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Long Memory in Stock Returns: Insights from the Indian Market

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ABSTRACT

Researchers have used different methods to detect the possibility of long-term dependence (long memory) in stock market returns. It is found that while the returns themselves do not exhibit any appreciable long memory, the absolute or squared returns display long memory features. In this study, three different tests, viz., rescaled range analysis, the modified rescaled range analysis and the Whittle test are used to test the presence of long memory in the Indian stock market. BSE SENSEX is used as a proxy for the Indian market. The raw, absolute and squared daily returns are used for the purpose of analysis. The Hurst coefficient H and the fractional differencing parameter d are computed for each of the three return series. The raw returns do not show any long-term dependence, but the absolute and squared returns do. These findings are in agreement with the stylized facts observed in financial time series.

Key words: Long memory processes, R/S analysis, fractional integration, Whittle test, Hurst coefficient

INTRODUCTION

Dependence structure across time plays an important role in the modeling of macroeconomic and financial data. Many macroeconomic and financial time series like nominal and real interest rates, real exchange rates, exchange rate forward premiums, interest rate differentials and volatility measures are found to be very persistent, i.e., an unexpected shock to the underlying variable has long lasting effects. In this context, intense interest has been shown in capturing the possibility of long memory (variously referred to as long-range dependence, or strong dependence or persistence) in financial time series of asset returns as shown by Robinson (2003). There exists significant evidence that long memory processes can provide a good description of many highly persistent financial time series.

Long-range versus short-range processes are often characterized in terms of their auto covariance functions. In case of short-range dependent processes, the coupling between values at different times decreases rapidly as the time difference increases. Either the auto-covariance drops to zero after a certain time lag or it eventually has an exponential decay. In contrast, for long-range processes, there is much stronger coupling so that the decay of the auto-covariance function is power-like i.e., the decay is slower than exponential.

The presence of long-term memory indicates that the market does not immediately react to the information coming into the financial market, but responds to such information gradually over a

period of time. Hence, past price changes can be used as significant information for the prediction of future price changes as discussed by Mantegna and Stanley (2000). This observation has important implications for market efficiency.

The study presented below considers SENSEX as a proxy for the Indian market. The period under consideration is from 1997 till March 2009. The daily return is used for all the analysis to make the series unit root free. The daily return is defined as:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where, P_t is the closing price on day t . The analysis considers both the raw return series (with sign) and the absolute and squared return series (without sign).

There is strong evidence that the return R_t has little or no autocorrelation, but the squared return R_t^2 or the absolute return $|R_t|$, exhibit noticeable autocorrelation. This implies no significant long-term memory in the returns of the financial time series, but existence of appreciable long-range dependence in the volatility time series, as discussed by Campbell *et al.* (1996).

The phenomenon of long-range dependence has a fairly long and chequered history. According to Beran (1994), long-memory processes have been observed in natural and human phenomena ranging from the level of rivers to the temperature of the earth. A test for long term dependence was first designed by Hurst (1951) to predict the pattern of flooding by the river Nile. The test statistic used for this purpose is known as the R/S statistic. According to Baillie *et al.* (1996), reports of long-memory in economic data range from macroeconomics to finance. In macroeconomics this includes GNP data discussed by Diebold and Rudebusch (1989), the consumer price index and other measures of inflation considered by Baillie *et al.* (1995) and Hassler and Wolters (1995) and the term structure of interest rates reviewed by Backus and Zin (1993).

There are not too many studies concerning the presence of long memory in the Indian stock market. It is the absence of significant amount of work on long-memory in the Indian context and the increasing importance of long-memory for policy decisions in finance and economics that prompted the present authors to undertake an investigation of long-term dependence in case of the Indian stock market. The objective of the present study is two-fold: (1) to introduce the basic technical formalism associated with the analysis of long-term dependence and (2) to use three different tests, viz., Rescaled Range (R/S) analysis, its modified form (due to Lo) and the Whittle test to analyze long-term dependence in case of raw, absolute and squared returns.

The auto-correlation of daily returns is typically found to be insignificant at lags between a few minutes and a month. This is one of the stylized facts of financial time series. On the other hand, the autocorrelation function of absolute returns remains positive over lags of several weeks and decays slowly to zero. This phenomenon occurs across different asset classes and time periods and is considered as another stylized fact. Similar behaviour is observed for squared returns and more generally for $|R_t|^\alpha$ according to Cont *et al.* (1997) and Ding and Granger (1996). However, as noted by Davidian and Carroll (1987), absolute realizations are more robust in the presence of fat-tailed observations found in financial time series compared to their squared counterparts. Moreover, empirical analysis of financial time series indicates that the long memory feature dominates for the absolute value over the squared realizations. This is the reason why the present authors used the raw, absolute and squared return to analyze long-term dependence in the Indian stock market.

