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Research Article

Testing Several Correlation Matrices Using Robust Approach

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Abstract

Background and Objective: The performance of classical Jennrich (J) statistic using classical estimators suffers from masking effects. To relieve the problem, robust estimators are recommended. In this study, a robust Jennrich statistic was proposed based on a S estimator (J_S) and M estimator (J_M) as alternative to the J statistic, which has good properties. **Methodology:** In the simulation study, the performance of proposed test is assessed in terms of a type I error and the power of test. The performance comparison between classical J, J_S and J_M statistics are conducted under several conditions. **Results:** The results of simulation study showed that J_S statistic has a competitive performance comparative to a J_M statistic and the J statistic. **Conclusion:** It was concluded that J_S statistic is robust for testing the equality of two or more difference correlation matrices when the data contains outlier.

Key words: Correlation matrix, hypothesis testing, M estimator, power of test, S estimator, type I error, outlier, data contamination

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Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Since three decades ago testing the stability of correlation matrices began receiving much attention in economic and financial studies. For instance, Lee¹ and Tang² studied the stability of correlation matrices among stock returns. Goetzmann *et al.*³ studied global market correlation matrices, Da Costa *et al.*⁴ showed that correlation stability is important in estate management and risk market management and Deblauwe and Le⁵ studied the stability of market risk and credit over different periods by using the J statistic⁶.

The Jennrich statistic is one method for testing the equality of two or more independent samples correlation matrices. In the multivariate statistical process control approach, when the data are in a time sequence, this test can be presented using a control chart to show the stability (or instability) of the correlation matrices. This test is computed repeatedly to measure the differences between the *i*-th sample and reference sample correlation matrices under the normality assumptions. To implement the Jennrich statistic, the Eq. 1 developed by Jennrich⁶ is as follows:

$$J = \sum_{i=1}^m \left\{ \frac{1}{2} \text{Tr}(Z_i^2) - (Z_i)' W^{-1} Z_i \right\} \quad (1)$$

Where:

$$Z_i = n_i^{\frac{1}{2}} R_p^{-1} (R_i - R_p)$$

$R_i = S_{d,i}^{-1/2} S_i S_{d,i}^{-1/2}$ is the *i*-th sample correlation

S_i = The *i*-th sample covariance matrices

R_p = The average of all sample correlation matrices

$W = I + R_p * R_p^{-1}$, the * is Hadamard product of two matrices

Z_d, S_d = A diagonal for Z_i, S_i , respectively

I_p = The identity matrix of size ($p \times p$)

When Σ is unknown, sample covariance matrix, S was used and this makes the classical statistic heavily sensitive to outliers because it depends on the normality of distribution. If the distributional property cannot be met, consequently, this will lead to imprecision in decisions on testing procedure⁷. An alternative estimation method to overcome the drawback is by applying the mahalanobis distance (MD). The MD can help to remove outliers from the data and then it can proceed with Jennrich statistic. However, a single or group of observations far from the main data can have severe impacts upon this distance because the scale and covariance are generally estimated

in a non-robust method⁸. In addition, data sets with outliers make this method subject to swamping and masking effects. These effects play an important role in the adequacy of the MD as a standard for finding outliers, masking effects might decrease the MD of an outliers. On the other hand, swamping might increase the MD of non-outlying observations^{9,10} and MD suffers from the masking effect.

One way to reduce the sensitivity of distributional assumptions is by using robust estimators in place of the classical estimator. In this paper, the authors explore S and M estimators because the computational complexity is less. The M estimator suggested by Huber¹¹ is a robust estimator and has a breakdown point of $1/(1+p)$, where p is the number of dimensions. The S estimator is based on Rousseeuw and Leory¹². The S estimator is defined as a solution of the location and scatter to minimize the determinant of the covariance matrix and it has high as breakdown¹³. This estimator highly resistant to outliers and gives the same values when there are no outliers¹⁴. The M and S estimators seek to minimize the impact of outliers. Because these two estimators are highly resistant to outliers and have high robust qualities, they are considered appropriate replacements. The purpose of this research is to suggest test used for testing the equality of two difference correlation matrices or more when the data contains outlier.

REVIEW ON ROBUST APPROACHES

When the assumption of normality is unfulfilled, some researchers have sought other methods to overcome the sensitivity to the violations of this assumptions, for example, in nonparametric method and simple transformation. This method has grown as a field of research and has become extensively popular in applications. Moreover, the nonparametric method is more suitable for non-normal symmetric data. In addition, the nonparametric procedure is less powerful than the parametric procedure and consequently a large sample size is required to reject a false hypothesis¹⁵. However, another method was recommended by Wilcox¹⁶ to overcome the effects of non-normality is that of simple transformations but these failed to deal effectively with outliers.

All violations mentioned above helped to improve the theory of robustness which opened the way to find the solutions in statistics¹¹. The robust procedures are useful and viable alternatives to classical methods for increasing the

sensitivity and the power of statistical tests and to control type I error rates at the nominal level^{15,17}. This procedure is used by calculating the estimator after removing the outliers from the data and other procedures by using robust estimators in place of traditional estimators. The use of this procedure is a notable in identifying the deviation of data or outliers¹⁸. Robust estimators are used to detect outliers and to provide results resistant to the existence of outliers¹⁹. The aim of robustness in statistics is to provide tools not only to assess the properties of robustness of classical procedures but also to produce estimators and tests that are robust to model deviations²⁰.

Beside S and M estimators, there are some other robustness estimators that have received attention from several researchers, these estimators include the minimum volume ellipsoid (MVE) and minimum covariance determinant (MCD), these two estimators were introduced by Rousseeuw²¹. The MVE is used widely in practice they have affine equivariance and also approximately have a 50% high breakdown point²². According to Yahaya *et al.*¹⁸, the computation of MVE may not be feasible and is very expensive. In addition, MVE fails to overcome with large sample size²³.

To relieve the complexity of MVE, Rousseeuw in 1985 introduced MCD. These estimators minimize the determinant of covariance matrices. The MCD has many advantages over MVE, the statistical efficiency is better because of MCD has convergence rates asymptotically normal. While MVE has lower a convergence rate and MCD has better accuracy²³. The use of MCD in data that can be computationally laborious when the data set is high dimension¹⁸.

To overcome with problems with MCD²³ developed a new algorithm called Fast MCD, but Fast MCS becomes less efficient in high dimensions because it includes the covariance matrix determinant and the inversion of covariance²⁴. To cope the weaknesses of FMCD, Herwindiati *et al.*²⁵ suggested the minimum vector variance (MVV). According to Yahaya *et al.*¹⁸, MVV is effective and based on vector variance (VV) wherein the MVV algorithm is simpler than FMCD because MVV has a lower computational complexity²⁶. The MVV is effective and efficient in the outliers labeling process but it takes a few more computations when the dimension is larger than 100²⁷. However, although those estimators are good, the computational complexity is difficult.

