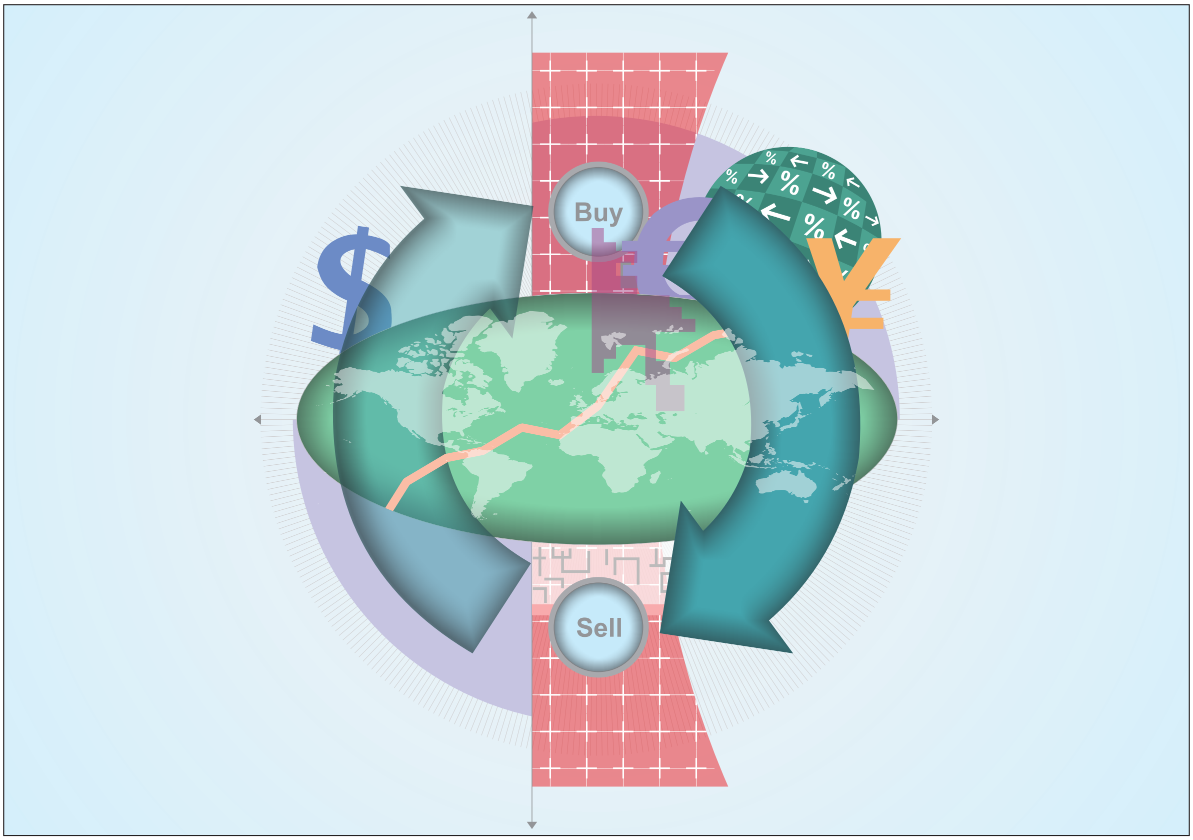

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Applying Point Elasticity of Demand Principles to Optimal Pricing in Management Accounting

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ABSTRACT

This study applies first principles in micro economics relating to price elasticity of demand to explore a new concept called the 'price elasticity of contribution' to help businesses adopt pricing strategies for the maximisation of contribution. The study derives a new proof for the maximisation of revenue and profit, using the point elasticity of demand equation to simplify the traditional management accounting methodology for algebraically calculating the price at which business revenue and profit are maximised. The study uses one example throughout to demonstrate the proof and to illustrate the concepts introduced.

Key words: Profit maximisation, price elasticity of contribution, point elasticity of contribution

INTRODUCTION

Marshall (1890) originally developed the concept of price elasticity of demand from marginal utility theory. Since then much has been written in business management text books about the importance of recognising the sensitivity of the quantity demanded of goods and services to changes in the prices charged for those goods and services and how useful this concept is in marketing and pricing policy.

The purpose of this study is to use Marshall's theory of price elasticity of demand and apply this model to management accounting and develop from this a simple and direct approach for determining the points at which revenue and profit are maximised.

