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H_2/H_∞ Control for Consensus of Networked Swarm Agents with Static Connected Topology Graph

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Abstract: Consensus of swarm agents had appeared in many applications and attracted more and more interests of researchers. In this study, the controller based on states feedback for consensus of networked multi-agents was designed. The topology graph composed by the agents were assumed to be static and connected. Both complete and incomplete graph cases were considered. The agents were considered as discrete linear systems. The sufficient conditions for the agents to achieve consensus were obtained to meet the H_2/H_∞ performance for external disturbances. The results were based on matrix analysis and linear matrix inequality. Finally, numerical examples were given to illustrate the effectiveness of the results.

Key words: Robust control, state feedback, distributed control, networked agents, discrete system, consensus

INTRODUCTION

Research on consensus of networked swarm agents had attracted the interests of scientists in mathematics, physics and engineering areas in recent years. The results on consensus had appeared widely in many applications including Unmanned Air Vehicles (UAVs), formation and automated highway systems (Xiaoqing *et al*., 2012).

The consensus phenomena had been found in nature such as fish schooling, bird flocking and herds migrating. Researchers were inspired by these nature phenomena. The computer model of flocks was designed (Reynolds, 1987). The famous model modeled by Reynolds was boid. Three rules such as collision avoidance, velocity matching and flock centering were followed in the swarm agents that exchange information with each other based on local information. A special version of boid was the Vicsek model proposed by Vicsek *et al.*(1995). Flocking behaviors had been analyzed in detail by Jadbabaie *et al.* (2003), Saber and Murray (2003, 2004), Moreau (2005), Ren and Beard (2005) and Saber *et al.* (2007). Moreover, consensus problems for networks of dynamic, autonomous agents with fixed and switching topologies were discussed (Housheng *et al*., 2011; Xiaoqing *et al*., 2012; Fenglan *et al*., 2012; Tao *et al*., 2011).

Some robust consensus algorithms with disturbances were analyzed based on matrices theory and graph theory to verify that under some control action the consensus would be achieved (Castro and Paganini, 2004). States feedback control law that guaranteed consensus for the closed-loop system was designed. Castro and Paganini (2004) propose a new way, that was think globally and act locally, to analyze swarm system by linear matrix inequality theory. The H_2 controller was designed to make the closed-loop system achieve consensus with non-consensus part of the system meeting the optimal H_2 performance (Li and Fang, 2009c).

It was very clear that disturbances always exist in swarm systems, especially in practical engineering systems. In order to guarantee the states of swarm agent reach consensus with H_2/H_∞ performance under various of external and internal disturbances, robust controller should be designed to make the consensus part of closed-loop system reach consensus with non-consensus part be Lyapunov stable meeting H_2/H_∞ performance for disturbances attenuation (Li and Fang, 2009c).

This study mainly focused on designing robust controllers for consensus of swarm agents. The agents were modeled as discrete dynamics. Future more, the topology graph composed by agents were considered to be complete or meeting certain topology structure.

PRELIMINARIES AND BACKGROUND

Model description: Consider the discrete swarm system composed of N interconnected agents, where each agent had the following dynamics:

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j\neq i} A_{ij}x_j(k) + B_{1i}u_i(k) + B_{2i}\omega_i(k), i \in \{1, 2, \cdots, N\} \quad (1)$$

where, $x_i(k)$ represented the ith individual's state variable at the k period which was assumed to be p dimensions,

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that was $x_i(k)\in R^p$. $x_j(k)$ represented the jth individual's state at k-period. $u_i(k)\in R^q$ was control input of the individual i at the period of k. $\omega_i(k)\in R^r$ represented the disturbances into ith individual at the k moment. $A_{ii}\in R^{p\times p}$, $A_{ij}\in R^{p\times p}$, $B_{1i}\in R^{p\times q}$, $B_{2i}\in R^{p\times r}$ were the corresponding coefficient matrices and A_{ij} showed the relationship between i and j, meaning there exists information exchange between neighbors, when the topology composed by agents were static, then A_{ij} was constant.

