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Research Article
On the LQ Optimization Subject to Descriptor System under Disturbance

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Abstract

Background and Objective: Recently, the linear quadratic (LQ) optimization subject to descriptor system received much attention from several researchers in field of descriptor system. In this study, it was aimed to establish a sufficient condition that guaranteeing the existence of pair the optimal control and optimal state of the LQ optimization problem subject to descriptor system under disturbance. **Materials and Methods:** In order to solve the considered problem, the LQ optimization problem subject to descriptor system transformed into the normal LQ optimization problem. The available results of the normal LQ optimization problem were utilized to find a sufficient condition for the existence of optimal solution for LQ optimization problem subject to descriptor system under disturbance. **Results:** The final results show that this sufficient condition constitutes a method to find the pair the optimal control and optimal state of the LQ optimization problem subject to descriptor system under disturbance. **Conclusion:** The impulse controllability and stabilizability of the constraint constitute the desired sufficient condition.

Key words: Descriptor system, LQ optimization, disturbance, impulse controllable, stabilizable

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Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Throughout this study the notations $\mathbb{R}^n$ denotes the set of all real vectors of n-dimension, $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices, I_r is the identity matrix of $r\times r$, O is the null matrices of suitable dimension, rank (A) is rank of matrix A and det (A) denotes determinant of matrix A.

Let us consider the following linear descriptor system:

$$\begin{aligned} L\dot{x} &= Ax + Bu + E\omega,\ x(0) = x_0 \\ y &= Cx + F\omega \end{aligned} \tag{1}$$

where, $x\in\mathbb{R}^n$, $u\in\mathbb{R}^m$, $\omega\in\mathbb{R}^q$, $y\in\mathbb{R}^r$ are the state, the control input, the disturbance input and the factual output, respectively. In the system (1), the matrices $L, A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{r\times n}$, $E\in\mathbb{R}^{n\times q}$, $F\in\mathbb{R}^{r\times q}$ and rank(L) = p<n. It is well-known that the system (1) has a solution if it is regular, i.e., there exist $\lambda\in\mathbb{C}$ such that det $(\lambda L-A)\neq 0$. If p = n then the Eq. 1 can be write as the following normal system:

$$\begin{aligned} \dot{x} &= L^{-1}Ax + L^{-1}Bu + L^{-1}E\omega,\ x(0) = x_0 \\ y &= Cx + F\omega \end{aligned} \tag{2}$$

in which its solution can be obtained easily.

The dynamical system (1) attracts interest because this kind of system appears in the modelling of many processes in various fields, e.g., in biology, chemistry[1] and especially it constitutes an economic model of Leontief input output relation[2]. Recently, Muhafzan[3] and Stevanovski[4] discussed the LQ optimization subject to (1) without disturbance input. The LQ optimization subject to normal system are discussed by Wu *et al.*[5]. In the other hand, the LQ optimization subject to descriptor system with disturbance are discussed by Chen[6] and Fang *et al.*[7]. The problem formulated in Chen[6] and Fang *et al.*[7] is to determine a control $u\in\mathbb{R}^m$ that satisfy (1) and to minimize the following quadratic performance index:

$$\zeta(u) = \int_0^\infty (y^T y + u^T Ru)\,dt \tag{3}$$

where, R is $m\times m$ definite positive matrix.

In any applications the desired output of a process is not always similar to the factual output, so there is a difference between the factual output and desired output. Let us denote the difference by ϵ, that is $\epsilon = (y-y_d)$, where, y_d denotes the desired output. Therefore we can define a new quadratic performance index to be minimized as follows:

$$\zeta_\epsilon(u) = \int_0^\infty \left(\gamma\epsilon^T S\epsilon + \dot{u}^T R\dot{u}\right) dt \tag{4}$$

where, $\dot{u}$ denotes the derivative of u over time and $\gamma>0$ is a weighted parameter.

