
9. Sharma, A.K., Shudhanshu and A. Bhardwaj, 2012. Manufacturing performance and evolution of TPM. Int. J. Eng. Sci. Technol., 4: 854-866.
10. Waghmode, L.Y. and A.D. Sahasrabudhe, 2012. Modelling maintenance and repair costs using stochastic point processes for life cycle costing of repairable systems. Int. J. Comput. Integr. Manuf., 25: 353-367.
11. Yang, Q., N. Zhang and Y. Hong, 2013. Reliability analysis of repairable systems with dependent component failures under partially perfect repair. IEEE Trans. Reliab., 62: 490-498.
12. Wu, Q. and S. Wu, 2011. Reliability Analysis of Two-Unit Cold Standby Repairable Systems under Poisson Shocks. Applied Math. Comput., 218: 171-182.
13. Yuan, L. and Z.D. Cui, 2013. Reliability analysis for the consecutive-k-out-of-n: F system with repairmen taking multiple vacations. Applied Math. Modell., 37: 4685-4697.
14. Hu, L., D. Yue and J. Li, 2010. Probabilistic analysis of a series-parallel repairable system with three units and vacation. Applied Math. Modell., 34: 2711-2721.
15. Guo, L., H. Xu, C. Gao and G. Zhu, 2011. Stability analysis of a new kind n-unit series repairable system. Applied Math. Modell., 35: 202-217.
16. Verma, W.C.N. and M.E.A. Papon, 2013. A study of reliability and failure rate for comparing BIT and non-BIT maintenance cycles in AONA. Int. J. Emerg. Technol. Adv. Eng., 3: 676-680.
17. Neil, M. and D. Marquez, 2012. Availability modelling of repairable systems using Bayesian networks. Eng. Applic. Artif. Intell., 25: 698-704.
18. El Genidy, M.M., 2014. Joint probability distribution function of the repairing time for the defective machines in the multiple queues system. Asian J. Math. Stat., 7: 35-39.
19. El Damsesy, M.A., M.M. El Genidy and A.M. El Gazar, 2014. Estimation of a system performance in Pareto distribution with two independent random variables. J. Applied Sci., 14: 2854-2856.