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Research Article
New Interval-valued Intuitionistic Fuzzy Soft Operators and their Properties

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Abstract

Background and Objective: The study on interval-valued intuitionistic fuzzy soft sets has developed quite rapidly. Related results are already widely used in real life. In this paper, two new operators on interval-valued intuitionistic fuzzy soft sets are defined and some their properties are studied. **Materials and Methods:** The definitions of related previous operators inspired the definition of the new interval-valued intuitionistic fuzzy soft operators. By combining the previous and the new operators, some related properties are constructed. **Results:** By giving sufficient conditions, it was obtained some interested results describing the relationship between the previous and the new interval-valued intuitionistic fuzzy soft operators. **Conclusion:** These results have enriched the study on interval-valued intuitionistic fuzzy soft operators, specifically the relationship between operations in interval-valued intuitionistic fuzzy soft sets.

Key words: Fuzzy sets, fuzzy soft sets, intuitionistic fuzzy soft sets, interval-valued fuzzy soft sets, fuzzy soft operators

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INTRODUCTION

Fuzzy set is a theory which has progressed in many knowledge fields and in various ways. One of them is research by Nazra[1] who studied on ideals and their fuzzifications in implicative semi groups as a study in fuzzy algebra. It is known that study on fuzzy sets or fuzzy soft sets are the studies that consider the membership (and non-membership) value of an element in a set. For that, it was proposed a concept on hesitant fuzzy soft set, intuitionistic fuzzy soft set, hesitant intuitionistic fuzzy soft set, etc[2-5].

The study on interval-valued intuitionistic fuzzy soft sets (IVIFSSs) is a study in the field of fuzzy sets where the membership and non-membership degree of an element in a set are intervals in interval [0,1]. Atanassov and Gargov[6] introduced the theory on interval-valued intuitionistic fuzzy sets (IVIFSs). By combining the concept of interval-valued fuzzy sets (IVFSs)[7] and soft sets. Yang *et al*.[8] constructed the concept of interval-valued fuzzy soft sets (IVFSSs).

Jiang *et al*.[9] presented the concept of IVIFSSs as a combination of two fuzzy set models, namely, intuitionistic fuzzy soft sets and IVFSSs. They defined some operations on the IVIFSSs called by complement, "and", "or", union, intersection, necessity and possibility operations. Then they derived and proved some basic properties of the IVIFSSs related to such operations. On the other hand, Shanthi *et al*.[10] have studied the other operators in IVIFSSs called concentration and dilatation operators.

The aim of this article is to propose two new operators on IVIFSSs and discuss their properties with respect to the "and", "or", union and intersection operations introduced by Jiang *et al*.[9]. This work as compound new operators defined and some operation well-known is very useful in dealing with decision making problems. Thus, these results might contribute useful concept to develop the properties of IVIFSSs.

MATERIALS AND METHODS

It is reviewed some definitions in Jiang *et al*.[9] such as: interval-valued intuitionistic fuzzy soft sets (IVIFSSs), operations "and", "or", union and intersection on IVIFSSs.

Definition 1: Let U be an universal set and E be a parameters sets. A pair is said an interval-valued intuitionistic fuzzy soft sets (IVIFSSs) over U, if F is a mapping given by F:A→IVIF (μ), where IVIF (μ) is the set of all interval-valued intuitionistic fuzzy sets (IVIFSs) over U and A⊆E. Concretely[9]:

$$\langle F, A\rangle = \left\{\langle e_i, F(e_i)\rangle \mid F(e_i)\in \mathrm{IVIF}(u), e_i \in A\right\}$$

Where:

$$F(e_i) = \left\{\left\langle x, \mu_{F(e_i)}(x), \gamma_{F(e_i)}(x)\right\rangle \mid x\in U,\right\}, e_i \in A$$

with:

$$\mu_{F(e_i)}(x) = \left[\underline{\mu}_{F(e_i)}(x), \overline{\mu}_{F(e_i)}(x)\right]$$

$$\gamma_{F(e_i)}(x) = \left[\underline{\gamma}_{F(e_i)}(x), \overline{\gamma}_{F(e_i)}(x)\right]$$

are membership and non-membership degree of x∈U, respectively on the set ⟨F, A⟩.

Definition 2: Suppose that ⟨F, A⟩ and ⟨G, B⟩ are two IVIFSSs over U. Then, operation "and" on ⟨F, A⟩ and ⟨G, B⟩ denoted by ⟨F, A⟩∧⟨G, B⟩ = ⟨H, A×B⟩ is an IVIFSS where[9]:

$$H(\alpha, \beta) = F(\alpha)\cap G(\beta), \forall(\alpha, \beta) \in A\times B$$

and:

$$H(\alpha, \beta)(x) = \Big\langle \left[\inf\left(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\right), \inf\left(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\right)\right]$$

$$\left[\sup\left(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\beta)}(x)\right), \sup\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\right)\right] \Big\rangle, \forall(\alpha, \beta)\in(A\times B), x\in U$$