MATERIALS AND METHODS

The study presented below considers SENSEX as a proxy for the Indian market. The period under consideration is from 1997 to March 2009. The daily return is used for all the analyses to make the series unit root free. The daily return is defined as:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where, P_t is the closing price on day t . The analysis considers the raw return series (with sign) as well as the absolute and squared return series (without sign).

Before performing tests for long memory in the respective indices, the data is AR-GARCH filtered first. The filtering ensures that the long term memory in the series can no longer be partially or fully explained by conditional heteroscedasticity. The tests for long memory are then performed on the residual series only.

Determination of the long-memory property of a time series is difficult because the strong autocorrelation of long-memory processes leads to large statistical fluctuations. Tests for long memory require large volumes of data and often lead to inconclusive or conflicting results. The next three sub-sections introduce the basic definitions necessary to understand long memory and discuss the different methodologies that are used to analyze long-term dependence, viz., the R/S and the modified R/S statistic, the Hurst exponent and Whittle's test for fractional integration.

Basic definitions: Roughly speaking, a random process is said to exhibit long-memory if it has an auto-correlation function which is not integrable. This implies that the autocorrelation function decays asymptotically as a function of the time lag. Such a strong autocorrelation implies a high degree of long-term predictability, meaning thereby that present stock prices can be significantly affected by prices from the distant past. Long-memory also has important consequences for diffusion processes and for the rate of convergence of statistical estimates.

Formally, a stationary process y_t is said to have long memory or long range dependence, if its autocorrelation function behaves like,

$$\rho(k) \rightarrow C_\rho k^{-\alpha} \text{ as } k \rightarrow \infty \quad (1)$$

where, C_ρ is a positive constant and α is a real number between 0 and 1. The degree of long-memory is given by the exponent α ; the smaller the value of α , the longer is the memory.

Thus, the autocorrelation function of a long memory process decays slowly at a hyperbolic rate. In fact, it decays so slowly that the autocorrelations are not summable, according to McLeod and Hippel (1978).

$$\sum_{k=-\infty}^{\infty} \rho(k) \rightarrow \infty \quad (2)$$

For a stationary process y_t , the autocorrelation function contains the same information as its spectral density. The spectral density is defined as:

$$f(\omega) = \frac{1}{2} \sum_{-\infty}^{\infty} \rho(k) e^{ik\omega} \quad (3)$$

where, ω is the fourier frequency. It can be shown using Eq. 1 that

$$f(\omega) \rightarrow C_f \omega^{\alpha-1} \text{ as } \omega \rightarrow 0 \quad (4)$$

where, C_f is a positive constant. Hence, for a long memory process, its spectral density tends to infinity at zero frequency.

Instead of using α , we often use the parameter H , linearly related to α and defined as:

$$H = 1 - \frac{\alpha}{2} \in (0.5, 1) \quad (5)$$

where, H is known as the Hurst exponent and it is used to measure the long memory in y_t . Larger the value of H , longer is the memory of the stationary process.

The R/S statistic and the modified R/S statistic: To investigate the nature and extent of long-term memory in a time series, we first employ a traditional and widely used test called the Rescaled Range (R/S) analysis. This test was first used by Hurst (1951) to test for long memory in the pattern of flooding by the river Nile. It was later modified by Mandelbrot (1972, 1975), Mandelbrot and Taqqu (1979) and Mandelbrot and Wallis (1968, 1969b, c).

The basic idea underlying the classical R/S test is to compare the minimum and maximum values of running sums of deviations from the sample mean, renormalized by the sample standard deviation. For long-memory processes the deviations are larger than for processes which do not have long-memory.

Consider a time series y_t for $t = 1, 2, \dots, T$.

