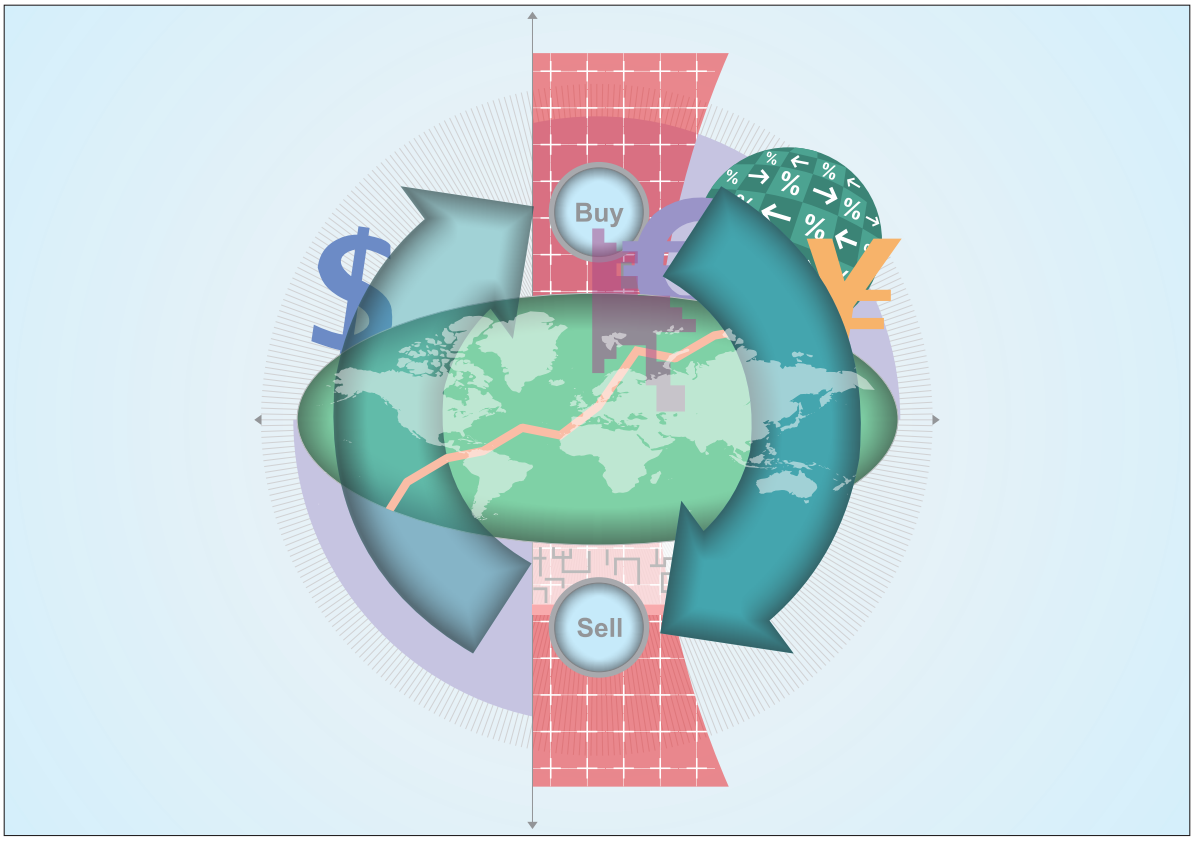

The International Journal of Applied Economics & Finance

ISSN 1991-0886

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OPEN ACCESS The International Journal of Applied Economics and Finance

ISSN 1991-0886
DOI: 10.3923/ijaef.2016.21.36

Case Report
Traditional Investment Tools Using Backtesting Simulations: The Case of Colombian Stock Market for the Period 2007-2013

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Abstract
This study intends to prove the viability of the mean-variance portfolio methodology introduced by Harry Markowitz in 1950 as a traditional concept that modern retail investors could use to improve the performance of their investments, over and above that offered by the average actively managed or index equity fund. Likewise, represents a final dossier with several outputs obtained by the design and running of an optimization model, which was developed by using basic financial concepts. This algorithm which is based on a back testing analysis, simulated four investment strategies by taking into account historical data on the Colombian stock market. From this application, recommended portfolios for each strategy are obtained and ran for the period between 2007 and 2013, which are compared in terms of return and risk to the most representative Colombian stock index (IGBC) behavior. Results achieved indicate that the designed algorithm is effective, yet demonstrate a more superior integral performance than the market. Its main contribution is the model's potential use for supporting actual investment decisions. It is believed there is sufficient evidence to support the use of traditional concepts and financial tools like the mean-variance optimization as a valid, value-adding mechanism for investors.