Definition 3: Let and be two IVIFSSs over U. Then operation "or" on ⟨F, A⟩ and ⟨G, B⟩ denoted by ⟨F, A⟩∨⟨G, B⟩ = ⟨O, A×B⟩ is an IVIFSS where[9]:

$$O(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall(\alpha, \beta)\in (A\times B)$$

and:

$$O(\alpha, \beta)(x) = \Big\langle \left[\sup\left(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\right), \sup\left(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\right)\right]$$

$$\left[\inf\left(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\beta)}(x)\right), \inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\right)\right] \Big\rangle, \forall(\alpha, \beta)\in(A\times B), x\in U$$

Definition 4: Union of two IVIFSSs ⟨F, A⟩ and ⟨G, B⟩ over U denoted by ⟨F, A⟩ ⋓ ⟨G, B⟩, is an IVIFSS⟨H, C⟩ where, C = A∪B and ∀ α∈C[9]:

$$\mu_{H(\alpha)}(x)=\begin{cases}\mu_{F(\alpha)}(x),\ \text{if}\ \alpha\in A-B,\ x\in U\\ \mu_{G(\alpha)}(x),\ \text{if}\ \alpha\in B-A,\ x\in U\\ \left[\sup\left(\underline{\mu}_{F(\alpha)}(x),\underline{\mu}_{G(\alpha)}(x)\right),\ \sup\left(\overline{\mu}_{F(\alpha)}(x),\overline{\mu}_{G(\alpha)}(x)\right)\right],\ \text{if}\ \alpha\in A\cap B,\ x\in U\end{cases}$$

$$\gamma_{H(\alpha)}(x)=\begin{cases}\gamma_{F(\alpha)}(x),\ \text{if}\ \alpha\in A-B,\ x\in U\\ \gamma_{G(\alpha)}(x),\ \text{if}\ \alpha\in B-A,\ x\in U\\ \left[\inf\left(\underline{\gamma}_{F(\alpha)}(x),\underline{\gamma}_{G(\alpha)}(x)\right),\inf\left(\overline{\gamma}_{F(\alpha)}(x),\overline{\gamma}_{G(\alpha)}(x)\right)\right],\ \text{if}\ \alpha\in A\cap B,\ x\in U\end{cases}$$

Definition 5: Intersection of two IVIFSSs ⟨F, A⟩ and ⟨G, B⟩ over U denoted by ⟨F, A⟩ ⋒ ⟨G, B⟩, is an IVIFSSs ⟨H, C⟩ where C = A∪B and ∀ α∈C[9]:

$$\mu_{H(\alpha)}(x)=\begin{cases}\mu_{F(\alpha)}(x),\ \text{if}\ \alpha\in A-B,\ x\in U\\ \mu_{G(\alpha)}(x),\ \text{if}\ \alpha\in B-A,\ x\in U\\ \left[\inf\left(\underline{\mu}_{F(\alpha)}(x),\underline{\mu}_{G(\alpha)}(x)\right),\inf\left(\overline{\mu}_{F(\alpha)}(x),\overline{\mu}_{G(\alpha)}(x)\right)\right],\ \text{if}\ \alpha\in A\cap B,\ x\in U\end{cases}$$

$$\gamma_{H(\alpha)}(x)=\begin{cases}\gamma_{F(\alpha)}(x),\ \text{if}\ \alpha\in A-B,\ x\in U\\ \gamma_{G(\alpha)}(x),\ \text{if}\ \alpha\in B-A,\ x\in U\\ \left[\inf\left(\underline{\gamma}_{F(\alpha)}(x),\ \gamma_{G(\alpha)}(x)\right),\sup\left(\overline{\gamma}_{F(\alpha)}(x),\overline{\gamma}_{G(\alpha)}(x)\right)\right],\ \text{if}\ \alpha\in A\cap B,\ x\in U\end{cases}$$

Definition 6: Assume that U is an initial universe and E is a set of parameters[9]. Let A, B ⊆E, ⟨F, A⟩ and ⟨G, B⟩ be two interval-valued intuitionistic fuzzy sets. ⟨F, A⟩ is an interval-valued intuitionistic fuzzy soft subset of ⟨G, B⟩, denoted by ⟨F, A⟩ ⋐ ⟨G, B⟩, if:

- A⊆B
- ∀α∈ A, F(α) is an interval-valued intuitionistic fuzzy subset of G(α), that is, for all x∈U and α∈A and:

$$\underline{\mu}_{F(\alpha)}(x)\leq\underline{\mu}_{G(\alpha)}(x)$$

$$\overline{\mu}_{F(\alpha)}(x)\leq\overline{\mu}_{G(\alpha)}(x)$$

$$\underline{\gamma}_{F(\alpha)}(x)\geq\underline{\gamma}_{G(\alpha)}(x)$$

$$\overline{\gamma}_{F(\alpha)}(x)\geq\overline{\gamma}_{G(\alpha)}(x)$$

RESULTS

Jiang *et al.*[9] defined some operators. In this paper it is defined two new operators. Then it is proved three theorems related to such new operators.