The demand function for goods and services: The demand for most goods and services is a downward sloping function. This reflects that the price and quantity demanded, other than in very special circumstances such as is the case with Veblen (1899) and Giffen goods, from Marshall (op. cit.) are inversely related. This is caused by both diminishing marginal utility and finite income which makes the consumer less willing and/or unable to pay as much to consume additional units of output.

If it is assumed that the relationship between price charged and quantity demanded is linear, it is possible to establish the equation of the demand curve from knowing two points on the graph. To explain this and all the other instructional points within this study, the same data from an illustrative example will be used throughout.

Illustrative example: Company A produces and sells a product called Z.

Table 1 shows two points on a demand curve relating to Product Z:

Having two points on the graph gives sufficient information to establish the equation of the demand curve.

Table 1: Schedule of prices and quantity demanded for Product Z

Graph co-ordinate	Price (y)	Quantity demanded (x)
Point 1	10	100
Point 2	7	400

The slope of the line can be measured between any two points on the demand function. In this instance the points given in Table 1, between the prices of $7 and $10, will be used.

Formulating the demand function equation from first principles: As a linear equation, the demand function takes the following form:

$$Q = I - m \times P$$

where, Q is Quantity; I is Intercept (constant); m is Slope or gradient of the function and P is price.

The slope (m) is (x2-x1)/(y2-y1) which is the change in quantity demanded/change in price as between the two points given.

Which means that m (slope) of the function for Product Z is:

$$(400-100)/(7-10) = 300/-3 = -100$$

Using the slope (m), it is possible to substitute the price and the associated quantity demanded at point 1 in Table 1 into the basic demand equation as follows:

$$100 = I - 100 \times 10$$

which restated in terms of I gives:

$$I = 1{,}100$$

This gives the completed demand function equation:

$$Q = 1{,}100 - 100P$$

And if Q is set to zero, the Y axis intercept can also be calculated:

$$0 = 1{,}100 - 100P$$

Therefore, P = 11.

The demand function is shown graphically in Fig. 1.

Price elasticity of demand: Price elasticity of demand (over a range of demand and prices) is defined as:

$$\frac{\text{Change in quantity demanded (\%)}}{\text{Change in price (\%)}}$$

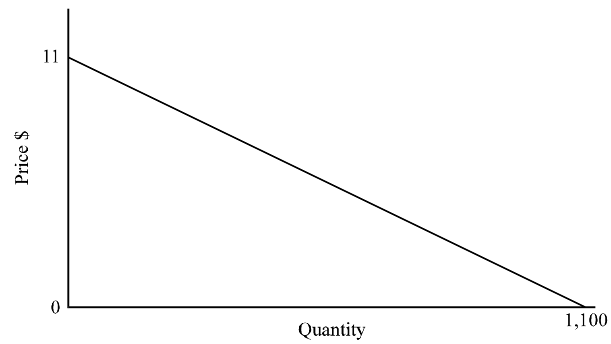

Fig. 1: Demand function for product Z

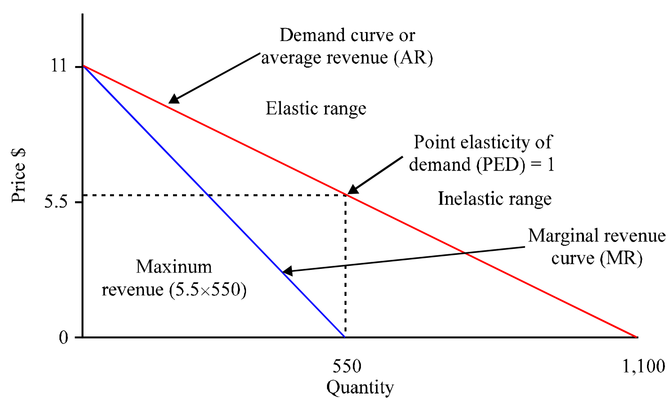

Fig. 2: Point elasticity of demand for product Z including the MR function

Because quantities demanded and prices are usually inversely related, the result of this formula is a negative number, confirming that the demand curve for a good or service is usually downward sloping. Depending on circumstances, such as competitive position, the availability of substitutes and complements and barriers to entry, the slope of the demand curve will vary in steepness-thereby affecting the elasticity of demand at various points along the curve.

A price elasticity of greater than one indicates that as price is reduced, total revenue (price x quantity demanded) will increase and vice versa.