Key words: Investment portfolios, investment strategies, stock market, back testing analysis, financial markets

Received: May 30, 2016 **Accepted:** June 07, 2016 **Published:** June 15, 2016

Citation: Orlando E. Contreras-Pacheco, Carlos E. Vecino-Arenas and Alejandra Barbosa Calderón, 2016. Traditional investment tools using backtesting simulations: The case of Colombian stock market for the period 2007-2013. Int. J. Applied Econ. Finance, 10: 21-36.

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Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

One of the biggest questions in the financial theory and practice has been how to perfect systems predicting future behaviors in order to make the best presents decisions. However, it has been established that it is from rigorous analysis of historical performance that the authors are able to reach a valid approximation of such predictions.

Managing investment portfolios, the applicability of this method is even more tangible when managing investment portfolios due to the existence of large historical databases enabling retrospective checks and back-testings, which validate or invalidate various decisional paths and revaluate investment strategies from current managers. For that reason, risk assumed and obtained returns are the two quintessential elements of such resolutive models.

Thus, the use of statistical tools becomes essential and appeals to different postulates such as those from Markowitz (1952) and Sharpe (1966) for the design of an optimization model, where the relationship to maximize risk-adjusted return and the input data are accessible to any potential investor by the gathering of historical data (daily closing prices of the shares on the stock market).

A model from the simulated formation of past portfolios is proposed, their behavior can be evaluated in terms of eventual returns obtained and risks assumed, yet compared to an indicator of basis to draw conclusions on the performance of the model according to a purely passive management model (in this case the IGBC).

Ultimately, the model is intended to prove that the application of basic theoretical postulates could have achieved better behaviors than the decision methods that were effectively conducted by administrators who followed the pattern of the market through replication of the structure of stock index concerned. This clearly has the potential to support future studies with a more robust investment model applicable to different scenarios and under various conditions to refine the results to the most appropriate possible level.

Without the possibility of direct comparison by methodological difference and time frame studied, this experiment partly represents, the study of Lizama and Ciudad (2013) who applied a similar model in the Chilean market and received interesting conclusions where (among other results) the previously exposed presumption was validated. In this study, the results in Colombia occur in the same way and suggests their theoretical and practical potential for application in other contexts.

Likewise, it is the continuation of different contributions by Contreras (2014) who started with a more detailed analysis, since this study only looked at a rebalancing strategy in investments and no comparisons were achieved in terms of the risk-return relationship with the selected referents.

The optimization model used for this purpose was written in the programming language R (free and open source code), which processed historic data of the Colombian stock market (December, 2001-2013) and provided a set of weights, which gave rise to different investment strategies that were analyzed separately and compared to the General Index of the Stock Exchange Market of Colombia (hereinafter IGBC), thus demonstrating the effectiveness of the model, leaving doors open for actual implementation.

From the consideration of risk aversion as a fundamental parameter of the behavior of financial managers, according to the contributions of Von Neumann and Morgenstern (1944, 1953), it has been possible to consolidate a body of knowledge that has permitted the use of mathematical statistical tools for the in-depth study of financial phenomena, including the ability to predict future events.

Thus, assuming certain conditions of symmetry, it has been admitted that the arithmetic average return to a historical period provides a good prognosis of the expected return on investment in the same way that the expected risk could be a factor on the volatility and thus the variability, of the results. Thus, the variance and standard deviation provide a measure of the uncertainty of the results, especially when it is assumed that the probability distribution is a normal curve.

Moreover, for purposes of the application of these concepts, the analysis of investment portfolios (not just of an asset) is made by using the calculation of weighted averages that depend on the performance of individual values comprising each of the portfolios analyzed, which subsequently for purposes of risk analysis would be separated into two parts, one that depends on the variance of returns for each asset present in each portfolio and another that depends on all possible correlations, that is their covariance.

For the present study the portfolio selection model of Markowitz (1952) and Sharpe (1966) is used, using the measurement of risk-adjusted performance proposed by Sharpe (1964), complemented by checking the effectiveness of the eventual investment through Jensen (1968) proposal.

The concept of diversification arose long before modern finance theory. It was not until 1952 that Harry Markowitz formally published a portfolio selection model that incorporated the principles of diversification (Due to this contribution, Markowitz won the Nobel prize in economics in 1990. It was the most representative of stock index created by the Colombia stock exchange (BVC), which it is the result of

weighing the most liquid and largest capitalized stocks traded on the exchange (i.e.) those having a higher turnover and frequency. This index was replaced by the new index COLCAP). His model was the 1st step in managing portfolios using the identification of a set of efficient portfolios or the efficient frontier of risky assets. The main idea of this set of risky portfolios is that for any level of risk, the only thing that matters is the portfolio with the highest expected return. Alternatively, the border is the set of portfolios that minimizes the variance for any goal of expected return.