Definition 7: The operation Δ on interval-valued intuitionistic fuzzy soft sets ⟨F, A⟩ is defined as:

$$\Delta\left\langle F,A\right\rangle=\left\{\left\langle x,\ \mu_{\Delta F(\alpha)}(x),\ \gamma_{\Delta F(\alpha)}(x)\right\rangle\ |\ x\in U,\ \alpha\in A\right\}$$

Where:

$$\mu_{\Delta F(\alpha)}(x)=\left[\overline{\gamma}_{F(\alpha)}(x),\ 1-\overline{\mu}_{F(\alpha)}(x)\right]$$

and:

$$\gamma_{\Delta F(\alpha)}(x) = \left[1 - \overline{\mu}_{F(\alpha)}(x),\ 1 - \underline{\mu}_{F(\alpha)}(x)\right]$$

Definition 8: Operation ∇ on an interval-valued intuitionistic fuzzy soft sets ⟨F, A⟩ is defined as:

$$\nabla \langle F, A\rangle = \left\{\left\langle x,\ \mu_{\Delta\, F(\alpha)}(x),\ \gamma_{\nabla\, F(\alpha)}(x)\right\rangle \ |\ x\in U,\ \alpha\in A\right\}$$

Where:

$$\mu_{\nabla F(\alpha)}(x) = \left[\overline{\mu}_{F(\alpha)}(x),\ 1 - \overline{\gamma}_{F(\alpha)}(x)\right]$$

and:

$$\gamma_{\nabla F(\alpha)}(x) = \left[1 - \overline{\gamma}_{F(\alpha)}(x),\ \underline{\gamma}_{F(\alpha)}(x)\right]$$

Theorem 1: Suppose that ⟨F, A⟩ and ⟨G, B⟩ are interval-valued intuitionistic fuzzy soft sets over U, then the following properties hold:

(1) If $\overline{\mu}_{F(\alpha)}$ (x) = $\overline{\mu}_{G(\alpha)}$ (x) for any α∈A∩B then Δ (⟨F, A⟩ ⋓ ⟨G, B⟩) = Δ ⟨F, A⟩ ⋒ Δ⟨G, B⟩
(2) If $\overline{\mu}_{F(\alpha)}$ (x) = $\overline{\mu}_{G(\alpha)}$ (x) for any α∈A∩B then Δ (⟨F, A⟩) ⋒ ⟨G, B⟩) = Δ ⟨F, A⟩ ⋓ Δ ⟨G, B⟩
(3) If $\overline{\gamma}_{F(\alpha)}$ (x) = $\overline{\gamma}_{G(\alpha)}$ (x) for any α∈A∩B then ∇ (⟨F, A⟩) ⋓ ⟨G, B⟩) = ∇ ⟨F, A⟩ ⋓ ∇ ⟨G, B⟩
(4) If $\overline{\gamma}_{F(\alpha)}$ (x) = $\overline{\gamma}_{G(\alpha)}$ (x) for any α∈A∩B then ∇ (⟨F, A⟩) ⋒ ⟨G, B⟩) = ∇ ⟨F, A⟩ ⋒ ∇ ⟨G, B⟩

Proof: Here it is given the proof of (1) and (3). The others are similar.

Suppose that ⟨F, A⟩ ⋓ ⟨G, B⟩ = ⟨H, C⟩, with C = A∪B and α∈C, by definition 4:

$$\mu_{H(\alpha)}(x) = \begin{cases} \mu_{F(\alpha)}(x),\ \text{if } \alpha \in A - B,\ x \in U \\ \mu_{G(\alpha)}(x),\ \text{if } \alpha \in B - A,\ x \in U \\ \left[\sup\left(\underline{\mu}_{F(\alpha)}(x),\ \underline{\mu}_{G(\alpha)}(x)\right),\ \sup\left(\overline{\mu}_{F(\alpha)}(x),\ \overline{\mu}_{G(\alpha)}(x)\right)\right], \text{if } \alpha\in A \cap B,\ x\in U \end{cases}$$

$$\gamma_{H(\alpha)}(x) = \begin{cases} \gamma_{F(\alpha)}(x),\ \text{if } \alpha \in A - B,\ x \in U \\ \gamma_{G(\alpha)}(x),\ \text{if } \alpha \in B - A,\ x \in U \\ \left[\inf\left(\underline{\gamma}_{F(\alpha)}(x),\ \underline{\gamma}_{G(\alpha)}(x)\right),\ \inf\left(\overline{\gamma}_{F(\alpha)}(x),\ \overline{\gamma}_{G(\alpha)}(x)\right)\right], \text{if } \alpha\in A \cap B,\ x\in U \end{cases}$$

Since Δ (⟨F, A⟩ ⋓ ⟨G, B⟩) = Δ ⟨H, C⟩ then by Definition 7:

$$\mu_{\Delta H(\alpha)}(x) = \begin{cases} \left[\overline{\gamma}_{F(\alpha)}(x),\ 1 - \overline{\mu}_{F(\alpha)}(x),\ \text{if } \alpha \in A - B,\ x \in U\right] \\ \left[\overline{\gamma}_{G(\alpha)}(x),\ 1 - \overline{\mu}_{G(\alpha)}(x),\ \text{if } \alpha \in B - A,\ x \in U\right] \\ \left[\inf\left(\overline{\gamma}_{F(\alpha)}(x),\ \overline{\gamma}_{G(\alpha)}(x)\right), 1 - \sup\left(\overline{\mu}_{F(\alpha)}(x),\ \overline{\mu}_{G(\alpha)}(x)\right)\right], \text{if } \alpha\in A \cap B,\ x\in U \end{cases}$$