The classical R/S statistic is defined as:

$$Q_\tau = \frac{1}{s_\tau} \left[\max_{1 \leq k \leq \tau} \sum_{j=1}^k (y_j - \bar{y}) - \min_{1 \leq k \leq \tau} \sum_{j=1}^k (y_j - \bar{y}) \right] \quad (6)$$

where, y_1, \dots, y_τ are sample observations and \bar{y} is the sample mean given by:

$$\bar{y} = \frac{1}{T} \sum_{i=1}^T y_i$$

The term within bracket shows the range of partial sum of deviations and it is rescaled by dividing with s_τ , the standard deviation which is defined as:

$$s_\tau \equiv \left[\frac{1}{T} \sum_j (y_j - \bar{y})^2 \right]^{\frac{1}{2}}$$

The R/S statistic provides the Hurst exponent H the value of which can be interpreted in different ways. When there is no long memory in a stationary time series, the R/S statistic converges to a random variable at the rate $T^{0.5}$. But, when the stationary process y_t has long memory, the R/S statistic converges to a random variable at the rate T^H , where H is the Hurst coefficient.

The H value of 0.5 signifies a random and statistically independent or uncorrelated series-a random walk where the present is not influencing the future. Such a process would increase with the square root of time. For H lying between 0.5 and 1, the series would be a persistent or trend-reinforcing series, where the current trend will be followed in the next period (volatility clustering). This process is mean-averting and is often referred to as fractional Brownian motion, or a biased random walk, in the terminology of nonlinear dynamics. If $\log Q$ is plotted against $\log k$, for a long memory process with sufficiently large lags, the points in the plot will be scattered around a straight line with slope $H > 0.5$. For a system with short memory, the points in the plot will be scattered around a straight line with slope $H = 0.5$. Moreover, as per the discussion of Peters (1989) higher value of H would mean stronger persistence and lesser white noise in the series. Finally, if H lies in between 0 and 0.5, the series will be anti-persistent or mean-reverting.

The R/S statistic is quite robust in itself. It has the ability to detect long range dependence in non-Gaussian time series with large skewness and kurtosis. Traditional methods like serial autocorrelation can be used to locate long range dependence only for models with near Gaussian effects. However, this method has severe shortcomings when used for non-Gaussian time series. According to Mandelbrot and Wallis (1969a), when an ACF analysis program is used blindly for such t.s., the degree of dependence is grossly underrated. Further, R/S analysis has its advantage over other measures of long run dependency like the variance time analysis. For stochastic processes with infinite variances, the variance time analysis becomes inapplicable. This problem can be overcome by using the R/S analysis. The spectral analysis also becomes inappropriate for the typical economic time series due to its inability to capture the non-periodicity. Again, R/S analysis can be used for such time series.

Nevertheless, R/S statistic has also been criticized on many counts. The classical R/S test has been proven to be too weak, i.e., it tends to indicate that a time series has long-memory when it does not really have so. Annis and Lloyd (1976) pointed out the small sample bias in the test. Lo (1991) criticized the R/S statistic for being unable to distinguish between short term and long term dependence and called it a severe shortcoming in the applications of the R/S analysis. This observation was backed strongly by Lo and McKinlay (1988, 1990), that showed significant short range dependence in stock returns.

In order to distinguish between long-range and short-range dependence, Lo (1991) modified the classical R/S statistic in such a way that its statistical behaviour is invariant over a general class of short memory processes, but deviates for long memory ones.

The modified R/S statistic is defined as:

$$\tilde{Q}_T = \frac{1}{\hat{\sigma}_T(q)} \left[\max_{1 \leq k \leq T} \sum_{j=1}^k (y_j - \bar{y}) - \min_{1 \leq k \leq T} \sum_{j=1}^k (y_j - \bar{y}) \right] \quad (7)$$

Where:

$$\begin{aligned} \hat{\sigma}_T^2(q) &\equiv \frac{1}{T} \sum_{j=1}^q (y_j - \bar{y})^2 + \frac{2}{T} \sum_{j=1}^q \omega_j(q) \left\{ \sum_{i=j+1}^T (y_i - \bar{y}) \{y_{i-j} - \bar{y}\} \right\} \\ &= \hat{\sigma}_y^2 + 2 \sum_{j=1}^q \omega_j(q) \hat{\gamma}_j \end{aligned}$$

Where:

$$\omega_j(q) \equiv 1 - \frac{j}{q+1}; \quad q < T$$

where, $\hat{\sigma}_y^2$ and $\hat{\gamma}_j$ are the usual sample variance and auto-covariance estimators of y_t .

\hat{Q}_T is different from Q_T only in the denominator, which is the square root of a consistent estimator of the variance of the partial sum. The estimator $\hat{\sigma}_T(q)$ involves not only sums of squared deviations of y_j , but also its weighted auto-covariances up to lag q . The weights $\omega_j(q)$ are those suggested by Newey and West (1987) and lead to a positive value of $\hat{\sigma}_T^2(q)$, an estimator of $2p$ times the (unnormalized) spectral density function of y_t at frequency zero using a Bartlett window.