The premise of this model is that the portfolio construction problem can be generalized to the case of many risky assets and risk-free assets.

Thus, its application consists of three parts. First, the risk-return combinations available from a set of risky assets are identified. Immediately after, the optimal portfolio of risky assets is identified by finding the portfolio weights. Finally, a complete portfolio is chosen when mixing risk-free assets with the optimal risky portfolio.

The first step is to determine the risk-return opportunities available to the investor. These are summarized in the minimum variance border of risky assets, which is represented by a graph of the minimal possible variance that can be obtained given the expected return of a portfolio. From the data of expected returns, variance and covariance, the minimum variance portfolio for any level of expected return can be calculated as shown in Fig. 1.

Note that all individual assets are at the lower right side of the border, which indicates that risky asset portfolios with only one asset are inefficient. The diversification of the investments leads to portfolios with a higher expected return and a lower standard deviation.

All portfolios are on the border of minimum variance from the global portfolio of minimum variance upwards, they provide the best risk-return combinations and are therefore candidates for an optimal portfolio. The part of the border that is above the global minimum variance portfolio is called the efficient border of risky assets. For any portfolio in the bottom of the minimum variance border, there is a portfolio with the same standard deviation and a higher return expected value positioned directly over it. Therefore, the lower border part of minimum variance is inefficient.

The second part of the optimization plan incorporates the risk-free asset. For this, the so-called Capital Asset Line (CAL) is sought with the highest risk-return ratio, which is the steepest slope, as shown in Fig. 2.

The Capital Asset Line (CAL) according to the optimal portfolio P is tangent to the efficient border. This CAL dominates all alternatives (in the graphic it is represented with a dotted line), so portfolio P is an optimal risk portfolio.

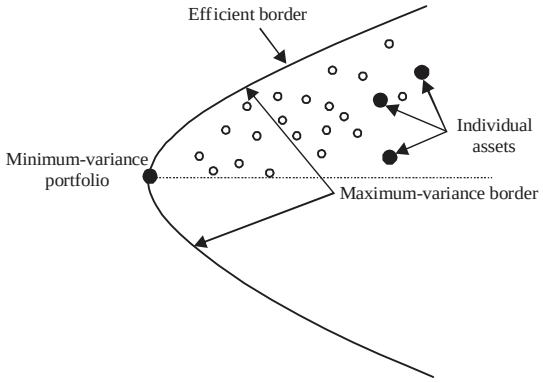


Fig. 1: Minimum-variance frontier of risky assets. Efficient frontier

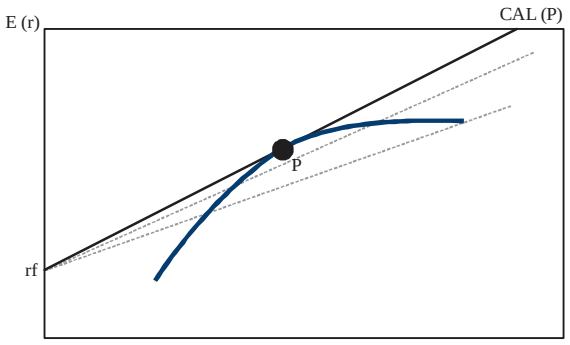


Fig. 2: Efficient border of risky assets with the optimal CAL

Finally, in the last part of the problem, the individual investor chooses an appropriate mix between and optimal risk portfolio P and risk-free assets.

Given the above, it should be considered that Markowitz develops his model based on the following assumptions:

- The performance of any asset or portfolio is described by a subjective random variable whose probability distribution for the reference period is known by the investor
- The risk of an asset or portfolio is measured by the variance (or standard deviation) of the representative random variable performance
- The investor will prefer those financial assets with higher returns for a given risk or a lower risk for a known performance. This decision rule is called rational investor behavior and assumes that investors are risk averse
- There are no market "imperfections" such as taxes and transference costs
- Considers the perfect divisibility of selected assets, which means that they can be purchased in any amount, even fractions of themselves

However, it must recognize that the process of Markowitz presents two problems: The model requires a lot of estimators to complete the covariance matrix and it does not provide guidance for projecting risk premiums of stocks that are essential to build the efficient border of risky assets. This is way it goes to an alternating pattern to simplify the estimation of the covariance matrix and to improve the analysis of risk premium stocks; this is the Single index model.

Evaluating performance based solely on the return obtained is not very useful. The returns must be adjusted by risk so that they can be compared significantly.