For a range of prices starting from the maximum Company A could charge for Product Z and sell nothing at all (where the demand curve intersects with the Y axis and where, P = 11), the marginal revenue obtained by reducing prices from that point will fall at twice the rate of price but remain positive until the point is reached where revenue is maximised. At this point marginal revenue reaches zero. A price elasticity of less than one indicates that as price is further reduced, total revenue will also fall. This second range of prices starts at the point at which marginal revenue is zero and every subsequent reduction in the price of Z will yield less total revenue until a price of zero is reached, at which total revenue also becomes zero by implication. A price elasticity of exactly one, known as 'unitary elasticity' indicates that any change in price between the two points being compared will have no effect on revenue. The mid-point is where revenue is maximised and where MR is equal to zero.

The point at which MR reaches zero corresponds to exactly half way down the demand curve, due to the MR curve always having *mathematically* twice the slope or gradient of the linear demand curve. This is shown in Fig. 2.

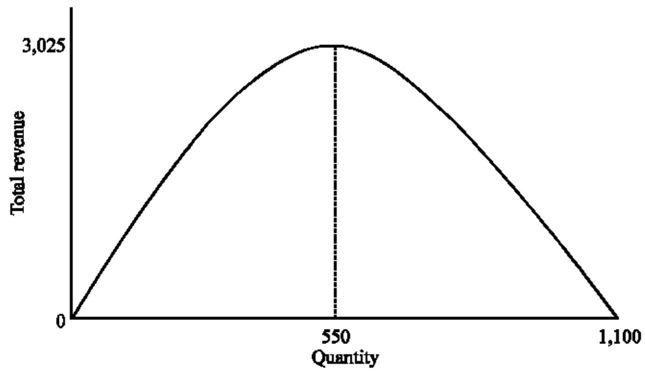

Fig. 3: Total revenue function for company A and product Z

Note that, the MR function reaches zero at the point precisely below where the point elasticity = 1 on the demand curve. The total revenue function is shown in Fig. 3.

Figure 3 confirms that, at the point where quantity demanded is equal to 550 units, total revenue is maximised and by implication marginal revenue is equal to zero. The maximum revenue is therefore (550×5.5) or $3,025.

Using the price elasticity of demand equation to maximise revenue: In some circumstances businesses may have a primary objective to maximise revenues, for example to maximise market share in a competitive market (Baumol, 1959).

The basic formula for elasticity of demand, shown earlier, measures change in quantity demanded against a change in price over a given range. To calculate elasticity of demand exactly, we should use the Point Elasticity of Demand (PED) formula:

$$PED = \text{Abs } dQ/dP \times (P/Q)$$

This formula always uses the absolute value of the derivative because PED is always described by economists as +1 despite the fact that in most instances, as discussed already, it is actually calculated as -1.

The PED formula gives the point elasticity of demand at a Price (P) and the corresponding Quantity (Q). This gives a more accurate figure for elasticity of demand than the basic equation, as it measures price elasticity over an infinitesimally small interval.

To use the PED formula to establish the point of revenue maximisation, it is necessary to differentiate the demand function which was formulated earlier:

$$Q = 1{,}100\text{-}100P$$

The absolute value of the derivative (dQ/dP) of quantity demanded (Q) with respect to Price (P) = 100 which, as already established, is the slope of the demand function (m).

Therefore, to maximise revenue, setting PED to 1 or 'unitary' in the original equation, gives the following equation:

$$1 = dQ/dP \times (P/Q)$$

Which is the same as 1 = 100×(P/Q) and can be restated as:

$$P = 0.01Q$$

If the demand function (1,100-100P) is substituted for Q in the above equation, the revenue maximising price can be established:

$$P = 0.01\times(1{,}100-100P);$$
$$P = 11-P$$

Therefore, P = 5.5

(Note that Fig. 2 confirms that this was the price at which PED = 1).

The method above requires the quantity demanded to be stated in terms of price and for the derivative of the demand function to be substituted into the simple PED equation to arrive at the point of revenue maximisation.

The above analysis therefore forms the basis of a very simple way to calculate the price at which revenue is maximised.

Because it is already known that, the derivative of the straight-line demand function is equal to the absolute slope of the function, it is possible to use the point elasticity of demand equation set equal to 1 to calculate the point of revenue maximisation directly from the demand equation.

Substituting the slope (m) of the demand function for dQ/dP in the PED equation, results in a simpler equation:

$$1 = m \text{ x } (P/Q)$$

The above equation can then be re-arranged and further simplified as follows:

$$P = f(Q)/m$$ (c) Gareth Owen 2011

Marginal costing: In management accounting, marginal costing is a way of classifying costs as output or time dependent. Output dependent costs (also known as variable or marginal costs) vary with output. Output is defined as the quantity of goods and services produced and sold.