$\hat{\sigma}_T(q)$ is consistent under the following conditions, according to Phillips' theorem 4.2 Phillips (1987).

- $\sup_t E[|\varepsilon_t|^{2\beta}] < \infty$ for some $\beta > 2$
- As T increases without bound, q also increases without bound such that $q \sim o(T^{1/4})$

Lo's modified R/S test has been criticized for being too stringent. It has been shown numerically that even for a synthetic long-memory time series with a moderate value of the Hurst coefficient, the Lo test cannot reject the null hypothesis of short-range dependence.

Testing for fractional integration: Whittle estimate of d : Long-memory processes lie between stationary $I(0)$ processes and non-stationary $I(1)$ processes. A long memory process can be considered as the bridge joining the non-stationary processes and the stationary ones-namely a fractionally integrated process. Using (1) the scaling property and (2) the frequency domain property, it has been shown by Granger and Joyeux (1980) and Hosking (1981) that a long memory process y_t can also be modeled parametrically by extending an integrated process to a fractionally integrated process. Fractional integration in a time series y_t can be described as:

$$(1-L)^d(y_t - \mu) = u_t \tag{8}$$

where, L denotes the lag operator, d is the fractional integration or fractional difference parameter, μ is the expectation of y_t and u_t is a stationary short memory disturbance with zero mean.

For some highly persistent economic and financial time series, integer difference of a time series is not always appropriate. To accommodate the long-memory effect we avoid taking an integer difference of y_t and allow d to be fractional. The fractional difference filter is defined as follows:

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-1)^k L^k \tag{9}$$

for any real $d > -1$ with binomial coefficients:

$$\binom{d}{k} = \frac{d!}{k!(d-k)!} = \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} \tag{10}$$

The fractional difference filter can be equivalently treated as an infinite order autoregressive filter. It can be shown that when $|d| > 1/2$, y_t is non-stationary; when $0 < d < 1/2$, y_t is stationary and has long memory; when $-1/2 < d < 0$, y_t is stationary and has short memory and is sometimes referred to as anti-persistent.

When a fractionally integrated series y_t has long memory, it may be shown that:

$$d = H - 0.5 \tag{11}$$

which means that d and H can be used interchangeably as the measure of long memory.

Whittle (1951) estimator is a non-parametric method used to estimate the fractional difference parameter d . In this method, d is estimated using a frequency domain maximum likelihood estimation of a fractionally integrated process Eq. 8. The parameters in Eq. 8 are estimated by minimizing a discretized version of

$$Q(\theta) = \int_{-\pi}^{\pi} \frac{I(\omega)}{f(\theta, \omega)} d\omega \tag{12}$$

where, θ is the vector of unknown parameters including the fractional difference parameter d , $I(\omega)$ is the periodogram of y_t and $f(\theta, \omega)$ is the theoretical spectral density of y_t .

RESULTS

The total sample size considered in this investigation was $T = 2381$.

The results obtained could be classified under three categories:

- Results from the unit root tests
- Results from visual representation of data
- Results from the application of the tests for long memory

Unit root test: The filtered raw return series (AR-GARCH filtered) were checked for possible presence of unit root. Augmented Dickey Fuller test and Philips Perron tests were carried out to check the stationarity of the raw return time series. The results were shown in Table 1.

Table 1: Result of unit root tests

	ADF test		Phillips Perron test	
	t-stat	p-value	t-stat	p-value
Null hypothesis				
Raw return	-45.23*	0	-45.19*	0
Absolute return	-12.467*	0	-51.068*	0
Squared return	t-stat = -8.66*	p = 0	t-stat = -46.65*	p = 0

*indicates significance at 1% level

The null and alternative hypotheses were as follows:

H_0 : Unit root is present

H_1 : Unit root is not present

The tests were carried out at 1% level of significance.

Values of t-statistic were computed and the p-values were determined. The extremely low p-values indicated that there was sufficient statistical evidence to reject the null hypothesis. In other words, it could be inferred that the raw return series was free from unit root.

Similar exercises were carried out for the absolute and squared return series. Once again, the low p-values pointed to the absence of unit root.

Thus all three series-raw, absolute and squared-were found to be unit root free according to both the tests.