MATERIALS AND METHODS

This section showed the robust approaches to test the equality of correlation matrices by using S and M estimators.

Robust control chart based on S and M estimation: In this section, the authors want to introduce robust control charts based on two robust estimates, namely, M and S estimators for identifying the outliers. The M estimator seeks to minimize the impact of the outliers. In multivariate analysis, choosing this estimator is important because it has a breakdown point. Maronna and Yohai²⁸ defined a solution of μ and E as a positive definite symmetric matrix V_n in R satisfying equations:

$$\frac{1}{n} \sum_i u_i d_i (x_i - \mu) = 0 \quad (2)$$

$$\frac{1}{n} \sum_i u_i d_i^2 (x_i - \mu)(x_i - \mu)^t = \Sigma \quad (3)$$

$$d_i = \left((x_i - \mu)^t \Sigma^{-1} (x_i - \mu) \right)^{1/2} \quad (4)$$

where, $i = 1, 2, \dots, n$, μ_1 and μ_2 real value function $[0, \infty)$. Our interest in the M estimator is due to its robust qualities. M estimators come from generalized maximum likelihood²⁹ and a M estimator of multivariate location and scatter are affine equivariante.

It was defined a robust J control chart by using M estimator as follows:

$$J_M = \sum_{i=1}^m \left\{ \frac{1}{2} \text{Tr}(Z_{M,i}^2) - (Z_{M,d})' W_M^{-1} Z_{M,d} \right\} \quad (5)$$

Equation 5 represents the robust Jennrich statistic by using M estimators, S_M is the robust M estimator. The sample correlation matrix by using M estimator $R_{M,i} = S_{M,d}^{-1/2} S_M S_{M,d}^{-1/2}$.

Let $p: R^+ \rightarrow R^+$ the function p is non-decreasing in positive values and has $p(0) = 0$ and is constant at $p(x) = p(c)$ for all $x \geq c$. The S estimator is based on the p function. This function is chosen because of its Gaussian efficiency and robustness:

$$b_0 = \frac{1}{n} \sum_i p(d_i) \quad (6)$$

Where:

$$d_i = \sqrt{(x_i - \mu)^t \Sigma^{-1} (x_i - \mu)}$$

$$b_0 = E[p(d_i)]$$

$$= \frac{v\chi^2(v+2, c^2)}{2} - \frac{v(v+2)\chi^2(v+4, c^2)}{2c^2} + \frac{v(v+2)(v+4)\chi^2(v+6, c^2)}{6c^4} + \frac{c^2}{6} [1 - \chi^2(v, c^2)]$$

$$d_i \sim N(0, 1)$$

The choice of c depends on the desired (BP) for the estimate. Rocke³⁰ introduced translated bi weight (t-bi weight) to provide the lowest sensitivity to outliers for a given (BP):

$$\rho(d_i) = \begin{cases} \frac{d_i^2}{2} - \frac{d_i^4}{2c^2} + \frac{d_i^6}{6c^4} & |d_i| \leq c \\ \frac{c^2}{6} & |d_i| > c \end{cases}$$

The basic criterion for a good robust estimators is the high breakdown and equally behavior with uncontaminated data sets¹⁴. The BP is the percentage of outliers in the sample that the estimators can handle, it was shown at 50%³¹. However, the BP of estimators take different values subject to the contaminated data set by which it is evaluated³².

It was defined that a robust J control chart by using S estimator as follows:

$$J_S = \sum_{i=1}^m \left\{ \frac{1}{2} \text{Tr}(Z_{S,i}^2) - (Z_{S,i})' W_S^{-1} Z_{S,i} \right\} \quad (7)$$

The Eq. 7 represents the robust the Jennrich statistic by using S estimators, S_S is the robust S estimator. The sample correlation matrix by using S estimator is $R_{S,i} = S_{S,i}^{-1/2} S_{S,i}^{-1/2}$.

Simulation study: To illustrate the performance of J_S and J_M , the authors need to get achieve a better understanding of its distribution to obtain appropriate of the control limits. The authors apply uses a simulation study to estimate the quantiles of J_S and J_M . This study focus on values of sample size and the number of dimensions as in Table 1. The authors generate $R = 5000$ sample of size N ($N = n_1, n_2 = n_1, n = n_1 + n_2$) from the standard multivariate normal distribution $MVN_p(0, I_p)$. The algorithm of test for estimating the type I error is as follows:

- Generate data based on nature characteristics from distribution $MVN_p(0, I_p)$
- Calculate the statistic to each one robust and traditional
- Compute the critical value (CV) under significance level 0.05 the value of test which is number 4750 ($0.05^* 5000 = 250$, now $5000 - 250 = 4750$)
- Repeat steps i-iii for 5000 replications

Table 1: Analysis of type I error rate for small variable

n	ξ	μ	p = 3			p = 5		
			J	J_S	J_M	J	J_S	J_M
10	0.0	0	0.0462	0.0484	0.0520	0.0496	0.0538	0.0478
		3	0.0212	0.0404	0.0232	0.0252	0.0330	0.0214
		5	0.0208	0.1016	0.0194	0.0202	0.0356	0.0180
	0.2	3	0.0170	0.0288	0.0182	0.0214	0.0272	0.0190
		5	0.1016	0.0222	0.0186	0.0178	0.0254	0.0228
20	0.0	0	0.0464	0.0486	0.0444	0.0500	0.0490	0.0434
		3	0.0228	0.0406	0.0208	0.0266	0.0290	0.0254
		5	0.0214	0.0868	0.0164	0.0220	0.0258	0.0234
	0.2	3	0.0212	0.0258	0.0232	0.0274	0.0324	0.0270
		5	0.0184	0.0182	0.0182	0.0244	0.0272	0.0244
30	0.0	0	0.0478	0.0464	0.0510	0.0448	0.0492	0.0496
		3	0.0272	0.0524	0.0208	0.0248	0.0334	0.0352
		5	0.0242	0.0100	0.0206	0.0248	0.0270	0.0272
	0.2	3	0.0234	0.0252	0.0236	0.0280	0.0282	0.0286
		5	0.0216	0.0258	0.0180	0.0286	0.0266	0.0256
50	0.0	0	0.0552	0.0504	0.0476	0.0482	0.0468	0.0536
		3	0.0318	0.0658	0.0262	0.0278	0.0308	0.0348
		5	0.0242	0.1612	0.0258	0.0290	0.0278	0.0360
	0.2	3	0.0282	0.0278	0.0268	0.0298	0.0280	0.0342
		5	0.0264	0.0302	0.0220	0.0284	0.0254	0.0400
100	0.0	0	0.0488	0.0476	0.0520	0.0478	0.0606	0.0456
		3	0.0302	0.0310	0.0320	0.0336	0.0454	0.0330
		5	0.2620	0.2044	0.0254	0.0324	0.0508	0.0324
	0.2	3	0.0284	0.0258	0.0258	0.0296	0.0334	0.0306
		5	0.0220	0.0278	0.0246	0.0322	0.0358	0.0286

- If the type I error is equal or large than the CV divided by number of replications

This study is conducted in research laboratory at Universiti Utara Malaysia started from mid of 2016 until end of the year to get the best and accurate results.