Let's consider this swarm system globally, the state space equation can be got:

$$x(k+1) = Ax(k) + B_1u(k) + B_2\omega(k) \tag{2}$$

Where:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \in R^{Np}, u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \in R^{Np}, B1 = \begin{bmatrix} B_{11} & & \\ & \ddots & \\ & & B_{1N} \end{bmatrix} \in R^{NP\times NQ}$$

$$B2 = \begin{bmatrix} B_{21} & & \\ & \ddots & \\ & & B_{2N} \end{bmatrix} \in R^{Np\times Nr}, A = = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \in R^{Np\times Np}$$

Consensus: Now the explanation and definition to consensus or agreement of discrete dynamic systems were given. The system composed of N agents whose dynamics were same as:

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j\neq i} A_{ij}x_j(k), i \in \{1, 2, \cdots, N\} \tag{3}$$

The equation can be rewritten as global equation:

$$x(k+1) = Ax(k) \tag{4}$$

Definition 1: Let β_i, i = 1, ..., Np be an orthonormal basis in R^{Np}. The Eq. 4 achieved consensus to the subspace:

$$S = \text{span } \{\beta_i, i = 1, \cdots, Np\} \tag{5}$$

If S was and minimal set such that for any initial condition, the state at k moment x(k) converged to a point in S. Then Lemma 1 was introduced.

Lemma 1: The states of Eq. 4 converged to consensus in S by definition 1, if and only if, $A\alpha = \alpha$; there existed X>0 such that:

$$(\alpha_\perp^T A\alpha_\perp)^T X(\alpha_\perp^T A\alpha_\perp) - X < 0 \tag{6}$$

where:

$$\alpha = \frac{1}{\sqrt{N}}[1 \ \ 1 \ \ \cdots \ \ 1]^T$$

let $\alpha_\perp$ be orthonormal complement of α, $\alpha^T_\perp$ be the transpose of $\alpha_\perp$, then $\alpha^T_\perp\alpha_\perp = I, \alpha^T_\perp\alpha = 0$. The Definition 1 and Lemma 1 can be concluded from continuous case to discrete case (Castro and Paganini, 2004).

State feedback control It was assumed that states of Eq. 1 can be measured directly, the state feedback controller can be:

$$u(k) = Kx(k) \tag{7}$$

With the controller the closed-loop system was:

$$x(k+1) = (A + B_1K)x(k) + B_2\omega(k) \tag{8}$$

$$z(k) = (C_1 + D_{12}K)x(k) \tag{9}$$

If the closed-loop was internal stable and closed-loop transfer function $\|T_{wz}(z)\|_\infty$ satisfies:

$$\| T_{wz}(z) \|_2 = \| (C_1 + D_{12}K)[zI - (A + B_2K)]^{-1}B_1 \|_\infty < \gamma \tag{10}$$

Then it was said that the control law of Eq. 7 was a state feedback γ-optimal H_∞ controller of Eq. 8.

If the closed-loop was internal stable and closed-loop transfer function $\|T_{wz}(z)\|_2$ satisfies:

$$\| T_{wz}(z) \|_2 = \| (C_1 + D_{12}K)[zI - (A + B_2K)]^{-1}B_1 \|_2 < \gamma \tag{11}$$

Then it was said that the control law of Eq. 7 was a state feedback H_2 controller of Eq. 8, where:

$$z(k) = (C_1 + D_{12}K)x(k) \tag{12}$$

is regulated output. The states were decomposed as:

$$x(k) = [\alpha_\perp \ \ \alpha]\begin{bmatrix} \theta(k) \\ \delta(k) \end{bmatrix} \tag{13}$$