Visual interpretation: To gain a visual interpretation of the results, the Auto-Correlation Function (ACF) was plotted against the time lag for the raw return time series. The lag was taken up to 200 (days in this case). The autocorrelation was found to be insignificant in case of the raw return time series (Fig. 1). This exercise was repeated in case of the absolute and squared return series. However, in case of these two time series, a discernible pattern of decaying autocorrelation was clearly visible in the respective figures (Fig. 2, 3). This implied that the absolute and squared return series exhibited long term dependence.

Numerical results from the tests of long memory: Once the stationarity of the time series was established in each of the three cases and a visual evidence of possible long memory established in the absolute and squared return series, more powerful and conclusive tests were carried out to

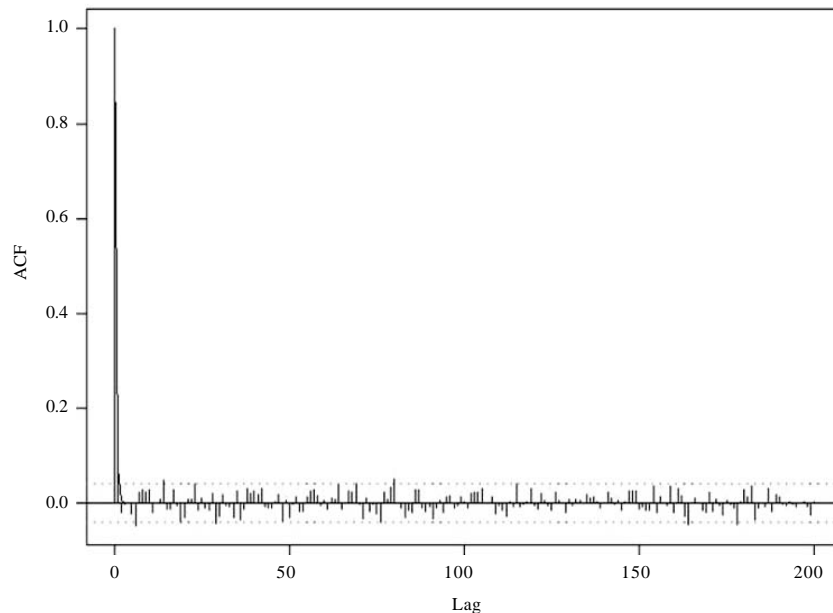


Fig. 1: ACF plot for raw daily return

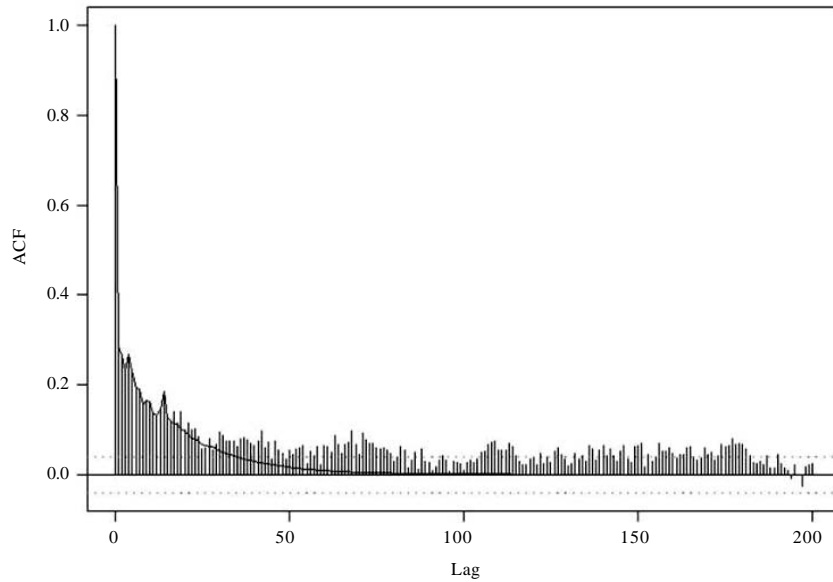


Fig. 2: ACF plot for absolute daily return

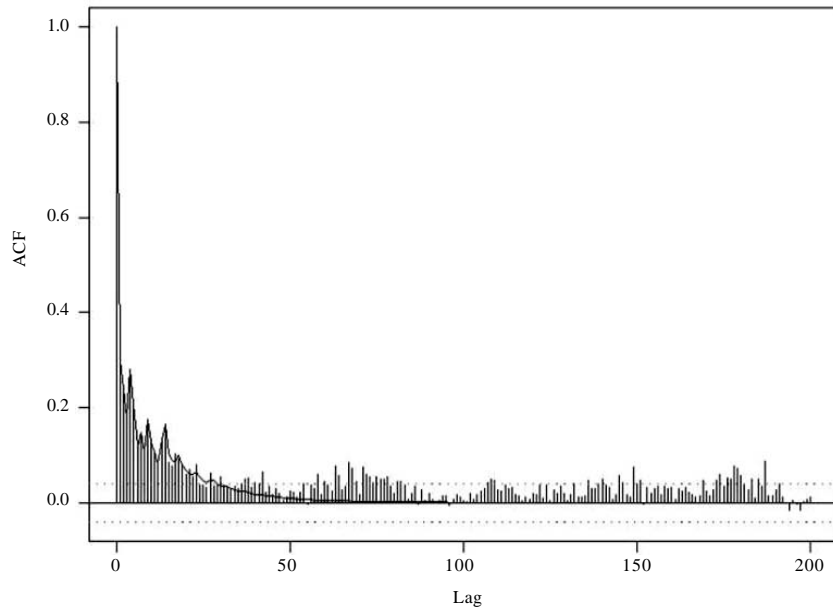


Fig. 3: ACF plot for squared daily return

check the presence of long memory in them. The tests used were the R/S, the modified R/S and Whittle's method for the determination of d . The results were summarized in Table 2.

The R/S and modified R/S tests were done using the following null and alternative hypotheses:

H_0 : There is no long term dependence in the return series

H_1 : There exists long term dependence in the return series

Table 2: Result for long memory tests

Results	R/S test	Modified R/S test	Whittle test	Hurst coefficient
Raw return	1.8118	1.7394	0.0273	0.5435
Absolute return	4.6457**	3.3484**	0.1307	0.6761
Squared return	3.9679**	3.1144**	0.1021	0.6970

Null hypothesis: No long term dependence, **Indicates significance at 1% level

The tests were carried out at the 1% level of significance.

For the raw return series, the following values were obtained:

R/S test statistic = 1.8118 and the modified R/S test statistic $\hat{Q}_T = 1.7394$

Both these values were less than the corresponding critical values at 1% level of significance which meant that there was sufficient statistical evidence to accept the null hypothesis. In other words, the raw return series showed no sign of long term memory. This was further supported by the value of the Hurst coefficient H (which was nearly equal to 0.5 in case of the raw return series).

For the absolute and squared return series, the same set of Null and Alternative hypothesis were used. The tests were again carried out at 1% level of significance. The following values were obtained:

Absolute return series:

R/S test statistic = 4.6457 and the modified R/S test statistic $\hat{Q}_T = 3.3484$

Squared return series:

R/S test statistic = 3.9679 and the modified R/S test statistic $\hat{Q}_T = 3.1144$

Values in both the cases were greater than the corresponding critical values at the 1% level of significance which indicated that there was sufficient statistical evidence to reject the null hypothesis and conclude that there was significant presence of long term memory in the absolute and squared return series. The same conclusion could be drawn from values of the Hurst coefficient, nearly equal to 0.7 in either case. Recall that a Hurst coefficient greater than 0.5 indicated a long memory process.

The following values of d were obtained from the Whittle test:

Absolute return series: $d = 0.130727$

Squared return series: $d = 0.1021621$

The foregoing values corroborated the long memory property in case of the absolute and squared return series. Moreover, in case of both the absolute as well as the squared return series, the relation $d = H - 1/2$ was almost satisfied, which was a further proof of the long memory property prevailing in these series.

DISCUSSION

Detection and characterization of long-memory processes in a time series is a non-trivial task. Several studies have been carried out in this area. One of the early studies was by Greene and Fielitz (1977), who considered the daily return series of 200 stocks and used the R/S analysis to

demonstrate the presence of long term memory in many of them. They found that in the presence of long term dependence, martingale process did not hold. Lo (1991) investigated long memory in the US stocks with some modifications in the R/S statistic and found that this modified statistic was unable to capture any presence of long memory in the US stock market. But according to Willinger *et al.* (1999), the modified R/S statistic showed a tendency to accept the null hypothesis of no long term memory and hinted at the lack of conclusiveness of Lo.

Barkoulas *et al.* (2000) used the spectral transmission method to find evidence of long term memory in the Greek stock market. Rege and Martin (2007) used the Hurst exponent to indicate the presence of long memory in the Portugese stock market. Cajueiro and Tabak (2004) demonstrated evidence for long term dependence in Hong Kong, Singapore and Chinese markets. Sadique and Silvapulle (2001) obtained long memory in Korean, Singapore, New Zealand and Malaysian markets. Lillo and Farmer (2004) found such evidence in London Stock Exchange data.