PERFORMANCE EVALUATION

To evaluate the performance of the test, different conditions were generated manipulating the number of variables (p), the number of observation (n) and various proportions of outliers (ξ) to add a level of contamination data and 0, 3 and 5 shifts values of mean. In this section, the authors compared the performance of three tests in terms of type I error rate and power of test respectively.

Type I error rate: The performance of the robust statistic is judged based on the type I error rate. To investigate the effect of contamination on the statistic performance the authors have to consider a contaminated model, used by Alfaro and Ortega³³ as follows:

$$(1-\xi) \text{MVN}_p(0, I_p) + \xi \text{MVN}_p(\mu, I_p) \quad (8)$$

where, ξ is the proportion of contamination data μ , the shift in mean and I_p is the identity matrix. It was assumed that contamination with the shift in the mean but no changes in the covariance matrix and consider levels of $\xi = 0, 0.1, 0.2$. The data was generated by using multivariate normal for independent variables. To examine the performance of the test in terms of type I error rates and the power of test, it was used the level of contaminations for the independent variables including no contamination, mild, moderate and extreme contamination the levels as follows:

- $\text{MVN}_p(0, I_p)$ no contamination
- $(0.9) \text{MVN}_p(0, I_p) + (0.1) \text{MVN}_p(3, I_p)$ Mild contamination
- $(0.8) \text{MVN}_p(0, I_p) + (0.2) \text{MVN}_p(3, I_p)$ Moderate contamination
- $(0.9) \text{MVN}_p(0, I_p) + (0.1) \text{MVN}_p(5, I_p)$ Moderate contamination
- $(0.8) \text{MVN}_p(0, I_p) + (0.2) \text{MVN}_p(5, I_p)$ Extreme contamination

The part of robustness in the hypothesis context is the ability of the procedures to control the type I error rate to

close to the significance level α . Robust statisticians are searching for test proceedings, which are fit to control the type I error at the significance value. The authors can consider a procedure robust if its empirical value of type I error is between $0.5\alpha \leq \hat{\alpha} \leq 1.5\alpha$, for the nominal level $\alpha = 0.05$; the empirical of the type I error rate should be in the interval $0.025 \leq \hat{\alpha} \leq 0.075$ ³⁴. Type I errors of less than 0.025 are considered conservative and, if the empirical is above 0.075 type I errors are considered liberal. However, if the significance level is 0.05, the test is considered robust if the empirical level of the type I error does not exceed the value 0.075. The data were generated by using Matlab 7.10.0.499 (R2010a). Table 1-3 which recorded the type I error for each condition are arranged based on the ascending number of variables, namely small ($p = 3$ and 5), medium ($p = 10$ and 15) and large number variables ($p = 20$).

In Table 1, type I error rate for the J statistic, the J_s statistic and the J_m statistic are recorded. The overall results from the table show that the J_s statistic is more robust compared to the J statistic and the J_m statistic. For $p = 3$, the value of the type I error of the J_s statistic falls within the robust interval except when $\mu = 5$. On the other hand, the J_m statistic for sample size $n = 10, 20$ and 30 is robust only under the condition when there are no outliers ($\mu = 0$). For the cases of $n = 50$ and 100 are not robust only when ($\mu = 5$) and the percentage of outliers 20%. The J statistic in the cases in which the sample size $n = 10, 20$ performs well under the condition no outliers. However, for the J statistic for sample size $n = 30, 50$ and 100 the type I error falls within robust interval for 9 conditions of the 15 conditions. Furthermore, for the dimension $p = 5$, the J_s statistic performs well in all conditions. On the other hand, the J statistic and the J_m statistic are robust in all conditions when the sample $n = 50$ and 100. For the sample size $n = 10, 20$ and 30, the J statistic falls within interval of robust for 8 of the 15 conditions while J_m statistic falls within the interval for robust for 9 conditions of the 15 conditions. Overall results in Table 1, show that J_s statistic under most conditions are well controlled.

The case of $p = 10$ is presented in Table 2 and the overall results on type I error show that the J_s statistic performs well and the results fall within the robust interval for all sample size under all conditions. On the other hand, the J statistic failed to control the type I error under conditions in which the percentage of outliers was 20% and $\mu = 5$. The performance of the J_m statistic is much better than the J statistic and the J_m statistic performs well

Table 2: Analysis of type I error rate for medium variable

n	ξ	μ	p = 10			p = 15		
			J	J_s	J_M	J	J_s	J_M
20	0.0	0	0.0468	0.0524	0.0460	0.0506	0.0458	0.0502
		3	0.0268	0.0352	0.0290	0.0286	0.0356	0.0274
	0.2	5	0.0272	0.0270	0.0270	0.0324	0.0358	0.0282
		3	0.0242	0.0306	0.0272	0.0248	0.0338	0.0266
		5	0.0210	0.0286	0.0242	0.0246	0.0346	0.0245
30	0.0	0	0.0518	0.0560	0.0578	0.0454	0.0520	0.0568
		3	0.0328	0.0298	0.0390	0.0286	0.0358	0.0396
	0.2	5	0.0342	0.0334	0.0346	0.0282	0.0336	0.0320
		3	0.0226	0.0312	0.0372	0.0246	0.0348	0.0356
		5	0.0246	0.0332	0.0245	0.0244	0.0342	0.0232
50	0.0	0	0.0462	0.0476	0.0486	0.0470	0.0468	0.0538
		3	0.0326	0.0358	0.0328	0.0412	0.0326	0.0340
	0.2	5	0.0318	0.0342	0.0264	0.0356	0.0330	0.0368
		3	0.0218	0.0348	0.0318	0.0380	0.0326	0.0334
		5	0.0202	0.0360	0.0237	0.0247	0.0344	0.0245
100	0.0	0	0.0466	0.0462	0.0464	0.0454	0.0468	0.0500
		3	0.0360	0.0378	0.0336	0.0374	0.0376	0.0424
	0.2	5	0.0386	0.0398	0.0358	0.0392	0.0332	0.0432
		3	0.0244	0.0364	0.0340	0.0326	0.0324	0.0450
		5	0.0248	0.0372	0.0247	0.2400	0.0320	0.0249