The closed-loop Eq. 8 was equivalent to:

$$\begin{aligned} \begin{bmatrix} \theta(k+1) \\ \delta(k+1) \end{bmatrix} &= \begin{bmatrix} \alpha_\perp^T(A + B_1K)\alpha_\perp\theta(k) \\ \alpha^T(A + B_1K)\alpha_\perp\theta(k) + \delta(k) \end{bmatrix} + \begin{bmatrix} \alpha_\perp^T \\ \alpha^T \end{bmatrix} B_2\omega(k) \\ z &= (C_1 + D_{12}K)[\alpha_\perp \ \ \alpha]\begin{bmatrix} \theta \\ \delta \end{bmatrix} \end{aligned} \tag{14}$$

the meaningful definition for performance variable was with respect to the stable part of the state x(k), namely:

$$\theta(k+1) = \alpha_{\perp}^{T}\ (A + B_1K)\alpha_{\perp}\theta(k) + \alpha_{\perp}^{T}\ B_2\omega \tag{15}$$

The meaningful definition for performance variable was:

$$z_{\theta}(k)\ (C_1 + D_{12}K)\alpha_{\perp}\theta(k) \tag{16}$$

The Lemma 2 was given by Li and Fang (2009a).

Lemma 2: Given $\gamma>0$, there existed a control law $u(k) = Kx(k)$, such that the states of Eq. 8 reach consensus in S with the transfer function T_{zw} from the disturbance ω in the non-consensus part to performance variable $z_{\theta}(k) = (C_1+D_{12}K)\alpha_{\perp}\theta(k)$ satisfying $\|T_{wz}(z)\|_2$, if and only if there existed X, Z, W, such that:

$$AX\alpha + B_1W\alpha = X\alpha$$
$$X = \alpha_{\perp}\alpha_{\perp}^{T}\ X\alpha_{\perp}\alpha_{\perp}^{T}\ + \alpha\alpha^{T}X\alpha\alpha^{T}$$
$$\begin{bmatrix} -\alpha_{\perp}^{T}\ X\alpha_{\perp} & \alpha_{\perp}^{T}\ (AX + B_1W)\alpha_{\perp} & 0 \\ \alpha_{\perp}^{T}\ (AX + B_1W)T\alpha_{\perp} & -\alpha_{\perp}^{T}\ X\alpha_{\perp} & ((C1X + D12W)\alpha_{\perp})^{T} \\ 0 & (C_1X + D_{12}W)\alpha_{\perp} & -I \end{bmatrix} < 0$$
$$\begin{bmatrix} -Z & (\alpha_{\perp}^{T}\ B_2)^{T} \\ \alpha_{\perp}^{T}\ B_2 & -\alpha_{\perp}^{T}\ X\alpha_{\perp} \end{bmatrix} < 0$$
$$\mathrm{Trace}(Z) < \gamma^2$$

If there were feasible solutions X*, W*, Z*, then $u(k) = W^*(X^*)^{-1}\ x(k)$ was a state feedback γ-optimal H_2 controller. The lemma 3 was given by Li and Fang (2009b).

Lemma 3: Given $\gamma>0$, there existed a control law $u(k) = Kx(k)$, such that the states of Eq. 8 reached consensus in S with transfer function T_{zw} from the disturbance ω in the non-consensus part to performance variable $z_{\theta}(k) = (C_1+D_{12}K)\alpha_{\perp}\theta(k)$ satisfying $\|T_{wz}(s)\|<\gamma$, if and only if there existed X>0 such that:

$$AX\alpha + B_1W\alpha = X\alpha$$
$$\begin{bmatrix} -\alpha_{\perp}^{T}\ X\alpha_{\perp} & \alpha_{\perp}^{T}\ (AX + B_1W)\alpha_{\perp} \\ \alpha_{\perp}^{T}\ (AX + B_1W)^{T}\alpha_{\perp} & -\alpha_{\perp}^{T}\ X\alpha_{\perp} \end{bmatrix} < 0$$
$$\begin{bmatrix} -\alpha_{\perp}^{T}\ X\alpha_{\perp} & 0 & \alpha_{\perp}^{T}\ (AX + B_1W)\alpha_{\perp} & ((C_1X + D_{12}W)\alpha_{\perp})^{T} \\ 0 & -\lambda^2 I & (\alpha_{\perp}^{T}\ B_2)^{T} & 0 \\ \alpha_{\perp}^{T}\ (AX + B_1W)\alpha_{\perp} & \alpha_{\perp}^{T}\ B_2 & -\alpha_{\perp}^{T}\ X\alpha_{\perp} & 0 \\ (C_1X + D_{12}W)\alpha_{\perp} & 0 & 0 & -I \end{bmatrix}$$
$$X = \alpha_{\perp}\alpha_{\perp}^{T}\ X\alpha_{\perp}\alpha_{\perp}^{T}\ + \alpha\alpha^{T}X\alpha\alpha^{T}$$