Ding *et al.* (1993) discovered long memory stochastic volatility in stock returns. Baillie *et al.* (1996) introduced the Fractionally Integrated Exponential GARCH model (FIEGARCH) to capture the long run dependence in the US stock market. Christensen *et al.* (2007) improved on this premise and built an even more powerful FIEGARCH-M (FIEGARCH in Mean) model to accommodate the long term memory in return volatility along with a short memory in the stock returns. It may be mentioned that some recent high-frequency studies point to the interesting question of how market efficiency coexists with the long-memory of order flow, as per the discussion of Bouchaud *et al.* (2004).

However, there are only few studies investigating long range dependence in the Indian financial markets. One such study was by Nath (2001), who used variance ratio test and the R/S analysis to test long term memory in the daily NIFTY returns. The R/S analysis indicated the presence of long memory but the variance ratio test failed to capture anything significant.

The present study was an attempt to analyze long memory features in the context of the Indian financial market and see how far the results agree with the stylized facts of financial time series. Recall that stylized facts are empirical observations that are so consistent and have been made in so many contexts (e.g., across a wide range of instruments, markets and time periods) that they are accepted as truths, to which theories must fit. Stylized facts are obtained by taking a common denominator among the properties observed in studies of different markets and instruments.

The study clearly showed that the raw return series did not exhibit any long term dependence but the absolute and squared returns both showed long-range behaviour-thus the results were in general agreement with the stylized facts of the financial time series.

In conclusion, it may be mentioned that it is possible to capture the long and short memory features of a stationary time series at the same time. The fractional ARIMA models (FARIMA or ARFIMA in short) are capable of modeling both the long memory and short run dynamics in a stationary time series. Such modeling could provide valuable insights into the features of time series data.

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REFERENCES

Annis, A.A. and E.H. Lloyd, 1976. The expected value of the adjusted rescaled Hurst range of independent normal summands. *Biometrika*, 63: 111-116.

- Backus, D.K. and S.E. Zin, 1993. Long-memory inflation uncertainty: Evidence from the term structure of interest rates. *J. Money Credit Banking*, 25: 681-700.
- Baillie, R.T., Chung, C.F. and M.A. Tieslau, 1995. Analyzing inflation by the fractional integrated arfima-garch model. *J. Applied Econ.*, 11: 23-40.
- Baillie, R.T., T. Bollerslev and H.O. Mikkelsen, 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. *J. Econ.*, 74: 3-30.
- Barkoulas, J.T., C.F. Baum and N. Travlos, 2000. Long memory in the Greek stock market. *Applied Financial Econ.*, 10: 177-184.
- Beran, J., 1994. *Statistics for Long-Memory Processes*. Chapman and Hall, New York.
- Bouchaud, J.P., Y. Gefen, M. Potters and M. Wyart, 2004. Fluctuations and response in financial markets: The subtle nature of random price changes. *Quantitative Finance*, 4: 176-190.
- Cajueiro, D.O. and B.M. Tabak, 2004. Evidence of long range dependence in Asian equity markets: the role of liquidity and market restrictions. *Physica A: Statistical Mech. Appli.*, 342: 656-664.
- Campbell, J.Y., A.W. Lo and MacKinlay, A. Craig, 1996. *The Econometrics of Financial Markets*. Princeton University Press, New Jersey, USA., ISBN13: 978-0-691-04301-2.
- Christensen, B.J., Nielsen, M. and J. Zhu, 2007. Long memory in Stock Market Volatility and the Volatility-in-Mean Effect: The FIEGARCH-M Model. CREATES Research Papers 2007-10, School of Economics and Management, University of Aarhus, Canada.
- Cont, R., J.P. Bouchaud and M. Potters, 1997. Scaling in Financial Data: Stable Laws and Beyond. In: *Scale Invariance and Beyond*, Dubrulle, B., F. Graner and D. Sornette (Eds.). Springer, Berlin.
- Davidian, M. and R.J. Carroll, 1987. Variance function estimation. *J. Am. Statistical Assoc.*, 82: 1079-1091.
- Diebold, F.X. and G.D. Rudebusch, 1989. Long-memory and persistence in aggregate output. *J. Monetary Econ.*, 24: 180-209.
- Ding, Z. and C.W.J. Granger, 1996. Modeling volatility persistence of speculative returns: A new approach. *J. Econ.*, 73: 185-215.
- Ding, Z., C.W.J. Granger and R.F. Engle, 1993. A long memory property of stock market returns and a new model. *J. Empirical Finance*, 1: 83-106.
- Granger, C.W.J. and R. Joyeux, 1980. An Introduction to long memory time series models and fractional differencing. *J. Time Seri. Anal.*, 1: 15-29.
- Greene, M.T. and B.D. Fielitz, 1977. Long-term dependence in common stock returns. *J. Financial Econ.*, 4: 339-349.
- Hassler, U. and J. Wolters, 1995. Long-memory in inflation rates: International evidence. *J. Business Econ. Statistics*, 13: 37-45.
- Hosking, J.R.M., 1981. Fractional differencing. *Biometrika*, 68: 165-176.
- Hurst, H., 1951. Long term storage capacity of reservoirs. *Trans. Am. Soc. Civil Eng.*, 116: 770-799.
- Lillo, F. and J.D. Farmer, 2004. The long memory of the efficient market. *Stud. Nonlinear Dyn. Econo.*, Vol. 8.
- Lo, A.W. and C. MacKinlay, 1988. Stock Market prices do not follow random walks: Evidence from a simple specification test. *Rev. Financial Stud.*, 1: 41-66.
- Lo, A.W. and C. MacKinlay, 1990. When are contrarian profits due to stock market overreaction. *Rev. Financial Stud.*, 3: 175-206.
- Lo, A.W., 1991. Long-term memory in stock market prices. *Econometrica*, 59: 1279-1313.