Table 3: Analysis of type I error rate for large variable

n	ξ	μ	p = 20		
			J	J_s	J_M
30	0.0	0	0.0542	0.0586	0.0522
		3	0.0298	0.0354	0.0326
	0.2	5	0.0332	0.0368	0.0304
		3	0.0257	0.0356	0.0310
		5	0.0237	0.0340	0.0236
40	0.0	0	0.0498	0.0448	0.0432
		3	0.0344	0.0316	0.0302
	0.2	5	0.0330	0.0338	0.0270
		3	0.0346	0.0334	0.0296
		5	0.0236	0.0336	0.0234
50	0.0	0	0.0550	0.0476	0.0484
		3	0.0340	0.0378	0.0344
	0.2	5	0.0336	0.0362	0.0348
		3	0.0324	0.0344	0.0389
		5	0.0230	0.0398	0.0372
100	0.0	0	0.0480	0.0528	0.0522
		3	0.0320	0.0492	0.0346
	0.2	5	0.0304	0.0408	0.0410
		3	0.0318	0.0336	0.0354
		5	0.0249	0.0364	0.0386

except for the percentage of outliers 20% and $\mu = 5$. For the case of $p = 15$ all the values of the J_s statistic fall within the interval. On the other hand, the J statistic for the sample size $n = 20$ and 30, 6 the 10 conditions fall within the interval. When the sample size is $n = 50$ and 100, 8 of the 10 conditions of the J_M statistic still perform as well as the case of $p = 10$.

For large number of variables $p = 20$, all the values of the type I error for the J_s statistic that fall within the interval and it still performs well under all conditions. On the other hand, the J_M statistic performs well and all the values are within the interval when the sample size $n = 50$ and 100 but when the sample size $n = 30$ and 40 it is robust except under the conditions the percentage of outliers 20% and $\mu = 5$. All the

values of the type I error for J statistic fall within the interval except the outliers 20% and $\mu = 5$ in Table 3.

Power of test: The investigation on the robustness of the improved the J statistic which is the J_S statistic and the J_M statistic has been shown in the previous section. The results showed that the J_S statistic performs well compared to the J_M statistic in terms of type I errors. In this study investigated the sensitivity analysis between the traditional statistic J statistic, the J_S statistic and the J_M statistic based on the power of test.

The power of test is defined as the probability of correctly rejecting the null hypothesis when it is false³⁵. Generally, the value of the power is between 0 and 1. The minimum value accepted is greater than 0.5³⁶. A value smaller than 0.5 is unexceptional. Usually, the desired power value is 0.8 and above³⁵. In this research, the statistic is considered to be powerful when the value is 0.8 and above and acceptable from 0.5-0.79. The values that were 0.5 and above are shaded in Table 4-8.

The results of examination are presented in Table 4-6. Each table represents the ascending number of variables $p = 3, 5, 10, 15$ and 20. The first column in each table displays the percentage of outliers (ξ) and followed by shift in the

matrix ρ and the mean shift; the following two columns present the power of J and J_S , respectively. The analysis is performed repeatedly for different sample sizes.

Table 4 and 5 display the power of test for $p = 3$ and 5. According to Table 4, for each sample size from $n = 10$ -100 the power value for the J statistic and the J_M statistic were always smaller than the J_S statistic.

When $p = 3$, there are 67, 59 and 53 values that fall within the interval of the power of test for the J_S statistic, the J_M statistic and the J statistic, respectively. When $p = 5$ there are 51, 25 and 37 values that fall within the interval of the power of test for the J_S statistic, the J_M statistic and the J statistic, respectively, in all conditions. We conclude that J_S statistic is more powerful compared to the two statistics.

For Table 6, for $p = 10$, there were 16, 10 and 14 values that fall within the interval of the power of test for the J_S statistic, the J_M statistic and the J statistic, respectively. According to Table 5, all the values fall within the interval when the sample size is large $n = 100$. When $p = 15$, from Table 7, there were 94, 0 and 1 values that fall within the interval of the power of test for the J_S statistic, the J_M statistic and the J statistic, respectively. It was concluded that the J_S statistic dominates the other two statistics. According to Table 8, for $p = 20$ (a large number of variables), there are