RESULTS

Here, some conditions were given to design H_2/H_{∞} controller for consensus of swarm discrete dynamic system with complete and certain topology graph. The models were described. Then two theorems were obtained.

Model description: The closed-loop system was:

$$\begin{aligned} x(k+1) &= Ax(k) + B_1u(k) + B_2\omega_1(k) + B_3\omega_2(k) \\ z_1(k) &= C_1x(k) + D_{10}u(k) \\ z_2(k) &= C_2x(k) + D_{20}u(k) \end{aligned} \tag{17}$$

where, ω_1 and ω_2 represented different disturbances. The matrices A, B_1, B_2, B_3 were coefficient matrices:

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \in R^{Np\times Np}, B_1 = \begin{bmatrix} B_{11} & & \\ & \ddots & \\ & & B_{1N} \end{bmatrix} \in R^{Np\times Nq}$$
$$B_2 = \begin{bmatrix} B_{21} & & \\ & \ddots & \\ & & B_{2N} \end{bmatrix} \in R^{Np\times Nr},\ B_3 = \begin{bmatrix} B_{31} & & \\ & \ddots & \\ & & B_{3N} \end{bmatrix} \in R^{Np\times Nr}$$

The control law was:

$$u(k) = Kx(k)$$

With the control law Eq. 17 can be written as:

$$\begin{aligned} x(k+1) &= (A + B_1K)x(k) + B_2\omega_1(k) + B_3\omega_2(k) \\ z_1(k) &= (C_1 + D_{10}K)x(k) \\ z_2(k) &= (C_2 + D_{20}K)x(k) \end{aligned} \tag{18}$$

The state variables can be decomposed as:

$$x = [a_{\perp}\quad a]\begin{bmatrix} \theta \\ \delta \end{bmatrix}$$

Then the Eq. 17 was equivalent to:

$$\begin{aligned} \begin{bmatrix} \theta(k+1) \\ \delta(k+1) \end{bmatrix} &= \begin{bmatrix} \alpha_{\perp}^{T}(A + B_1K)\alpha_{\perp}\theta(k) \\ \alpha^{T}(A + B_1K)\alpha_{\perp}\theta(k) + \delta(k) \end{bmatrix} + \begin{bmatrix} \alpha_{\perp}^{T} \\ \alpha^{T} \end{bmatrix} B_2\omega_1(k) + \begin{bmatrix} \alpha_{\perp}^{T} \\ \alpha^{T} \end{bmatrix} B_3\omega_2(k) \\ z_1(k) &= (C_1 + D_{10}K)[\alpha_{\perp}\quad \alpha]\begin{bmatrix} \theta(k) \\ \delta(k) \end{bmatrix} \\ z_2(k) &= (C_2 + D_{20}K)[\alpha_{\perp}\quad \alpha]\begin{bmatrix} \theta(k) \\ \delta(k) \end{bmatrix} \end{aligned} \tag{19}$$

The stable part of Eq. 17 was:

$$\theta(k+1) = \alpha_{\perp}^{T}(A + B_1K)\alpha_{\perp}\theta(k) + \alpha_{\perp}^{T}B_2\omega_1(k) + \alpha_{\perp}^{T}B_3\omega_2(k)$$

the meaningful performance functions were as follows:

$$z_{1\theta}(k) = (C_1 + D_{10}K)\alpha_\perp\theta(k)$$
$$z_{2\theta}(k) = (C_2 + D_{20}K)\alpha_\perp\theta(k)$$

The state feedback control law was expected to make the states of close-loop reach consensus with the H_∞ norm and H_2 norm satisfying $||T\omega_1 z_1||\infty<\gamma_1$, $||T\omega_2 z_2||\infty<\gamma_2$.