Time dependent costs are also known as fixed or period costs. They vary only with time elapsed and have no dependency on production or output. Using marginal costing it is possible to establish the 'contribution' that a unit of product or service makes towards meeting fixed costs in a given financial period. This is established by subtracting the unit variable cost from the price to arrive at a 'contribution' per unit.

For example, Product Z produced by Company A can be sold for $10 but each unit costs $5 to produce in materials and other output related costs. This leaves $5 left over to contribute towards the fixed costs which have to be incurred and paid for within the period, regardless of output. If it is assumed that the periodic fixed costs of Company A were $2,000 then in order to 'break-even', in this case, 400 units would need to be produced and sold. For each unit sold above the break-even point of 400, a profit of $5 is contributed. Note that, in this basic management accounting example it is assumed that in the short run and over a normal range of output, the variable cost per unit remains constant. Micro economic theory however, indicates that variable costs per unit will change over a wide range of output as a consequence of increasing and diminishing returns, resulting in an 'S' shaped total variable cost curve but from a management accounting perspective, it is reasonable to assume linearity over the relevant range.

While the price elasticity of demand concept is useful in terms of establishing where revenue is maximised at the point where elasticity of demand equals 1 or is 'unitary', the analysis so far does not provide information about maximising profitability where a business incurs significant variable costs. In other words the above PED equation can only establish the point of profit maximisation in the special case where all (or the vast majority) of the costs of a business are fixed and none of the costs are output dependent, thereby coinciding with the point of revenue maximisation already discussed.

The price elasticity of contribution: It is possible to develop a more useful business pricing concept by adapting the price elasticity of demand formula so that it can measure the percentage change in contribution against the percentage change in price:

$$\frac{\text{Change in total contribution (\%)}}{\text{Change in price (\%)}}$$

To measure the sensitivity of profitability to price changes, where a proportion of a business's costs are variable, it is necessary to examine the change in contribution in response to price changes.

As already stated, Company A sells a product Z which costs \$5 in variable costs per unit to produce and sell. Table 2 shows the relationship between prices charged, contribution earned, quantity demanded and total revenue generated at the two different prices and levels of output.

The elasticity of contribution over the two prices and related quantities given in Table 1 can be expressed using the following formula:

$$\frac{\text{Change in total contribution (C2 - C1)/C1 (\%)}}{\text{Change in price (P2 - P1)/P1 (\%)}}$$

$$\frac{(800-500)/500}{(7-10)/10} = \frac{0.60}{-0.3} = -2$$

Note that, the absolute price elasticity of contribution between these two prices is greater than one, meaning that a reduction in price from \$10 to \$7 results in an increase in total contribution. Therefore, over this range, elasticity of contribution is defined as being elastic. This is useful information when a business decides what price to set or whether to offer discounts.

The measure of elasticity of contribution is therefore much more relevant for businesses which incur variable costs in producing or providing goods and services, than the conventional price elasticity of demand measure.

A price elasticity of contribution of greater than one indicates that as price is reduced, total contribution (unit contribution x quantity demanded) will increase and vice versa until marginal contribution reaches zero.

Table 2: Schedule of prices, contribution and quantity demanded of Product Z

Graph co-ordinate	Price (P)	Marginal cost (MC)	Unit contribution (y)	Quantity demanded (x)	Total contribution (C)	Revenue
Point 1	10	5	5	100	500	1,000
Point 2	7	5	2	400	800	2,800

Along this range of prices it may be worth reducing or discounting prices to increase overall profit for that product or service. A price elasticity of contribution of less than one indicates that as price is further reduced, total contribution will also fall. This second half of the average contribution function starts at the point at which marginal contribution has reached zero and every subsequent reduction in price yields less and less total contribution until a unit contribution of zero is reached.

A price elasticity of contribution of exactly one is known as ‘unitary elasticity’ and indicates that any change in price between the two points being compared will have no effect on contribution. This is the point referred to above where contribution is maximised and where marginal contribution is equal to zero.

The point at which marginal contribution reaches zero corresponds to exactly half way down the average contribution function, due to the marginal contribution curve having mathematically twice the slope or gradient of the average contribution curve.

Therefore, by linking the marginal costing concept to elasticity of demand, it is possible to derive a measure of how sensitive total contribution is to any change in the price of a good or service. As already discussed, this model can give useful information on profitability at various prices and output and can support a business with its pricing policy and help to assess the ‘bottom-line’ financial impact of such a policy.