Table 4: Analysis of power of test for $p = 3$

ξ	ρ	μ	n = 10			n = 20			n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0	0.0	0	0.040	0.0500	0.0512	0.0494	0.0500	0.0482	0.0528	0.0500	0.0812	0.0536	0.0502	0.0548	0.0578	0.0500	0.0500
		0.1	0.073	0.0648	0.0878	0.1108	0.1004	0.1574	0.1694	0.1156	0.2432	0.2454	0.1644	0.5404	0.4856	0.3290	0.6134
		0.2	0.142	0.1188	0.2272	0.3194	0.2400	0.5334	0.4992	0.3410	0.7678	0.7646	0.5560	0.9254	0.9816	0.8900	0.9966
		0.3	0.282	0.2084	0.4474	0.6450	0.4764	0.8894	0.8638	0.6748	0.9848	0.9852	0.9056	0.9996	0.9998	0.9982	1
		0.4	0.468	0.3393	0.6758	0.8936	0.7256	0.7902	0.9864	0.9090	0.9994	0.9998	0.9922	1	1	1	1
		0.5	0.661	0.4966	0.8362	0.9838	0.9084	0.9920	0.9996	0.9807	1	1	1	1	1	1	1
		0.6	0.826	0.6502	0.9226	0.9986	0.9810	0.9996	1.0000	0.9992	1	1	1	1	1	1	1
0.1	0.0	0.7	0.941	0.7756	0.9744	1	0.9840	1	1	1	1	1	1	1	1	1	1
	3	0.0	0.018	0.032	0.047	0.025	0.035	0.034	0.026	0.031	0.052	0.029	0.034	0.067	0.031	0.034	0.106
		0.1	0.019	0.035	0.036	0.026	0.050	0.040	0.038	0.063	0.108	0.052	0.097	0.090	0.083	0.209	0.143
		0.2	0.016	0.046	0.031	0.037	0.114	0.048	0.091	0.175	0.225	0.185	0.314	0.187	0.310	0.638	0.740
		0.3	0.017	0.077	0.038	0.083	0.237	0.089	0.251	0.434	0.554	0.493	0.749	0.893	0.656	0.986	0.794
		0.4	0.019	0.115	0.039	0.257	0.437	0.590	0.618	0.736	0.777	0.932	0.962	0.967	1	1	0.959
		0.5	0.024	0.171	0.043	0.460	0.619	0.696	0.912	0.928	0.967	0.999	0.999	0.996	1	1	1
0.1	0.0	0.6	0.049	0.249	0.053	0.878	0.884	0.981	0.998	0.994	0.999	1	1	1	1	1	1
		0.7	0.106	0.323	0.177	0.987	0.988	0.926	1	1	1	1	1	1	1	1	1
	5	0.0	0.016	0.033	0.111	0.025	0.038	0.092	0.025	0.035	0.109	0.026	0.040	0.157	0.028	0.04	0.198
		0.1	0.021	0.036	0.085	0.023	0.057	0.070	0.027	0.070	0.105	0.036	0.116	0.146	0.045	0.242	0.215
		0.2	0.017	0.05	0.085	0.022	0.123	0.074	0.041	0.198	0.115	0.094	0.451	0.174	0.287	0.388	0.292
		0.3	0.018	0.055	0.077	0.028	0.253	0.089	0.105	0.469	0.174	0.363	0.782	0.508	0.876	0.991	0.551
		0.4	0.009	0.076	0.083	0.063	0.450	0.119	0.258	0.762	0.597	0.818	0.965	0.874	0.997	1	0.854
		0.5	0.006	0.166	0.071	0.145	0.695	0.593	0.754	0.939	0.646	0.988	1	0.989	1	1	0.979
		0.6	0.002	0.235	0.090	0.246	0.885	0.678	0.981	0.996	0.989	1	1	1	1	1	0.999
		0.7	0.001	0.341	0.076	0.647	0.979	0.712	1	0.999	1	1	1	1	1	1	1

Table 4: Continue

ξ	ρ	μ	n = 10			n = 20			n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0.2	0.0	3	0.021	0.017	0.023	0.022	0.023	0.022	0.024	0.022	0.024	0.026	0.029	0.027	0.030	0.031	0.038
	0.1		0.02	0.016	0.016	0.021	0.023	0.018	0.029	0.024	0.026	0.045	0.039	0.054	0.032	0.065	0.045
	0.2		0.014	0.015	0.024	0.034	0.023	0.022	0.037	0.035	0.040	0.132	0.092	0.253	0.331	0.328	0.157
	0.3		0.012	0.014	0.015	0.060	0.027	0.026	0.093	0.08	0.094	0.485	0.302	0.647	0.523	0.809	0.598
	0.4		0.010	0.011	0.012	0.156	0.047	0.058	0.160	0.242	0.254	0.895	0.692	0.947	0.96	0.992	0.962
	0.5		0.007	0.007	0.012	0.421	0.14	0.564	0.490	0.550	0.627	0.997	0.949	0.998	0.999	1	0.999
	0.6		0.011	0.006	0.017	0.795	0.377	0.839	0.853	0.873	0.923	1	0.999	1	1	1	1
	0.7		0.024	0.006	0.025	0.977	0.753	0.996	1	0.992	0.998	1	1	1	1	1	1
	0.0	5	0.021	0.020	0.021	0.019	0.029	0.020	0.026	0.026	0.028	0.026	0.033	0.027	0.023	0.036	0.03
	0.1		0.017	0.020	0.015	0.022	0.026	0.017	0.021	0.025	0.021	0.032	0.039	0.044	0.031	0.036	0.035
	0.2		0.013	0.018	0.019	0.017	0.021	0.018	0.025	0.028	0.025	0.069	0.073	0.13	0.27	0.063	0.302
	0.3		0.014	0.015	0.014	0.019	0.018	0.016	0.041	0.045	0.042	0.302	0.228	0.546	0.834	0.267	0.921
	0.4		0.01	0.012	0.012	0.041	0.017	0.014	0.150	0.132	0.161	0.770	0.603	0.869	0.998	0.754	0.999
	0.5		0.008	0.008	0.010	0.161	0.039	0.325	0.265	0.402	0.361	0.989	0.929	0.996	1	0.989	1
	0.6		0.002	0.003	0.005	0.524	0.165	0.640	0.765	0.795	0.807	1	0.999	1	1	1	1
	0.7		0.001	0.001	0.002	0.929	0.523	0.954	0.822	0.986	0.922	1	1	1	1	1	1