The complete graph case

Theorem 1: For Eq. 17, given $\gamma_1>0$, $\gamma_2>0$, then there existed a state feedback control law $u(k) = Kx(k)$, such that the states of Eq. 17 converged to a point in S with H_∞ norm of the transfer function $T\omega_1 z_1$ from $\omega_1(k)$ to $z_{1\theta}(k)$ no greater than γ_1, and H_2 norm of transfer function $T\omega_2 z_2$ from $\omega_2(k)$ to $z_{2\theta}(k)$ no greater than γ_2, if there existed such that:

$$AX\alpha + B_1 W\alpha = X\alpha$$
$$X = \alpha_\perp\alpha_\perp^T X\alpha_\perp\alpha_\perp^T + \alpha\alpha^T X\alpha\alpha^T$$
$$\begin{bmatrix} -\alpha_\perp^T X\alpha_\perp & \alpha_\perp^T (AX + B_1 W)\alpha_\perp & 0 \\ \alpha_\perp^T (AX + B_1 W)T\alpha_\perp & -\alpha_\perp^T X\alpha_\perp & ((C1X + D12W)\alpha_\perp)^T \\ 0 & (C_1 X + D_{12} W)\alpha_\perp & -I \end{bmatrix} < 0$$
$$\begin{bmatrix} -Z & (\alpha_\perp^T B_2)^T \\ \alpha_\perp^T B_2 & -\alpha_\perp^T X\alpha_\perp \end{bmatrix} < 0$$
$$\mathrm{Trace}(Z) < \gamma^2$$
$$\begin{bmatrix} -\alpha_\perp^T X\alpha_\perp & 0 & \alpha_\perp^T (AX + B_1 W)\alpha_\perp & ((C_1 X + D_{12} W)\alpha_\perp)^T \\ 0 & -\lambda^2 I & (\alpha_\perp^T B_2)^T & 0 \\ \alpha_\perp^T (AX + B_1 W)\alpha_\perp & \alpha_\perp^T B_2 & -\alpha_\perp^T X\alpha_\perp & 0 \\ (C_1 X + D_{12} W)\alpha_\perp & 0 & 0 & -I \end{bmatrix}$$

The feasible solution is X^*, W^*, then the control law was $u = W^*(X^*)^{-1}$ x.

Proof: According to Lemma 2 and 3, the states of Eq. 18 could reach consensus with H_2/H_∞ performance if there existed positive definitive matrices X_1, X_2 and matrices W_1, W_2, Z, such that:

$$AX_1\alpha + B_1 W_1\alpha = X_1\alpha$$
$$X = \alpha_\perp\alpha_\perp^T X_1\alpha_\perp\alpha_\perp^T + \alpha\alpha^T X_1\alpha\alpha^T$$
$$\begin{bmatrix} -\alpha_\perp^T X_1\alpha_\perp & \alpha_\perp^T (AX_1 + B_1 W_1)\alpha_\perp & 0 \\ \alpha_\perp^T (AX_1 + B_1 W_1)^T\alpha_\perp & -\alpha_\perp^T X_1\alpha_\perp & ((C_1 X + D_{12} W_1)\alpha_\perp)^T \\ 0 & (C_1 X + D_{12} W_1)\alpha_\perp & -I \end{bmatrix} < 0$$
$$\begin{bmatrix} -Z & (\alpha_\perp^T B_3)^T \\ \alpha_\perp^T B_3 & -\alpha_\perp^T X_1\alpha_\perp \end{bmatrix} < 0$$
$$\mathrm{Trace}(Z) < \gamma_2^2$$
$$AX_2\alpha + B_1 W_2\alpha = X_2\alpha$$
$$\begin{bmatrix} -\alpha_\perp^T X_2\alpha_\perp & \alpha_\perp^T (AX_2 + B_1 W_2)\alpha_\perp \\ \alpha_\perp^T (AX_2 + B_1 W_2)^T\alpha_\perp & -\alpha_\perp^T X_2\alpha_\perp \end{bmatrix} < 0$$
$$\begin{bmatrix} -\alpha_\perp^T X_2\alpha_\perp & 0 & \alpha_\perp^T (AX_2 + B_1 W_2)\alpha_\perp & ((C_1 X_2 + D_{12} W_2)\alpha_\perp)^T \\ 0 & -\lambda_1^2 I & (\alpha_\perp^T B_2)^T & 0 \\ \alpha_\perp^T (AX_2 + B_1 W_2)\alpha_\perp & \alpha_\perp^T B_2 & -\alpha_\perp^T X_2\alpha_\perp & 0 \\ (C_1 X_2 + D_{12} W_2)\alpha_\perp & 0 & 0 & -I \end{bmatrix}$$
$$X_2 = \alpha_\perp\alpha_\perp^T X_2\alpha_\perp\alpha_\perp^T + \alpha\alpha^T X_2\alpha\alpha^T$$

It was very difficult to solve X_1 and X_2 as they were coupled together. Let $X = X_1 = X_2$, then $W = W_1 = KX_1 = W_2 = KX_2$. The theorem 1 was proved.

Comment: The solution to the above equations was based on the constraint, the results may be more conservative, however, the conservativeness can make compute more effective. When the H_2/H_∞ control law was determined, the H_∞ and H_2 performance of closed-loop can be analyzed. Usually the H_∞ norm will be strictly less than γ_1, H_2 norm will be strictly less than the optimal obtained by solution to the H_2/H_∞ problem.

Certain topology case: In general, the solutions obtained from theorem 1 would had a full block state feedback matrix K, i.e., each agent can receive information from every agent in the swarm system with certain topology structure. But this case was more practical.

The network topology was considered (Fig. 1). The direction of arrow represented the direction of information flow. The individual labelled 1 can get the information from 2 and 3. While 2 can get information from 1 and 3 can visit 2. The feedback matrix K may not be full, that was, K may satisfy certain requirement:

$$K = \begin{bmatrix} K_{11} & 0 & K_{13} \\ K_{21} & K_{22} & 0 \\ 0 & K_{32} & K_{33} \end{bmatrix}, K_{ij} \in R^{q\times p}$$

Then the H_2/H_∞ controller should be designed to make states system reach consensus meeting H_2/H_∞ performance with the special structure of K. According to theorem 1, $K = W^*(X^*)^{-1}$, in order to simply the problem, the matrix X should be with a special structure. $X \in R^{3p\times 3p}$ was assumed to be a block diagonal matrix diag (X_1, X_2, X_3), where, $X_i \in R^{p\times p}$. So, K would have the same structure with W. If:

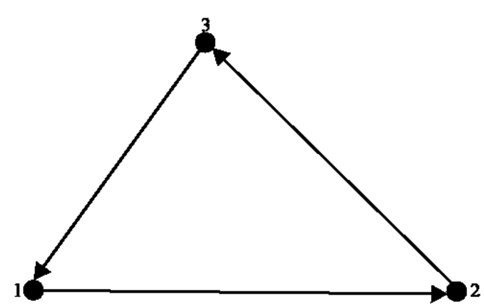

Fig. 1: Certain topology by 3 individuals

$$W=\begin{bmatrix}W_{11}&0&W_{13}\\W_{21}&W_{22}&0\\0&W_{32}&W_{33}\end{bmatrix}$$

Then:

$$K=WX^{-1}=\begin{bmatrix}W_{11}&0&W_{13}\\W_{21}&W_{22}&0\\0&W_{32}&W_{33}\end{bmatrix}\begin{bmatrix}X_1^{-1}&0&0\\0&X_2^{-1}&0\\0&0&X_3^{-1}\end{bmatrix}$$
$$=\begin{bmatrix}W_{11}X_1^{-1}&0&W_{13}X^{-1}{}_3\\W_{21}X_1^{-1}&W_{22}X^{-1}{}_2&0\\0&W_{32}X^{-1}{}_2&W_{33}X^{-1}{}_3\end{bmatrix}$$