Under the assumption that the desired output and disturbance input are constant vectors and the system (1) is regular, the problem addresses in this paper is to find the optimal control $u\in\mathbb{R}^m$ and the optimal state $x\in\mathbb{R}^n$ that satisfy the descriptor system (1) such that the performance index (4) are minimized and $\epsilon = (y-y_d)\to 0$ when $t\to\infty$. This problem formulation constitute a novel aspect in the LQ optimization subject to descriptor system area. The novelty lies in the inclusion of condition $\epsilon = (y-y_d)\to 0$ when $t\to\infty$ and the quadratic performance index $\zeta_\epsilon(u)$ in Eq. 4. We denote this optimization problem as Ω. Henceforth, we denote the optimal control and the optimal state by u^{opt} and x^{opt}, respectively. The pair u^{opt} and x^{opt} are called the optimal solution of problem Ω.

MATERIALS AND METHODS

In order to find the desired result, let us define a new vector z such that $\dot{z} = \epsilon$. From the relation $\epsilon = (y-y_d)$, the system (1) can be written as follows:

$$\begin{bmatrix} L & O \\ O & I_r \end{bmatrix}\begin{bmatrix} \dot{x} \\ \epsilon \end{bmatrix} = \begin{bmatrix} A & O \\ C & O \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ O \end{bmatrix}u + \begin{bmatrix} E \\ F \end{bmatrix}\omega - \begin{bmatrix} O \\ I_r \end{bmatrix}y_d \tag{5}$$

Since the disturbance ω and the desire output y_d are constant vectors, the differentiation of Eq. 5 for t result:

$$\begin{bmatrix} L & O \\ O & I_r \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} A & O \\ C & O \end{bmatrix}\begin{bmatrix} \dot{x} \\ \epsilon \end{bmatrix} + \begin{bmatrix} B \\ O \end{bmatrix}\dot{u} \tag{6}$$

By defining:

$$\dot{u} := \vartheta,\ \begin{bmatrix} \dot{x} \\ \epsilon \end{bmatrix} := \varsigma,\ \bar{L} := \begin{bmatrix} L & O \\ O & I_r \end{bmatrix},\ \bar{A} := \begin{bmatrix} A & O \\ C & O \end{bmatrix},\ \bar{B} := \begin{bmatrix} B \\ O \end{bmatrix},\ Q := \begin{bmatrix} O & O \\ O & \gamma S \end{bmatrix} \tag{7}$$

the problem is Ω equivalent to the following optimization problem:

$$\min_\vartheta\ \zeta_\epsilon(\vartheta) = \int_0^\infty \left(\varsigma^T Q\varsigma + \vartheta^T R\vartheta\right) dt \tag{8}$$

$$\text{s.t. } \bar{L}\dot{\varsigma} = \bar{A}\varsigma + \bar{B}\vartheta,\ \varsigma(0) = \varsigma_0 \tag{9}$$

It is obvious that rank (L) = p+r<n+r. Since the system (1) is regular, the system (9) is also regular, i.e., there exist $\bar{\lambda} \in \mathbb{C}$ such that det $\left(\bar{\lambda}\bar{L} - \bar{A}\right) \neq 0$. It is also obvious that Eq. 8 and 9 constitute a LQ optimization problem subject to descriptor system without disturbance with state ς and control ϑ. By using the theory in Cobb[8] and Duan[9], the optimal control for optimization problem Eq. 8 and 9 exist if and only if system (9) is impulse controllable and stabilizable[7].

Under the assumption controllable impulse of the system (9), there exist a matrix $K \in \mathbb{R}^{m\times(n+r)}$ such that:

$$\deg\det\left(\bar{\lambda}\bar{L} - \left(\bar{A} + \bar{B}K\right)\right) = \operatorname{rank}\bar{L} \tag{10}$$

By choosing a feedback control $\vartheta = K\varsigma + v$, for some new control $v \in \mathbb{R}^m$ and apply it to system (9), we have:

$$\bar{L}\dot{\varsigma} = \left(\bar{A} + \bar{B}K\right)\varsigma + \bar{B}v \tag{11}$$