$$\gamma_{\Delta H(\alpha)}(x) = \begin{cases} \left[1 - \overline{\mu}_{F(\alpha)}(x),\ \overline{\mu}_{F(\alpha)}(x)\right],\ \text{if } \alpha \in A - B,\ x \in U \\ \left[1 - \overline{\mu}_{G(\alpha)}(x),\ \overline{\mu}_{G(\alpha)}(x)\right],\ \text{if } \alpha \in B - A,\ x \in U \\ \left[1 - \sup\left(\overline{\mu}_{F(\alpha)}(x),\ \overline{\mu}_{G(\alpha)}(x)\right), \sup\left(\underline{\mu}_{F(\alpha)}(x),\ \underline{\mu}_{G(\alpha)}(x)\right)\right], \text{if } \alpha\in A \cap B,\ x\in U \end{cases}$$

By applying operation Δ on ⟨F, A⟩ and ⟨G, B⟩:

$$\Delta\langle F, A\rangle = \left\{ x,\ \mu_{\Delta F(\alpha)}(x),\ \gamma_{\Delta F(\alpha)}(x)\ \ |\ x\in U,\ \alpha\in A\right\}$$

$$= \left\{ x, \left[\bar{\gamma}_{F(\alpha)}(x),\, 1 - \bar{\mu}_{F(\alpha)}(x)\right], \left[1 - \bar{\mu}_{F(\alpha)}(x), \underline{\mu}_{F(\alpha)}(x)\right] \mid x \in U,\, \alpha \in A \right\}$$

$$\Delta\langle G, B\rangle = \left\{ x,\, \mu_{\Delta G(\alpha)}(x),\, \gamma_{\Delta G(\alpha)}(x) \mid x \in U,\, \alpha \in B \right\}$$

$$= \left\{ x, \left[\bar{\gamma}_{G(\alpha)}(x),\, 1 - \bar{\mu}_{G(\alpha)}(x)\right], \left[1 - \bar{\mu}_{G(\alpha)}(x), \underline{\mu}_{G(\alpha)}(x)\right] \mid x \in U,\, \alpha \in B \right\}$$

Suppose that $\Delta\langle F, A\rangle \Cap \Delta\langle G, B\rangle = \langle O, C\rangle$, with $C = A \cup B$ and $A \in C$. By definition 5:

$$\mu_{0(\alpha)}(x)$$

$$= \begin{cases} \left[\bar{\gamma}_{F(\alpha)}(x), 1 - \bar{\mu}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A - B,\, x \in U \\ \left[\bar{\gamma}_{G(\alpha)}(x), 1 - \bar{\mu}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B - A,\, x \in U \\ \left[\inf\left(\bar{\gamma}_{F(\alpha)}(x), \bar{\gamma}_{G(\alpha)}(x)\right),\, \inf\left(1 - \bar{\mu}_{F(\alpha)}(x),\, 1 - \bar{\mu}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B,\, x \in U \end{cases}$$

$$= \begin{cases} \left[\bar{\gamma}_{F(\alpha)}(x), 1 - \bar{\mu}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A - B,\, x \in U \\ \left[\bar{\gamma}_{G(\alpha)}(x), 1 - \bar{\mu}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B - A,\, x \in U \\ \left[\inf\left(\bar{\gamma}_{F(\alpha)}(x), \bar{\gamma}_{G(\alpha)}(x)\right), 1 - \sup\left(\bar{\mu}_{F(\alpha)}(x), \bar{\mu}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B,\, x \in U \end{cases}$$

$$\gamma_{0(\alpha)}(x)$$

$$= \begin{cases} \left[1 - \bar{\mu}_{F(\alpha)}(x),\, \underline{\mu}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A - B,\, x \in U \\ \left[1 - \bar{\mu}_{G(\alpha)}(x),\, \underline{\mu}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B - A,\, x \in U \\ \left[\sup\left(1 - \bar{\mu}_{F(\alpha)}(x), 1 - \bar{\mu}_{G(\alpha)}(x)\right),\, \sup\left(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B,\, x \in U \end{cases}$$

$$= \begin{cases} \left[1 - \bar{\mu}_{F(\alpha)}(x),\, \underline{\mu}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A - B,\, x \in U \\ \left[1 - \bar{\mu}_{G(\alpha)}(x),\, \underline{\mu}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B - A,\, x \in U \\ \left[1 - \inf\left(\bar{\mu}_{F(\alpha)}(x),\, \bar{\mu}_{G(\alpha)}(x)\right),\, \sup\left(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B,\, x \in U \end{cases}$$

By assumption, $\bar{\mu}_{F(\alpha)}(x) = \bar{\mu}_{G(\alpha)}(x)$ for any $\alpha \in A \cap B$. Therefore:

$$1 - \inf\left(\bar{\mu}_{F(\alpha)}(x), \bar{\mu}_{G(\alpha)}(x)\right) = 1 - \sup\left(\bar{\mu}_{F(\alpha)}(x), \bar{\mu}_{G(\alpha)}(x)\right)$$