Corollary 1(structured H_2/H_∞ control for consensus): Let χ = {K: $K_{ij} = 0 \epsilon R^{q\times p}$, for (i, j)$\epsilon$I} be a given structure. Given $\gamma_1>0$, $\gamma_2>0$, there existed a state feedback law $u(k) = K_x x(k)$ that made Eq. 17 reach consensus in S with the transfer function $T_{\omega 1z1}$ from $\omega_1(k)$ to $z_{1\theta}(k)$ and the transfer function $T_{\omega 2z2}$ from $\omega_2(k)$ to $z_{2\theta}(k)$ satisfying $||T_{\omega 1z1}||_\infty<\gamma_1$,$||T_{\omega 2z2}||_2<\gamma_2$ if there existed matrice Z.W = W_x and $X_i>0$, i 1, .., N, such that:

$$AX\alpha+B_1W\alpha=X\alpha$$
$$X=\alpha_\perp\alpha_\perp^T X\alpha_\perp\alpha_\perp^T+\alpha\alpha^T X\alpha\alpha^T$$
$$\begin{bmatrix}-\alpha_\perp^T X\alpha_\perp & \alpha_\perp^T(AX+B_1W)\alpha_\perp & 0\\ \alpha_\perp^T(AX+B_1W)^T\alpha_\perp & -\alpha_\perp^T X\alpha_\perp & ((C_1X+D_{12}W)\alpha_\perp)^T\\ 0 & (C_1X+D_{12}W)\alpha_\perp & -I\end{bmatrix}<0$$
$$\begin{bmatrix}-Z & (\alpha_\perp^T B_3)^T\\ \alpha_\perp^T B_3 & -\alpha_\perp^T X\alpha_\perp\end{bmatrix}<0$$
$$\mathrm{Trace}(Z)<\gamma_2^2$$
$$\begin{bmatrix}-\alpha_\perp^T X\alpha_\perp & 0 & \alpha_\perp^T(AX+B_1W)\alpha_\perp & ((C_1X+D_{12}W)\alpha_\perp)^T\\ 0 & -\lambda_1^2 I & (\alpha_\perp^T B_2)^T & 0\\ \alpha_\perp^T(AX+B_1W)\alpha_\perp & \alpha_\perp^T B_2 & -\alpha_\perp^T X\alpha_\perp & 0\\ (C_1X+D_{12}W)\alpha_\perp & 0 & 0 & -I\end{bmatrix}$$

Then the control law was:

$$K_x=W_x\ \mathrm{diag}(X_1^{-1},\dots,X^{-1}{}_N)$$

Proof: The results can be obtained directly by substituting the matrix W, X with W_χ and diag(X_1, ..., X_N) in theorem 1.

Numerical simulations: Consider the swarm systems with 3 integrators each agent having the same dynamics:

$$x_i(k+1)=A_{ii}x_i(k)+\sum_{j\neq i}A_{ij}x_j(k)+B_{1i}u_i(k)+B_{2i}\omega_1(k), i\in\{1,2,3\}$$

where, let A_{ii} = I, B_{1i} = I, B_{2i} = I, p = 1, N = 3 then:

$$x(k+1)=Ax(k)+B_1u(k)+B_2\omega_1(k)+B_3\omega_2(k)$$
$$\begin{bmatrix}x_1(k+1)\\x_2(k+1)\\x_3(k+1)\end{bmatrix}=\begin{bmatrix}100\\010\\001\end{bmatrix}\begin{bmatrix}x_1(k)\\x_2(k)\\x_3(k)\end{bmatrix}+\begin{bmatrix}100\\010\\001\end{bmatrix}\begin{bmatrix}u_1(k)\\u_2(k)\\u_3(k)\end{bmatrix}+\begin{bmatrix}100\\010\\001\end{bmatrix}\begin{bmatrix}\omega_{11}(k)\\\omega_{12}(k)\\\omega_{13}(k)\end{bmatrix}+\begin{bmatrix}100\\010\\001\end{bmatrix}\begin{bmatrix}\omega_{21}(k)\\\omega_{22}(k)\\\omega_{23}(k)\end{bmatrix}$$

When, A = I, B_1 = B_2 = B_3 = I.