Formulating the average contribution function: To illustrate how to determine the point (or price) where profit is maximised, it is useful to obtain the equation for the average contribution function.

As a linear equation, the average contribution function takes the following form:

$$Q = I - m \times C$$

where, Q is a quantity demanded; I is the Intercept (quantity demanded when average contribution per unit = 0); m is Slope of the average contribution function and C is contribution per unit demanded.

The slope (m) is (x2-x1)/(y2-y1) which on this occasion is simply the change in quantity demanded/change in unit contribution as between the two points given in Table 2 which means that m (slope) of the function is:

$$(400-100)/(2-5) = 300/-3 = -100$$

Note that, this is the same slope as the slope of the demand curve calculated earlier.

Using the slope (m), it is possible to substitute the price and the associated quantity demanded at Point 1 in Table 2 into the average contribution equation as follows:

$$100 = I-100\times 5$$

which restated in terms of I gives:

$$I = 600$$

And this gives the completed average contribution function:

$$Q = 600-100C$$

Fig. 4: Average contribution function for product Z

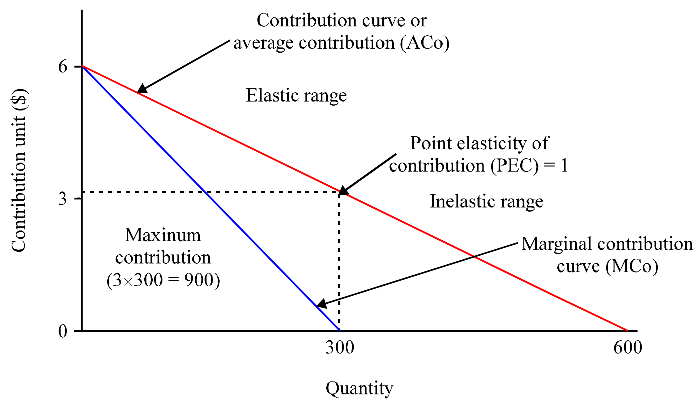

Fig. 5: Point elasticity of contribution for Product Z including the marginal contribution function

And if Q is set to zero, the Y axis intercept can also be calculated:

$$0 = 600\text{-}100C$$

Therefore, C = 6.

This function is shown graphically in Fig. 4.

Figure 5 shows the average and marginal contribution functions and the point elasticity of contribution:

Figure 5 shows that, the Average Contribution (AC_0) and the Marginal Contribution (MC_0) curves have exactly the same slopes as the AR and MR curves shown in Fig. 2. The contribution is maximised when $MC_0 = 0$ and where Point Elasticity of Contribution (PEC) = 1.

Figure 6 shows the total contribution function.

Figure 6 confirms that, at the point where quantity demanded is equal to 300 units, total contribution is maximised and by implication marginal contribution is equal to zero. The maximum contribution available from Product Z is therefore, (300×3) or $900.

Profit maximisation and the MC = MR method: From micro economic first principles it can be established that profits are maximised when Marginal Cost (MC) equates to Marginal Revenue (MR).

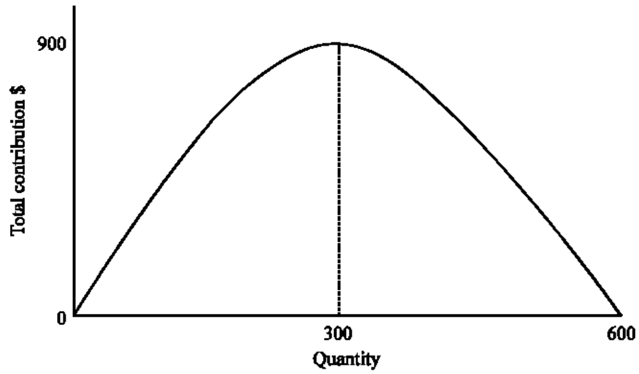

Fig. 6: Total contribution function for company A and product Z

This is because up to this level of output it is always worth producing and selling more goods, as the extra revenue obtained exceeds the additional costs incurred.

In management accounting, establishing MC = MR usually involves the following steps:

1. Formulate the demand function in the form Q = I-m x P (as demand is a downward sloping function)*
2. Restate the demand function in terms of Price (P)*
3. Multiply the price function by the Quantity demanded (Q) to derive the Total Revenue function (TR)
4. Derive the Marginal Revenue (MR) by finding the first derivative of the Total Revenue (TR) function
5. Set MR = MC and solve for Q
6. Substitute Q into the price function (P) obtained in Step 2 and solve for P

*Steps 1 and 2 are unnecessary if the demand function is already given and stated in terms of price rather than quantity demanded. Some management accounting text books also abbreviate this process further by combining Steps 3 and 4 by simply doubling the slope of the demand curve expressed in terms of price, to arrive at the marginal revenue function more directly.