Table 5: Analysis of Power of test for $p = 5$

ξ	ρ	μ	n = 10			n = 20			n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0	0.0	0	0.0500	0.0500	0.0470	0.0474	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
	0.1		0.0644	0.0724	0.0986	0.1164	0.1118	0.1204	0.0470	0.1696	0.1840	0.3878	0.2816	0.3970	0.0460	0.6110	0.0480
	0.2		0.1278	0.1206	0.2214	0.3756	0.3408	0.5770	0.1990	0.5786	0.6270	0.9530	0.8492	0.9992	0.7468	0.9974	0.8088
	0.3		0.2442	0.1970	0.1994	0.7264	0.6742	0.7294	0.7012	0.9104	0.9502	1	0.9968	1	1	1	1
	0.4		0.3984	0.2904	0.5080	0.9404	0.8964	0.9456	0.9702	0.9948	0.9976	1	1	1	1	1	1
	0.5		0.5452	0.4030	0.64462	0.9944	0.9820	0.9960	1	1	1	1	1	1	1	1	1
	0.6		0.6938	0.5294	0.7868	0.9999	0.9980	1	1	1	1	1	1	1	1	1	1
0.1	0.0	3	0.022	0.028	0.028	0.033	0.03	0.035	0.031	0.031	0.032	0.033	0.036	0.0354	0.056	0.022	0.055
	0.1		0.020	0.029	0.027	0.024	0.034	0.028	0.035	0.046	0.030	0.040	0.044	0.0440	0.089	0.020	0.088
	0.2		0.023	0.031	0.033	0.028	0.048	0.032	0.039	0.095	0.042	0.100	0.103	0.1022	0.299	0.023	0.298
	0.3		0.026	0.033	0.032	0.033	0.067	0.031	0.055	0.233	0.071	0.388	0.579	0.5786	0.891	0.026	0.890
	0.4		0.025	0.032	0.031	0.032	0.108	0.035	0.088	0.430	0.171	0.886	0.929	0.9286	0.999	0.025	0.998
	0.5		0.020	0.026	0.036	0.036	0.212	0.037	0.253	0.847	0.512	0.998	1	0.9999	1	0.02	1
	0.6		0.013	0.017	0.024	0.080	0.406	0.083	0.662	0.979	0.893	1	1	1	1	0.013	1
0.1	0.0	5	0.005	0.015	0.021	0.242	0.647	0.545	0.964	1	0.996	1	1	1	1	0.005	1
	0.1		0.016	0.032	0.038	0.026	0.032	0.030	0.029	0.035	0.035	0.031	0.040	0.0400	0.057	0.016	0.056
	0.2		0.020	0.028	0.029	0.028	0.033	0.030	0.029	0.042	0.031	0.035	0.037	0.0364	0.068	0.02	0.067
	0.3		0.016	0.028	0.035	0.026	0.034	0.028	0.031	0.058	0.031	0.048	0.049	0.0484	0.138	0.016	0.137
	0.4		0.020	0.025	0.036	0.023	0.035	0.039	0.023	0.105	0.029	0.109	0.298	0.2980	0.502	0.020	0.501
	0.5		0.024	0.023	0.029	0.020	0.033	0.021	0.034	0.267	0.037	0.415	0.735	0.7350	0.939	0.024	0.938
	0.6		0.013	0.014	0.031	0.012	0.042	0.019	0.079	0.595	0.604	0.926	0.991	0.9908	1	0.013	0.999
0.2	0.0	3	0.021	0.017	0.038	0.024	0.026	0.032	0.024	0.028	0.030	0.032	0.037	0.0366	0.038	0.021	0.037
	0.1		0.021	0.016	0.056	0.024	0.025	0.025	0.033	0.025	0.034	0.038	0.040	0.0394	0.047	0.021	0.047
	0.2		0.019	0.015	0.157	0.025	0.025	0.026	0.027	0.025	0.030	0.067	0.114	0.1132	0.142	0.019	0.141
	0.3		0.021	0.014	0.653	0.021	0.021	0.022	0.04	0.023	0.043	0.229	0.501	0.5001	0.626	0.021	0.725
	0.4		0.018	0.014	0.989	0.023	0.015	0.029	0.05	0.019	0.052	0.718	0.921	0.9202	0.989	0.018	1
	0.5		0.012	0.011	1	0.016	0.010	0.017	0.105	0.023	0.147	0.989	1	0.9996	1	0.012	1
	0.6		0.011	0.008	1	0.008	0.006	0.009	0.362	0.104	0.563	1	1	1	1	0.011	1
0.2	0.0	5	0.006	0.004	1	0.01	0.001	0.011	0.851	0.497	0.960	1	1	1	1	0.006	1
	0.1		0.018	0.017	0.041	0.026	0.027	0.033	0.026	0.028	0.030	0.032	0.040	0.0396	0.037	0.018	0.036
	0.2		0.022	0.015	0.024	0.025	0.026	0.027	0.030	0.026	0.034	0.034	0.040	0.0392	0.044	0.022	0.043
	0.3		0.018	0.016	0.026	0.021	0.023	0.026	0.025	0.024	0.027	0.040	0.044	0.0434	0.092	0.018	0.091
	0.4		0.023	0.015	0.026	0.022	0.019	0.024	0.029	0.018	0.032	0.068	0.103	0.1022	0.290	0.023	0.289
	0.5		0.017	0.014	0.027	0.015	0.014	0.017	0.029	0.013	0.024	0.264	0.502	0.5015	0.884	0.017	0.883
	0.6		0.013	0.012	0.029	0.012	0.009	0.022	0.016	0.008	0.017	0.843	0.977	0.9762	1	0.013	0.999
0.2	0.0	5	0.013	0.009	0.025	0.003	0.005	0.012	0.037	0.003	0.039	0.999	1	1	1	0.013	1
	0.7		0.007	0.005	0.019	0.002	0.001	0.008	0.184	0.019	0.36	1	1	1	1	0.007	1

Table 6: Analysis of power of test for $p = 10$

ξ	ρ	μ	n = 20			n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0	0.0	0	0.0552	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0468
	0.1		0.1456	0.1304	0.1510	0.2482	0.2102	0.2538	0.5434	0.4828	0.5570	0.9474	0.9196	0.9468
	0.2		0.2854	0.2200	0.2948	0.6772	0.5508	0.6858	0.9299	0.9714	0.9866	1	1	1
	0.3		0.2730	0.2002	0.3516	0.9238	0.7996	0.9950	1	1	1	1	1	1
	0.4		0.2010	0.1186	0.2670	0.9944	0.9478	0.9998	1	1	1	1	1	1
	0.5		0.1136	0.0468	0.1872	0.9996	0.9906	1	1	1	1	1	1	1
	0.6		0.1188	0.0108	0.1668	1	0.9994	1	1	1	1	1	1	1
0.1	0.7		0.2632	0.0038	0.3284	1	1	1	1	1	1	1	1	1
	0.0	3	0.030	0.028	0.039	0.032	0.03	0.036	0.036	0.033	0.036	0.04	0.033	0.0416
	0.1		0.029	0.026	0.022	0.03	0.027	0.03	0.038	0.035	0.043	0.053	0.050	0.0582
	0.2		0.024	0.017	0.014	0.023	0.017	0.024	0.03	0.031	0.032	0.123	0.117	0.1996
	0.3		0.013	0.008	0.005	0.01	0.006	0.02	0.021	0.02	0.022	0.507	0.457	0.5968
	0.4		0.005	0.003	0.012	0.004	0.002	0.014	0.009	0.007	0.012	0.988	0.950	0.9932
	0.5		0.001	0.001	0.008	0.001	0.001	0.011	0.003	0.002	0.018	1	1	1
0.1	0.6	0	0	0	0.004	0	0	0.021	0.01	0.007	0.013	1	1	1
	0.7		0.001	0	0.002	0.034	0	0.009	0.002	0.066	0.012	1	1	1
	0.0	5	0.031	0.025	0.03	0.027	0.029	0.029	0.035	0.033	0.04	0.035	0.038	0.0410
	0.1		0.024	0.021	0.027	0.016	0.025	0.029	0.032	0.032	0.042	0.041	0.039	0.0488
	0.2		0.021	0.015	0.021	0.002	0.016	0.019	0.023	0.022	0.029	0.044	0.043	0.0514
	0.3		0.012	0.008	0.014	0.001	0.006	0.009	0.01	0.01	0.013	0.063	0.049	0.1100
	0.4		0.003	0.003	0.006	0.001	0.001	0.003	0.003	0.003	0.003	0.173	0.111	0.6500
0.2	0.5		0.002	0.001	0.01	0.001	0.001	0.008	0.001	0.001	0.002	0.865	0.636	1
	0.6	0	0	0	0.006	0.001	0	0.001	0	0	0.002	1	0.997	1
	0.7		0	0	0.003	0	0	0.002	0	0	0.001	1	1	1
	0.0	3	0.033	0.029	0.03	0.033	0.025	0.034	0.031	0.031	0.04	0.038	0.033	0.0444
	0.1		0.028	0.026	0.031	0.029	0.021	0.031	0.032	0.029	0.036	0.037	0.032	0.0420
	0.2		0.023	0.018	0.02	0.021	0.016	0.024	0.025	0.023	0.034	0.073	0.034	0.0770
	0.3		0.009	0.01	0.013	0.01	0.007	0.011	0.013	0.012	0.014	0.152	0.037	0.1938
0.2	0.4		0.004	0.003	0.017	0.003	0.002	0.034	0.004	0.003	0.011	0.786	0.059	0.8508
	0.5		0.001	0.001	0.014	0.001	0	0.031	0.001	0.001	0.002	0.999	0.381	0.9999
	0.6	0	0	0	0.011	0	0	0.024	0	0	0.003	1	0.990	1
	0.7		0	0	0.008	0	0	0.011	0	0	0.04	1	1	1
	0.0	5	0.033	0.027	0.029	0.03	0.027	0.035	0.032	0.03	0.041	0.035	0.033	0.0380
	0.1		0.034	0.026	0.029	0.032	0.023	0.028	0.032	0.029	0.041	0.037	0.032	0.0370
	0.2		0.022	0.019	0.023	0.017	0.016	0.02	0.022	0.022	0.039	0.036	0.027	0.0408
0.2	0.3		0.014	0.01	0.011	0.009	0.008	0.009	0.01	0.011	0.012	0.038	0.021	0.0384
	0.4		0.007	0.004	0.015	0.003	0.002	0.024	0.002	0.003	0.006	0.036	0.013	0.0406
	0.5		0.001	0.001	0.012	0.001	0	0.002	0	0.002	0.001	0.412	0.008	0.5498
	0.6	0	0	0	0.011	0	0	0.002	0	0	0.001	0.998	0.074	0.9996
	0.7		0	0	0.002	0	0	0.002	0	0	0.001	1	0.982	1