Thus we have the normal decomposition of the form:

$$Q_1\bar{L}P_1 = \begin{bmatrix} I_{p+r} & O \\ O & O \end{bmatrix},\; Q_1\left(\bar{A} + \bar{B}K\right)P_1 = \begin{bmatrix} \bar{A}_1 & O \\ O & I_{n-p} \end{bmatrix},\; Q_1\bar{B} = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}$$

for some nonsingular matrices[8] Q_1, $P_1 \in \mathbb{R}^{(n+r)\times(n+r)}$. Denoting:

$$\varsigma = P_1\begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix},\; \varsigma_1 \in \mathbb{R}^{p+r},\; \varsigma_2 \in \mathbb{R}^{n-p} \tag{12}$$

the system (11) is equivalent to:

$$\dot{\varsigma}_1 = \bar{A}_1\varsigma_1 + \bar{B}_1 v,\; \varsigma_1(0) = \varsigma_{10} \tag{13}$$

$$0 = \varsigma_2 + \bar{B}_2 v,\; \varsigma_2(0) = \varsigma_{20} \tag{14}$$

Using Eq. 13, the objective function (8) becomes:

$$\zeta_\epsilon(v) = \int_0^\infty \begin{bmatrix} \varsigma_1 \\ v \end{bmatrix}^T \begin{bmatrix} \hat{Q} & H \\ H^T & \tilde{R} \end{bmatrix} \begin{bmatrix} \varsigma_1 \\ v \end{bmatrix} dt \tag{15}$$

where:

$$\begin{bmatrix} \hat{Q} & H \\ H^T & \tilde{R} \end{bmatrix} = \begin{bmatrix} I & O \\ O & -\bar{B}_2 \\ O & I \end{bmatrix}^T \Gamma \begin{bmatrix} I & O \\ O & -\bar{B}_2 \\ O & I \end{bmatrix} \tag{16}$$

$$\Gamma = \begin{bmatrix} P_1 & O \\ KP_1 & I \end{bmatrix}^T \begin{bmatrix} Q & O \\ O & R \end{bmatrix} \begin{bmatrix} P_1 & O \\ KP_1 & I \end{bmatrix} \tag{17}$$

By using the substitution:

$$v = w - \tilde{R}^{-1}H^T\varsigma_1 \tag{18}$$

where:

$$w = \vartheta - K\varsigma + \tilde{R}^{-1}H^T\varsigma_1 \tag{19}$$

the LQ optimization problem (8) and (9) can be converted into the following normal LQ optimization problem:

$$\min_w \zeta_\epsilon(w) = \int_0^\infty \left(\varsigma_1^T\tilde{Q}\varsigma_1 + w^T\tilde{R}w\right) dt \tag{20}$$

$$\text{s.t. } \dot{\varsigma}_1 = \bar{A}_1\varsigma_1 + \bar{B}_1 w,\; \varsigma_1(0) = \varsigma_{10} \tag{21}$$

where:

$$\tilde{Q} = \hat{Q} - H\tilde{R}^{-1}H^T \tag{22}$$

and:

$$\tilde{A}_1 = \bar{A}_1 - \bar{B}_1\tilde{R}^{-1}H^T \tag{23}$$

RESULTS AND DISCUSSION

It is obvious that Eq. 20-23 constitute the normal LQ optimization problem with state ς_1 and control w. Based on the theory of normal LQ optimization problem, the optimal control for Eq. 20 and 21 exist and unique if the system (21) is stabilizable[3]. Note that the stabilizability of the system (9) implies the system (13) is stabilizable. Since Eq. 21 is closed-loop system resulted in by applying a state feedback with gain matrix $-\tilde{R}^{-1}H^T$ to the system (13) and the fact that the state feedback does not change stabilizability, we get the system (21) is stabilizable.