Whichever method is used, equating MC to MR is quite a lengthy process even with linear functions, as assumed in this case. The process can be considerably abbreviated, using basic principles of micro economics relating to price elasticity of contribution, as has been explained earlier for the revenue maximisation problem.

Maximising profit using the point elasticity equation: The above process can be simplified by recognising that MR-MC = MC_o which means that contribution is maximised when $MC_o = 0$. Because it has already been shown that, the Point Elasticity of Contribution (PEC) = 1 when $MC_o = 0$, it is therefore possible to use an adapted version of the PED equation to arrive at the same solution much more directly than trying to equate MC with MR algebraically.

As already explained, the PED equation to maximise revenue was:

$$1 = m \times (P/Q)$$

Therefore, to maximise contribution the formula can be refined as follows:

$$1 = m \times (P\text{-}MC/Q)$$

Note that, this equation uses unit contribution (P-MC) rather than Price (P) as was used in the PED equation as this will yield the price for contribution rather than revenue maximisation, directly from a demand function expressed in terms of quantity.

The above equation can then be re-arranged in terms of P and further simplified as follows:

$$P = f\,(Q)/m + C \text{ (c) Gareth Owen 2011}$$

The simplicity of this alternative method can now be illustrated by comparing the steps required to calculate the price for profit maximisation using the traditional method against those required when using the refined point elasticity of contribution formula.

As already stated, the equation of the demand curve for Product Z is:

$$Q = 1{,}100\text{-}100P$$

and the marginal cost per unit = $5.

Traditional method of finding the optimum price for profit maximisation:

Step 1: **Restate the demand equation in terms of P:** (*Please note that this step can be avoided if the demand equation is already stated in terms of P):

$$100P = 1{,}100\text{-}Q$$

Therefore, P = 11-0.01Q

Step 2: **Derive Total Revenue (TR) multiplying Step 1 by Q:** (*Please note that this and Step 3 below can be combined into one step if the slope of the demand equation in Step 1 above is multiplied by two):

$$TR = 11q\text{-}0.01Q^2$$

Step 3: Find 1st derivative of Step 2 to calculate MR:

$$MR = 11\text{-}0.02Q$$

Step 4: Set MC = MR and restate in terms of Q:

$$5 = 11\text{-}0.02Q$$

Therefore, Q = 300.

Step 5: Substitute Q into the price function in Step 1 to obtain P:

$$P = 11\text{-}0.01Q$$
$$P = 11\text{-}\ 3$$

Therefore, to maximise profit P = 8.

Alternative method; using P = (Q/m) + MC to determine optimal price when given the equation for a demand function:

Step 1: Using the demand function given above; solve for P (price) when/PEC = 1:

$$P = ((1{,}100\text{-}100P)/100) + 5$$
$$P = 11\text{-}P + 5$$

Therefore, P = 8.

As can be seen this method is much more straightforward than using the MC = MR methods.

CONCLUSIONS

For many years traditional management accounting text books when dealing with optimal pricing decisions have calculated the point of profit maximisation using MC = MR, which while firmly based on fundamental micro economic theory, is an indirect method to use. This method is indirect because it requires a linear function (demand) to be converted into a quadratic function (TR) and then differentiated to obtain another straight line function (MR). This function then needs to be equated to MC before Q can be derived. Q then also needs to be substituted into the demand curve to finally obtain P. Because management accountants will know that MR-MC is the same as Marginal Contribution (MC_o), it can be said that profit is maximised when $MC_o = 0$ and by implication where the Point Elasticity of Contribution (PEC) = 1.

Therefore, using related fundamental principles of micro economics, this study has demonstrated that by applying a variation of the price elasticity of demand concept, the point elasticity equation can be adapted to easily and quickly set the prices for both revenue and profit maximisation in management accounting.

Using this new concept of price elasticity of contribution is a much more useful concept than PED for most businesses which have a mix of fixed and variable costs. For such businesses, they need information to support pricing decisions that maximise contribution rather than revenue, unless their prime objective is to maximise market share. Price elasticity of contribution indicates over what range of prices contribution can be increased and at which point profit is maximised. This is particularly useful for businesses who wish to use discounting to increase profitability rather than just to increase market share, regardless of profitability.

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