Hence:

$$\gamma_{O(\alpha)}(x)=\begin{cases}\left[1-\overline{\mu}_{F(\alpha)}(x),\ \underline{\mu}_{F(\alpha)}(x)\right], \text{ if } \alpha\in A-B,\ x\in U\\ \left[1-\overline{\mu}_{G(\alpha)}(x),\ \underline{\mu}_{G(\alpha)}(x)\right], \text{ if } \alpha\in B-A,\ x\in U\\ \left[1-\sup\left(\overline{\mu}_{F(\alpha)}(x),\ \overline{\mu}_{G(\alpha)}(x)\right),\ \sup\left(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha\in A\cap B,\ x\in U\end{cases}$$

Suppose that ⟨F, A⟩ ⋓ ⟨G, B⟩ = ⟨H, C⟩ with C = A∪B and A∈C. By definition 4:

$$\mu_{H(\alpha)}(x)=\begin{cases}\mu_{F(\alpha)}(x), \text{ if } \alpha\in A-B,\ x\in U\\ \mu_{G(\alpha)}(x), \text{ if } \alpha\in B-A,\ x\in U\\ \left[\sup\left(\underline{\mu}_{F(\alpha)}(x),\ \underline{\mu}_{G(\alpha)}(x)\right),\ \sup\left(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha\in A\cap B,\ x\in U\end{cases}$$

$$\gamma_{H(\alpha)}(x)=\begin{cases}\gamma_{F(\alpha)}(x), \text{ if } \alpha\in A-B,\ x\in U\\ \gamma_{G(\alpha)}(x), \text{ if } \alpha\in B-A,\ x\in U\\ \left[\inf\left(\underline{\gamma}_{F(\alpha)}(x),\ \underline{\gamma}_{G(\alpha)}(x)\right),\ \inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha\in A\cap B,\ x\in U\end{cases}$$

Since ∇(⟨F, A⟩ ⋓ ⟨G, B⟩) = ∇⟨H, C⟩ by definition 8:

$$\mu_{\nabla H(\alpha)}(x)=\begin{cases}\left[\overline{\mu}_{F(\alpha)}(x), 1-\overline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha\in A-B,\ x\in U\\ \left[\overline{\mu}_{F(\alpha)}(x), 1-\overline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha\in B-A,\ x\in U\\ \left[\sup\left(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\alpha)}(x)\right),\ 1-\inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha\in A\cap B,\ x\in U\end{cases}$$

$$\gamma_{\nabla H(\alpha)}(x)=\begin{cases}\left[1-\overline{\gamma}_{F(\alpha)}(x), 1-\underline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha\in A-B,\ x\in U\\ \left[1-\overline{\gamma}_{G(\alpha)}(x), 1-\underline{\gamma}_{G(\alpha)}(x)\right], \text{ if } \alpha\in B-A,\ x\in U\\ \left[1-\inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right),\ 1-\inf\left(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha\in A\cap B,\ x\in U\end{cases}$$

By definition:

$$\nabla\langle F, A\rangle = \left\{\left\langle x,\ \mu_{\nabla F(\alpha)}(x),\ \gamma_{\nabla F(\alpha)}(x)\right\rangle \mid x\in U,\ \alpha\in A\right\}$$

$$=\left\{ x, \left[\overline{\mu}_{F(\alpha)}(x),\ 1-\overline{\gamma}_{F(\alpha)}(x)\right], \left[1-\overline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{F(\alpha)}(x)\right] \mid x\in U,\ \alpha\in A\right\}$$

$$\nabla\langle G, B\rangle = \left\{\left\langle x,\ \mu_{\nabla G(\alpha)}(x),\ \gamma_{\nabla G(\alpha)}(x)\right\rangle \mid x\in U,\ \alpha\in B\right\}$$

$$=\left\{ x, \left[\overline{\mu}_{G(\alpha)}(x),\ 1-\overline{\gamma}_{G(\alpha)}(x)\right], \left[1-\overline{\gamma}_{G(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right] \rangle \mid x\in U,\ \alpha\in A\right\}$$

Suppose that $\nabla\langle F, A\rangle \Cup \nabla\langle G, B\rangle = \langle O, C\rangle$ with $C = A \cup B$ and $\alpha \in C$. Then:

$$\mu_{0(\alpha)}(x)$$

$$= \begin{cases} \left[\overline{\mu}_{F(\alpha)}(x), 1-\overline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A-B, x \in U \\ \left[\overline{\mu}_{G(\alpha)}(x), 1-\overline{\gamma}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B-A, x \in U \\ \left[\sup\left(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\alpha)}(x)\right), \sup\left(1-\overline{\gamma}_{F(\alpha)}(x), 1-\overline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B, x \in U \end{cases}$$

$$= \begin{cases} \left[\overline{\mu}_{F(\alpha)}(x), 1-\overline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A-B, x \in U \\ \left[\overline{\mu}_{G(\alpha)}(x), 1-\overline{\gamma}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B-A, x \in U \\ \left[\sup\left(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\alpha)}(x)\right), 1-\inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B, x \in U \end{cases}$$

$$\gamma_{0(\alpha)}(x)$$

$$= \begin{cases} \left[1-\overline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A-B, x \in U \\ \left[1-\overline{\gamma}_{G(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B-A, x \in U \\ \left[\inf\left(1-\overline{\gamma}_{F(\alpha)}(x), 1-\overline{\gamma}_{F(\alpha)}(x)\right), \inf\left(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B, x \in U \end{cases}$$

$$= \begin{cases} \left[1-\overline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A-B, x \in U \\ \left[1-\overline{\gamma}_{G(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B-A, x \in U \\ \left[1-\sup\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{F(\alpha)}(x)\right), \inf\left(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right)\right], \text{ if } \alpha \in A \cap B, x \in U \end{cases}$$