The performance function:

$$z_1(k)=(C_1+D_{10}K)\alpha_\perp\theta(k)$$
$$z_2(k)=(C_2+D_{20}K)\alpha_\perp\theta(k)$$

When:

$$\alpha=(\sqrt{3})^{-1}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

The following results can obtain:

$$\alpha_\perp=\begin{bmatrix}\frac{-1}{\sqrt{2}}&\frac{-1}{\sqrt{6}}\\0&\frac{2}{\sqrt{6}}\\-\frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{6}}\end{bmatrix}\text{and } C_1=C_2\begin{bmatrix}\alpha_\perp^T\\0\end{bmatrix},D_{10}=D_{20}\begin{bmatrix}0\\1\end{bmatrix}$$

The inequalities of Theorem 2 and 3 were feasible on both continuous and discrete cases.

The complete graph case: According to theorem 1, the solutions can be got by MATLAB, the solutions were feasible.

$$X=\begin{bmatrix}11.2266&10.2266&10.2266\\10.2266&11.2266&10.2266\\10.2266&10.2266&11.2266\end{bmatrix},Z=\begin{bmatrix}0.2500&27.3965&27.3965\\27.3965&0.2500&27.3965\\27.3965&27.3965&0.2500\end{bmatrix}$$
$$W=\begin{bmatrix}-0.6667&0.3333&0.3333\\0.3333&-0.6667&0.3333\\0.3333&0.3333&0.6667\end{bmatrix}$$

then the control law is:

$$u(k)=W*(X*)^{-1}x(k)=\begin{bmatrix}-0.6667&0.3333&0.3333\\0.3333&0.6667&0.3333\\0.3333&0.3333&0.6667\end{bmatrix}x(k)$$

When $||T_{z\omega}||_2$ = 0.8165<γ = 1,$||T_{z2\omega 2}||_2$ = 1.000$\leq$ γ_2 = 1.

Certain graph case: When the topology graph was same as Fig. 1, Collorally 1 was used to design the control law. The solutions were feasible:

$$X=\begin{bmatrix}0.5000 & 0 & 0\\ 0 & 0.5000 & 0\\ 0 & 0 & 0.5000\end{bmatrix}, Z=\begin{bmatrix}0.2500 & 17.6432 & 17.6432\\ 17.6432 & 0.2500 & 17.6432\\ 17.6432 & 17.6432 & 0.2500\end{bmatrix}$$

$$W=\begin{bmatrix}-0.2500 & 0 & 0.2500\\ 0.2500 & -0.2500 & 0\\ 0 & 0.2500 & -0.2500\end{bmatrix}$$

the control law is:

$$u(k)=W_\chi^{*}\ (X^{*})^{-1}x(k)=\begin{bmatrix}-0.5000 & 0 & 0.5000\\ 0.5000 & -0.5000 & 0\\ 0 & 0.5000 & -0.5000\end{bmatrix}x(k),$$

$$\| T_{z1\omega1} \|_\infty = 0.9428 < \gamma_1 = 1, \| T_{z2\omega2} \|_2 = 1.1668 < \gamma_2 = 2$$

CONCLUSIONS

This study provided a theoretical analysis for design H_2/H_∞ controller of consensus of swarm discrete dynamic systems. Both the complete and certain topology structure were considered. Theorem 1 and corollary 1 were concluded. the controller can make the states of system obtain consensus and the non-consensus meeting H_2/H_∞ performance. The results were drawn by LMI. A specific example was used to test the validity of the results.

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