- Mandelbrot, B. and J. Wallis, 1968. Noah, Joseph and operational hydrology. *Water Resour. Res.*, 4: 909-918.
- Mandelbrot, B. and J. Wallis, 1969a. Computer experiments with fractional Gaussian noises Parts 1, 2, 3. *Water Resour. Res.*, 5: 228-267.
- Mandelbrot, B. and J. Wallis, 1969b. Some long run properties of geophysical records. *Water Resour. Res.*, 5: 321-340.
- Mandelbrot, B. and J. Wallis, 1969c. Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence. *Water Resour. Res.*, 5: 967-988.
- Mandelbrot, B.B., 1972. A statistical methodology for non-periodic cycles: From the covariance to R/S analysis. *Annals Econ. Social Measurement*, 1: 259-290.
- Mandelbrot, B. and M. Taqqu, 1979. Robust R/S analysis of long-run serial correlation. *Bull. Int. Statistical Inst.*, 48: 59-104.
- Mandelbrot, B.B., 1975. Limit theorems on the self-normalized range for weakly and strongly dependent processes. *Probability Theory Related Fields*, 31: 271-285.
- Mantegna, R.N and H.E. Stanley, 2000. *An Introduction to Econophysics*. Cambridge University Press, UK., ISBN-13: 978-0521620086.
- McLeod, A.I. and A.W. Hippel, 1978. Preservation of the rescaled adjusted range, 1: A reassessment of the Hurst phenomenon. *Water Resour. Res.*, 14: 491-508.
- Nath, G.C., 2001. Long memory and Indian stock market-an empirical evidence UTIICM Conference Paper. http://www.utiicm.com/Cmc/PDFs/2001/golakanath_longmemory_indian_stock_market.pdf.
- Newey, W. and K. West, 1987. A simple positive definite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55: 703-705.
- Peters, E.E., 1989. Fractal structure in the capital markets. *Financial Analysts J.*, 45: 32-37.
- Phillips, P.C.B., 1987. Time series regression with a unit root. *Econometrica*, 55: 277-301.
- Rege, S. and S.G. Martin, 2007. Portuguese stock market: A long memory process?. Proceedings of the 1st Portuguese Economic Journal Conference Acores, June 22-23, CEEAplA and Departamento de Economia e Gestao, Universidade dos Acores, pp: 1-10.
- Robinson, P.M., 2003. Long Memory Time Series. In: *Time Series with Long Memory; Advanced Texts in Econometrics*, Peter, M.R. (Ed.). OUP, UK., ISBN-13: 978-0199257300, pp: 4-32.
- Sadique, S. and P. Silvapulle, 2001. Long-term memory in stock market returns: International evidence. *Int. J. Finance Econ.*, 6: 59-67.
- Whittle, P., 1951. *Hypothesis Testing in Time Series Analysis*. Hafner Publisher, New York.
- Willinger, W., M.S. Taqqu and V. Teverovsky, 1999. Stock market prices and long-range dependence. *Finance Stochastics*, 3: 1-13.