Since:

$$\overline{\gamma}_{F(\alpha)}(x) = \overline{\gamma}_{G(\alpha)}(x)$$

for any $\alpha \in A \cap B$:

$$1-\sup\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right) = 1-\inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right)$$

Therefore:

$$\gamma_{O(\alpha)}(x) = \begin{cases} \left[1-\overline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{F(\alpha)}(x)\right], \text{ if } \alpha \in A-B, x \in U \\ \left[1-\overline{\gamma}_{G(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right], \text{ if } \alpha \in B-A, x \in U \\ \left[1-\inf\left(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\alpha)}(x)\right), \inf\left(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\alpha)}(x)\right)\right], \text{if } \alpha \in A \cap B, x \in U \end{cases}$$

The theorem is proved.

Theorem 2: Let ⟨F, A⟩ and ⟨G, B⟩ be interval-valued intuitionistic fuzzy soft sets over U, then the following properties hold:

(1) If $\overline{\mu}_{F(\alpha)}(x) = \overline{\mu}_{G(\beta)}(x)$ then $\Delta\,(\langle F, A\rangle \wedge \langle G, B\rangle) = \Delta\langle F, A\rangle \vee \Delta\langle G, B\rangle$
(2) If $\overline{\mu}_{F(\alpha)}(x) = \overline{\mu}_{G(\beta)}(x)$ then $\Delta\,(\langle F, A\rangle \vee \langle G, B\rangle) = \Delta\langle F, A\rangle \wedge \Delta\langle G, B\rangle$
(3) If $\overline{\gamma}_{F(\alpha)}(x) = \overline{\gamma}_{G(\beta)}(x)$ then $\nabla\,(\langle F, A\rangle \wedge \langle G, B\rangle) = \nabla\langle F, A\rangle \wedge \nabla\langle G, B\rangle$
(4) If $\overline{\gamma}_{F(\alpha)}(x) = \overline{\gamma}_{G(\beta)}(x)$ then $\nabla\,(\langle F, A\rangle \vee \langle G, B\rangle) = \nabla\langle F, A\rangle \vee \nabla\langle G, B\rangle$

Proof: It will be given the proof (1) and (3). While (2) and (4) can be proved analogously. Assume that ⟨F, A⟩^⟨G, B⟩ = ⟨H, A×B⟩, with H(α, β) = F(α)∩G (β), (α, β)∈(A×B). By definition 2:

$$H(\alpha,\beta)(x) = \Big\langle \Big[\inf\big(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\big), \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big)\Big], \Big[\sup\big(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\beta)}(x)\big), \sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big)\Big]\Big\rangle$$

$$\forall(\alpha, B)\in(A\times B),\ x\in U$$

By definition 7:

$$\Delta\,(\langle F, A\rangle \wedge \langle G, B\rangle) = \Delta\,\langle H, A\times B\rangle$$

$$= \Big\{\Big\langle x, \Big[\sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big), 1 - \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big)\Big] \Big[1 - \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big), \inf\big(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\big)\Big]\Big\rangle \,\Big|\, (\alpha,\beta)\in(A\times B),\ x\in U\Big\}$$

Suppose that:

$$\Delta\langle F, A\rangle = \Big\{\Big\langle x, \big[\overline{\gamma}_{F(\alpha)}(x), 1 - \overline{\mu}_{F(\alpha)}(x)\big], \big[1 - \overline{\mu}_{F(\alpha)}(x), \underline{\mu}_{F(\alpha)}(x)\big]\Big\rangle \,\Big|\, x\in U, \alpha\in A\Big\}$$

$$\Delta\langle G, B\rangle = \Big\{\Big\langle x, \big[\overline{\gamma}_{G(\beta)}(x), 1 - \overline{\mu}_{G(\beta)}(x)\big], \big[1 - \overline{\mu}_{G(\beta)}(x), \underline{\mu}_{G(\beta)}(x)\big]\Big\langle \,\Big|\, x\in U, \beta\in B\Big\}$$

By definition 3:

$$\Delta\langle F, A\rangle \vee \Delta\langle G, B\rangle = \Big\{\Big\langle x, \Big[\sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big), \sup\big(1 - \overline{\mu}_{F(\alpha)}(x), 1 - \overline{\mu}_{G(\beta)}(x)\big)\Big] \Big[\inf\big(1 - \overline{\mu}_{F(\alpha)}(x), 1 - \overline{\mu}_{G(\beta)}(x)\big), \inf\big(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\big)\Big]\Big\rangle \,\Big|\, (\alpha,\beta)\in(A\times B)\ x\in U\Big\}$$

$$= \Big\{\Big\langle x, \Big[\sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big), 1 - \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big)\Big], \Big[1 - \sup\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big), \inf\big(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\big)\Big]\Big\rangle \,\Big|\, (\alpha,\beta)\in(A\times B),\ x\in U\Big\}$$