Table 7: Analysis of power of test for $p = 15$

ξ	ρ	μ	n = 20			n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0	0.0	0	0.0500	0.0500	0.0500	0.050	0.050	0.050	0.050	0.050	0.050	0.056	0.05	0.056
	0.1		0.1074	0.0914	0.1896	0.138	0.175	0.243	0.366	0.471	0.2116	0.048	0.95	0.274
	0.2		0.0996	0.0784	0.2248	0.214	0.170	0.827	0.5270	0.816	0.7656	0.984	1	1
	0.3		0.0372	0.0284	0.3883	0.059	0.043	0.996	0.903	0.888	0.9911	1	1	1
	0.4		0.0166	0.0052	0.5276	0.014	0.002	1	0.980	0.957	1	1	1	1
	0.5		0.0066	0.0002	0.6078	0.012	0	1	0.999	0.997	1	1	1	1
	0.6		0.0014	0.0000	0.6404	0.008	0	1	1	0.999	1	1	1	1
0.1	0.7		0.0004	0.0000	0.7696	0.006	0	1	1	1	1	1	1	1
	0.0	3	0.031	0.020	0.090	0.033	0.028	0.095	0.042	0.034	0.0943	0.042	0.0380	0.050
	0.1		0.027	0.018	0.187	0.030	0.024	0.264	0.035	0.026	0.2640	0.038	0.0348	1
	0.2		0.016	0.012	0.444	0.021	0.012	0.676	0.014	0.012	0.6755	0.031	0.0196	1
	0.3		0.006	0.005	0.791	0.02	0.002	0.962	0.008	0.001	0.9612	0.008	0.0030	1
	0.4		0.002	0.002	0.970	0.012	0.001	0.999	0.003	0	0.9986	0.001	0.0000	1
	0.5		0.001	0.001	0.999	0.008	0	1	0.002	0	1	0	0.0000	1
0.1	0.6		0.001	0	1	0.004	0	1	0.001	0	1	0.013	0.0000	1
	0.7		0	0	1	0.001	0	1	0.001	0	1	0.691	0.0090	1

Table 7: Continue

ξ_s	ρ	μ	n = 20			n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0.1	0.0	5	0.034	0.020	0.273	0.029	0.026	0.336	0.039	0.034	0.3353	0.040	0.0410	0.060
	0.1		0.029	0.018	0.29	0.023	0.024	0.371	0.033	0.027	0.3709	0.038	0.0340	0.189
	0.2		0.017	0.013	0.501	0.014	0.013	0.655	0.014	0.014	0.6542	0.015	0.0130	0.658
	0.3		0.01	0.008	0.742	0.013	0.004	0.921	0.009	0.002	0.9204	0.008	0.0014	0.768
	0.4		0.004	0.003	0.937	0.011	0.001	0.995	0.003	0.001	0.9948	0.004	0.0000	0.896
	0.5		0.002	0.001	0.994	0.009	0.001	1	0.002	0	0.9999	0.002	0.0000	0.980
	0.6		0.001	0.001	1	0.003	0	1	0.001	0	1	0.001	0.0000	1
0.2	0.0	3	0.001	0	1	0.001	0	1	0.001	0	1	0.001	0.0000	1
	0.1		0.03	0.025	0.067	0.028	0.026	0.071	0.044	0.037	0.0702	0.043	0.0410	0.058
	0.2		0.023	0.023	0.162	0.027	0.023	0.241	0.034	0.029	0.2408	0.038	0.0366	0.236
	0.3		0.018	0.016	0.38	0.011	0.003	0.607	0.014	0.012	0.6070	0.02	0.0166	0.671
	0.4		0.01	0.007	0.725	0.004	0.001	0.919	0.007	0.003	0.9189	0.009	0.0018	0.789
	0.5		0.003	0.004	0.939	0.002	0	0.996	0.004	0	0.9959	0.005	0.0000	0.9876
	0.6		0.002	0.001	0.997	0.003	0	1	0.002	0	0.9998	0.003	0.0000	0.9999
0.2	0.0	5	0.001	0	1	0.001	0	1	0.001	0	1	0.002	0.0000	1.0000
	0.1		0.001	0	1	0.001	0	1	0.001	0	1	0.001	0.0000	1.0000
	0.2		0.031	0.027	0.066	0.03	0.027	0.078	0.044	0.037	0.0772	0.04	0.0418	0.0675
	0.3	252	0.021	0.019	0.162	0.025	0.022	0.235	0.034	0.03	0.2341	0.034	0.0340	0.2126
	0.4		0.011	0.01	0.345	0.016	0.015	0.542	0.015	0.014	0.5416	0.013	0.0152	0.6754
	0.5		0.005	0.005	0.632	0.015	0.004	0.874	0.003	0.004	0.8733	0.008	0.0016	0.8754
	0.6		0.002	0.002	0.896	0.01	0.002	0.986	0.002	0.001	0.9860	0.002	0.0000	0.9756
0.2	0.0		0.001	0.001	0.987	0.004	0	1	0.001	0	0.9999	0.002	0.0000	0.9999
	0.1		0.001	0.001	1	0.002	0	1	0.001	0	1	0.001	0.0000	1.0000
	0.2		0.001	0	1	0.001	0	1	0.001	0	1	0	0.0000	1.0000
	0.3		0.001	0	1	0.001	0	1	0.001	0	1	0	0.0000	1.0000
	0.4		0.001	0	1	0.001	0	1	0.001	0	1	0	0.0000	1.0000
	0.5		0.001	0	1	0.001	0	1	0.001	0	1	0	0.0000	1.0000
	0.6		0.001	0	1	0.001	0	1	0.001	0	1	0	0.0000	1.0000