It follows that the solution of the problem (20) and (21) is given by:

$$w = -\tilde{R}^{-1}\bar{B}_1^T P\,\varsigma_1 \tag{24}$$

where, ς_1 is the solution of the following initial value problem:

$$\dot{\varsigma}_1 = \left(\bar{A}_1 - \bar{B}_1\tilde{R}^{-1}\left(\bar{B}_1^T + H^T\right)\right)\varsigma_1,\; \varsigma_1(0) = \varsigma_{10} \tag{25}$$

and the matrix P is the unique symmetric positive definite solution of the following algebraic Riccati equation:

$$\begin{aligned}&\left(\bar{A}_1-\bar{B}_1\tilde{R}^{-1}H^T\right)^T P+P\left(\bar{A}_1-\bar{B}_1\tilde{R}^{-1}H^T\right)-\\&P\bar{B}_1\tilde{R}^{-1}\bar{B}_1^T P+\hat{Q}-H\tilde{R}^{-1}H^T=O\end{aligned} \quad (26)$$

with the minimum value is $\varsigma_{10}^T P\varsigma_{10}$. Moreover, the solution of initial value problem (25) is stable, in the sense that $\varsigma_1(t)\to 0$ if $t\to\infty$.

Using Eq. 18 we have:

$$v=-\tilde{R}^{-1}(\bar{B}_1^T P+H^T)\varsigma_1 \quad (27)$$

and using Eq. 19, 12 and 14, we have:

$$\dot{u}=\vartheta=\left(KP_1\begin{bmatrix}I_{p+r}\\ \bar{B}_2\tilde{R}^{-1}\left(\bar{B}_1^T P+H^T\right)\end{bmatrix}-\tilde{R}^{-1}\left(\bar{B}_1^T P+H^T\right)\right)\varsigma_1 \quad (28)$$

Moreover, from Eq. 7 and 12 we also have:

$$\begin{bmatrix}\dot{x}\\ \epsilon\end{bmatrix}=P_1\begin{bmatrix}I_{p+r}\\ \bar{B}_2\tilde{R}^{-1}\left(\bar{B}_1^T P+H^T\right)\end{bmatrix}\varsigma_1 \quad (29)$$

Therefore, the optimal solution of the optimization problem Ω are u^{opt} and x^{opt}, where, u^{opt} satisfies (28) and x^{opt} satisfies (29). Moreover, since $\varsigma_1(t)\to 0$ when $t\to\infty$ we also obtain $\epsilon=(y-y_d)\to 0$ when $t\to\infty$.

Thus we have proved the following Theorem that constitutes the main result of this study.

Theorem 3.1: If the system (9) is impulse controllable and stabilizable, then the optimal control u^{opt} and the optimal state x^{opt} for the optimization problem Ω exist and unique, where, u^{opt} satisfies the Eq. 28 and x^{opt} satisfies (29). Moreover, $y\to y_d$ when $t\to\infty$.

This result constitute a new contribution in field of the LQ optimization subject to descriptor system area that require condition $\epsilon=(y-y_d)\to 0$ when $t\to 0$. It is obvious that this result different to the result of Chen[6] and Fang *et al.*[7]. This difference is due to the additional condition $\epsilon=(y-y_d)\to 0$ when $t\to\infty$ in our problem. The findings show that this sufficient condition constitutes a method to find the pair the optimal control and optimal state of the LQ optimization problem subject to descriptor system under disturbance.

CONCLUSION

A sufficient condition that guaranteeing the existence of pair the optimal control and optimal state of the LQ optimization problem subject to descriptor system under disturbance has been established, namely as given in Theorem 3.1. The findings show that this sufficient condition constitutes a method to find the pair the optimal control and optimal state of the LQ optimization problem subject to descriptor system under disturbance. This result constitute a new contribution in the dynamic optimization area.

SIGNIFICANCE STATEMENT

The study discovers a sufficient condition that guaranteeing the existence of pair the optimal control and optimal state of LQ optimization problem subject to descriptor system under disturbance. The sufficient condition can be beneficial for both practitioners and researches. This study will help the researcher to solve the model optimization of the form linear quadratic governed by the descriptor system.

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