Since:

$$\overline{\mu}_{F(\alpha)}(x) = \overline{\mu}_{G(\beta)}(x)$$

then:

$$1 - \sup\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big) = 1 - \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big)$$

Therefore:

$$\Delta\langle F, A\rangle \vee \Delta\langle G, B\rangle = \Big\{\Big\langle x, \Big[\sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big), 1 - \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big)\Big], \Big[1 - \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big), \inf\big(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\big)\Big]\Big\rangle \,\Big|\, (\alpha,\beta)\in(A\times B),\ x\in U\Big\}$$

Assume that ⟨F, A⟩^⟨G, B⟩ = ⟨H, A×B⟩, where H (α, β) = F(α) ∩ G(β), (α, β) ∈ (A×B). By definition 2:

$$H(\alpha,\beta)(x) = \Big\langle \Big[\inf\big(\underline{\mu}_{F(\alpha)}(x), \underline{\mu}_{G(\beta)}(x)\big), \inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big)\Big], \Big[\sup\big(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\beta)}(x)\big), \sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big)\Big]\Big\rangle$$

$$\forall(\alpha, \beta) \in (A\times B),\ x \in U$$

By definition 8:

$$\Delta\big(\langle F, A\rangle \wedge \langle G, B\rangle\big) = \Delta\langle H, AXB\rangle =$$
$$\Big\{\Big\langle x, \Big[\inf\big(\overline{\mu}_{F(\alpha)}(x), \overline{\mu}_{G(\beta)}(x)\big), 1 - \sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big)\Big], \Big[1 - \sup\big(\overline{\gamma}_{F(\alpha)}(x), \overline{\gamma}_{G(\beta)}(x)\big), \sup\big(\underline{\gamma}_{F(\alpha)}(x), \underline{\gamma}_{G(\beta)}(x)\big)\Big]\Big\rangle \,\Big|\, (\alpha,\beta)\in(A\times B),\ x\in U\Big\}$$

Now assume that:

$$\nabla\langle F,A\rangle=\left\{\left\langle x,\mu_{\nabla F(\alpha)}(x),\gamma_{\nabla F(\alpha)}(x)\right\rangle \mid x\in U,\alpha\in A\right\}$$

$$=\left\{\left\langle x,\left[\bar{\mu}_{F(\alpha)}(x),1-\bar{\gamma}_{F(\alpha)}(x)\right],\left[1-\bar{\gamma}_{F(\alpha)}(x),\underline{\gamma}_{F(\alpha)}(x)\right]\right\rangle \mid x\in U,\alpha\in A\right\}$$

and:

$$\nabla\langle G,B\rangle=\left\{\left\langle x,\mu_{\nabla G(\beta)}(x),\gamma_{\nabla G(\beta)}(x)\right\rangle \mid x\in U,\beta\in B\right\}$$

$$=\left\{\left\langle x,\left[\bar{\mu}_{G(\beta)}(x),1-\bar{\gamma}_{G(\beta)}(x)\right],\left[1-\bar{\gamma}_{G(\beta)}(x),\underline{\gamma}_{G(\alpha)}(x)\right]\right\rangle \mid x\in U,\beta\in B\right\}$$

By definition 2:

$$\nabla\langle F,A\rangle\wedge\nabla\langle G,B\rangle=\left\{\left\langle x,\left[\inf\left(\bar{\mu}_{F(\alpha)}(x),\bar{\mu}_{G(\beta)}(x)\right),\inf\left(1-\bar{\gamma}_{F(\alpha)}(x),1-\bar{\gamma}_{G(\beta)}(x)\right)\right],\right.\right.$$
$$\left.\left[\sup\left(1-\bar{\gamma}_{F(\alpha)}(x),1-\bar{\gamma}_{G(\beta)}(x)\right),\sup\left(\underline{\gamma}_{F(\alpha)}(x),\underline{\gamma}_{G(\alpha)}(x)\right)\right]\right\rangle\mid$$
$$\left.(\alpha,\beta)\in(A\times B),x\in U\right\}$$

$$\left\{\left\langle x,\left[\inf\left(\bar{\mu}_{F(\alpha)}(x),\bar{\mu}_{G(\beta)}(x)\right),1-\sup\left(\bar{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{G(\beta)}(x)\right)\right],\right.\right.$$
$$\left.\left[1-\inf\left(\bar{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{G(\beta)}(x)\right),\sup\left(\underline{\gamma}_{F(\alpha)}(x),\underline{\gamma}_{G(\alpha)}(x)\right)\right]\right\rangle\mid$$
$$\left.(\alpha,\beta)\in(A\times B),x\in U\right\}$$

Since:

$$\bar{\gamma}_{F(\alpha)}(x)=\bar{\gamma}_{G(\beta)}(x)$$

It is obtained:

$$1-\inf\left(\bar{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{G(\beta)}(x)\right)=1-\sup\left(\bar{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{G(\beta)}(x)\right)$$