Table 8: Analysis of power of test for $p = 20$

ξ_s	ρ	μ	n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0	0.0	0	0.050	0.050	0.050	0.050	0.050	0.050	0.056	0.05	0.056
	0.1		0.138	0.131	0.243	0.366	0.355	0.243	0.048	0.95	0.274
	0.2		0.059	0.042	0.827	0.225	0.192	0.827	0.984	1	1
	0.3		0.036	0.001	0.996	0.008	0.006	0.996	1	1	1
	0.4		0.014	0	1	0.003	0	1	1	1	1
	0.5		0.012	0	1	0.002	0	1	1	1	1
	0.6		0.008	0	1	0.001	0	1	1	1	1
0.1	0.0	3	0.006	0	1	0.001	0	1	1	1	1
	0.1		0.034	0.030	0.093	0.035	0.032	0.045	0.045	0.038	0.050
	0.2		0.028	0.026	0.284	0.024	0.022	0.294	0.029	0.027	0.230
	0.3		0.017	0.011	0.740	0.005	0.004	0.788	0.006	0.004	0.675
	0.4		0.008	0.002	0.977	0	0	0.989	0.003	0	0.876
	0.5		0.006	0.001	1	0	0	1	0.002	0	0.898
	0.6		0.004	0	1	0	0	1	0.002	0	0.955
0.1	0.0	5	0.003	0	1	0	0	1	0.001	0	1
	0.1		0.001	0	1	0	0	1	0	0	1
	0.2		0.028	0.029	0.046	0.036	0.05	0.550	0.043	0.038	0.065
	0.3		0.028	0.026	0.353	0.024	0.032	0.346	0.031	0.028	0.253
	0.4		0.013	0.015	0.681	0.006	0.022	0.690	0.006	0.006	0.743
	0.5		0.008	0.004	0.943	0.002	0.006	0.910	0.004	0	0.855
	0.6		0.002	0.001	0.997	0.002	0.001	0.950	0.002	0	0.899
0.2	0.0	3	0.002	0	1	0.001	0	1	0.001	0	0.944
	0.1		0.001	0	1	0.001	0	1	0.001	0	0.979
	0.2		0.001	0	1	0	0	1	0	0	1
	0.3		0.036	0.029	0.087	0.034	0.034	0.049	0.044	0.043	0.058
	0.4		0.025	0.024	0.252	0.022	0.024	0.259	0.029	0.028	0.263
	0.5		0.013	0.009	0.609	0.007	0.007	0.605	0.014	0.005	0.601
	0.6		0.009	0.003	0.905	0.002	0	0.910	0.009	0	0.915
0.2	0.0	5	0.005	0.001	0.993	0.002	0	0.950	0.003	0	0.996
	0.1		0.002	0	1	0.001	0	1	0.002	0	1
	0.2		0.001	0	1	0	0	1	0.001	0	1
	0.3		0.001	0	1	0	0	1	0.001	0	1
	0.4		0.001	0	1	0	0	1	0.001	0	1
	0.5		0.001	0	1	0	0	1	0.001	0	1
	0.6		0.001	0	1	0	0	1	0.001	0	1

Table 8: Continue

ξ	ρ	μ	n = 30			n = 50			n = 100		
			J	J_M	J_S	J	J_M	J_S	J	J_M	J_S
0.2	0.0	5	0.035	0.031	0.039	0.038	0.033	0.040	0.045	0.042	0.068
	0.1		0.028	0.026	0.045	0.024	0.026	0.029	0.037	0.028	0.125
	0.2		0.016	0.014	0.035	0.010	0.010	0.049	0.029	0.007	0.354
	0.3		0.009	0.005	0.041	0.003	0.001	0.066	0.013	0.001	0.833
	0.4		0.009	0.001	0.145	0.002	0	0.151	0.008	0	0.895
	0.5		0.004	0	0.959	0.001	0	0.959	0.003	0	1
	0.6		0.003	0	1	0	0	1	0.001	0	1
	0.7		0.001	0	1	0	0	1	0	0	1

65, 0 and 0 values that fall within the interval of the power of test for the J_S statistic, the J_M statistic and the J statistic, respectively. The J_S statistic still dominates the other two statistics.

CONCLUSION

The Jennrich statistic is known as a popular test for testing two or several correlation matrices, however, under conditions of non-normality, this test is known to under perform. Alternative test statistics are recommended to produce active methods regardless of the conditions. In this study, we proposed alternative methods to the Jennrich statistic by using a robust estimator known as the S estimator for scatter matrix. The S estimator not only has the properties of the well-known M estimator such as the affine equivariant and a high breakdown point and has a better computational. The performance of the proposed robust test by using the S estimator (J_S) and by using the M estimator (J_M) was compared with the J statistic in terms of the type I error rate and the power of test. The simulation study showed that statistic performs well in terms of controlling type I errors and the power of test.

SIGNIFICANCE STATEMENTS

The study discovers the robust approach that can be beneficial for testing the equality of two or more difference correlation matrices at the time when the data contains outlier. This study will help the researcher to uncover the critical areas of multivariate outlier that many researchers were not able to explore. The new theory on robust J statistic may be arrived.

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