Thus:

$$\nabla\langle F,A\rangle\wedge\nabla\langle G,B\rangle=\left\{\left\langle x,\left[\inf\left(\bar{\mu}_{F(\alpha)}(x),\bar{\mu}_{G(\beta)}(x)\right),1-\sup\left(\bar{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{G(\beta)}(x)\right)\right],\right.\right.$$
$$\left.\left[1-\sup\left(\bar{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{G(\beta)}(x)\right),\sup\left(\underline{\gamma}_{F(\alpha)}(x),\underline{\gamma}_{G(\beta)}(x)\right)\right]\right\rangle\mid$$
$$\left.(\alpha,\beta)\in(A\times B),x\in U\right\}$$

Theorem 3: Assume that $\langle F, A\rangle$ is an interval-valued intuitionistic fuzzy soft sets over U and then the following properties hold:

- $\langle F, A\rangle \Subset \nabla\langle F, A\rangle$
- $\Delta\,\Delta\,\Delta\,\langle F, A\rangle = \Delta\,\langle F, A\rangle$

Proof: Suppose that:

$$\langle F,A\rangle=\left\{\left\langle x,\left[\underline{\mu}_{F(\alpha)}(x),\bar{\mu}_{F(\alpha)}(x)\right],\left[\underline{\gamma}_{F(\alpha)}(x),\bar{\gamma}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}$$

$$\nabla\langle F,A\rangle=\left\{\left\langle x,\left[\bar{\mu}_{F(\alpha)}(x),1-\bar{\gamma}_{F(\alpha)}(x)\right],\left[1-\bar{\gamma}_{F(\alpha)}(x),\underline{\gamma}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}$$

and:

$$\Delta\langle F,A\rangle=\left\{\left\langle x,\left[\bar{\gamma}_{F(\alpha)}(x),1-\bar{\mu}_{F(\alpha)}(x)\right],\left[1-\bar{\mu}_{F(\alpha)}(x),\underline{\mu}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}$$

By definition $\bar{\mu}_{F(\alpha)}(x)\leq 1-\bar{\gamma}_{F(\alpha)}(x)$. Since, $\underline{\mu}_{F(\alpha)}(x)\leq\bar{\mu}_{F(\alpha)}(x)$, $\underline{\gamma}_{F(\alpha)}(x)\geq 1-\bar{\gamma}_{F(\alpha)}(x)$ and $\bar{\gamma}_{F(\alpha)}(x)\geq\underline{\gamma}_{F(\alpha)}(x)$. It is obtained $\langle F, A\rangle \Subset \nabla\langle F, A\rangle$.

Suppose that:

$$\Delta\langle F,A\rangle=\left\{\left\langle x,\left[\bar{\gamma}_{F(\alpha)}(x),1-\bar{\mu}_{F(\alpha)}(x)\right],\left[1-\bar{\mu}_{F(\alpha)}(x),\underline{\mu}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}$$

Then by definition 7:

$$\Delta\Delta\Delta\langle F,A\rangle=$$
$$\Delta\Delta\left\{\left\langle x,\left[\bar{\gamma}_{F(\alpha)}(x),1-\bar{\mu}_{F(\alpha)}(x)\right],\left[1-\bar{\mu}_{F(\alpha)}(x),\underline{\mu}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}=$$

$$\Delta\left\{\left\langle x,\left[\underline{\mu}_{F(\alpha)}(x),1-(1-\bar{\mu}_{F(\alpha)}(x))\right],\left[1-(1-\bar{\mu}_{F(\alpha)}(x)),\bar{\gamma}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}=$$

$$\Delta\left\{\left\langle x,\left[\underline{\mu}_{F(\alpha)}(x),\bar{\mu}_{F(\alpha)}(x)\right],\left[\bar{\mu}_{F(\alpha)}(x),\bar{\gamma}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}=$$

$$\left\{\left\langle x,\left[\bar{\gamma}_{F(\alpha)}(x),1-\bar{\mu}_{F(\alpha)}(x)\right],\left[1-\bar{\mu}_{F(\alpha)}(x),\underline{\mu}_{F(\alpha)}(x)\right]\right\rangle\mid x\in U,\alpha\in A\right\}=\langle F,A\rangle$$

DISCUSSION

By defining two new operators Δ and ∇, in the results section, it was proposed the sufficient conditions to obtain some relations between the new operators and the operations $\wedge$, $\vee$, $\Cup$ and $\Cap$ defined by Jiang *et al.*[9]. On the other hand, it was also obtained some properties of the new operator as stated in Theorem 3. These results are certainly very different from the existing articles[9-13], because operators studied are different. On the other hand, the study on operators in IVIFSS is a generalization of those in hesitant intuitionistic fuzzy soft sets[4,6]. Therefore this work has contributed to the development of the study on interval-valued intuitionistic fuzzy soft operators. For further, it is interesting to study concerning operators of interval-valued hesitant intuitionistic fuzzy soft sets.

CONCLUSION

It was proposed the two new operators on interval-valued intuitionistic fuzzy soft sets. Refer to on these operators and the "and", "or", union and intersection operations, it is obtained some properties.

SIGNIFICANCE STATEMENT

These results have extended and enriched those of previous study on IVIFSS, particularly with regard to their operators. Therefore the results of this study can be utilized by researchers so that can be beneficial for decision-making problem in the fields of science and engineering.

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