The methods of risk-adjusted performance using the mean-variance criteria arose simultaneously with the CAPM. Sharpe (1966), Treynor (1965) and Jensen (1968) all recognized the implications of CAPM in developing a management performance rating. In this sense, the performance of a portfolio is the result of decisions made by the manager, including the choice of the assets. So, evaluating the performance reveals the skills and added value of the manager.

However, despite the growing interest in measuring performance under new types of measurements, among which stand the information ratio of Treynor and Black (1973) and the M^2 of Modigliani and Modigliani (1997) classic measurements from the 60 sec are still widely used in complex processes of analysis, perhaps because of their simplicity and effectiveness.

When choosing a risky portfolio, it can be argued that an investor would presumably be interested in both an expected excess return over the risk-free return as well as the risk thereof in which it incurred. In the same way, it can determine with certainty which rate would be obtained if a "risk-free" instrument is purchased and held to maturity, even under the assumption of nominal changes in emissions of such instruments.

Thus, investors value risky assets so that the risk premium goes along with the risk of that expected excess return and therefore it is best to measure risk by the standard deviation of excess return not over the total.

The importance of this suggests that the attractiveness of an investment portfolio is measured by the ratio of the risk premium to the standard deviation of the excess return so that the Sharpe Ratio (SR) is given as:

$$SR = \frac{R_p - r_f}{\sigma\left(R_p - r_f\right)}$$

Where:

R_p = Return of the portfolio

r_f = Risk free profitability

$\sigma (R_p-r_f)$ = Standard deviation of the excess return of the portfolio

The SR has become an industry standard because it is a simple measure to calculate and can be used to compare different types of investment strategies with relative ease. Despite these benefits, it is important to note that it is not perfect and in relation to its weaknesses and main criticisms the reader can say that:

- It does not take into account whether the volatility is upward or downward, trying all kinds of volatility in the same way. That is, a system whose profits have much deviation from each other will result in a poor SR although its strategy might be better than another more "constant". For an investor looking for potential investment strategies, rising volatility is not necessarily a bad thing, however, SR does not make the difference and thus volatility is penalized in the equation. This potentially could lead investors to believe that the investment is unattractive because the ratio sees positive and negative volatility equally
- A negative SR indicates that the strategy or asset being analyzed has lower performance than the risk-free rate. Keeping constant the remaining variables, the ratio of the portfolio should decrease if the risk increases. Looking at the equation, the authors can see that this is true only when the SR is positive. Therefore, with a negative SR, increasing the risk results in a higher ratio. It is therefore concluded that the SR should not be used as a measure to compare portfolios that have a negative value in its calculation
- The SR considers only the standard deviation as a measure of risk. This can be problematic when it is calculated for assets with asymmetric returns because the standard deviation is appropriate as a measure of risk strategies with an approximately symmetric distribution of returns

The analyses found in the literature on the historical and retrospective performance of the Colombia stock market, especially for what it has to do with the study of the IGBC are mainly limited to statistical diagnostics and in some cases demonstrate its non-linearity in mean (Ospina and Caicedo, 2008; Alonso and Garcia, 2009) validate the existence of certain behavioral patterns considering the leverage effect and the effect of the week day and the time effect, among others.

Despite this, according to Rivera's (Rivera, 2009) study that concludes particular days impact the change in share prices, the results show that stock markets in Colombia have not been efficient in information, which means share prices do not reflect their true value and therefore the market is highly exposed to speculative movements.

More recently, Fernández (2010) studied the dynamics of the IGBC and its volatility. The results obtained suggest that the EGARCH model can better capture the stylized facts of the behavior of the Colombian market than the GARCH model. Similarly, the importance of asymmetric models to estimate the volatility on time series was evident.

Some attempts to estimate future behavior are also perceived. Some of them base their analysis on models such as ARCH or the empirical model of continuous time that according to Grajales and Pérez (2008) are reasonably adequate to estimate the volatility of the index in question, with the potential addition of determining the temporal assessment of the distribution and subsequent volatility in the valuation of derivatives on these indexes.

There is also sophistication in the analysis with Londoño *et al.* (2010) use of artificial neural networks where, in addition to the effectiveness of the method over traditional statistical models, also demonstrate the effectiveness of the conventional APT model (based on macroeconomic fundamental data) to predict the future behavior of the Colombian stock market.

Meanwhile, Alonso and Torres (2014) continued the study of Alonso and Arcos (2006) and through the use of a wide battery of statistical tests were able to document the presence of five stylized facts on the behavior of yields of the IGBC in its first 10 years (July 2001-2011). These are: (1) No soft market efficiency is presented, (2) Heavy distribution queues of returns, (3) Normality added of the distribution of returns, (4) Non-constant and grouped volatility in yields and (5) The Taylor effect.

Either way, some important contributions are worth mentioning that are related to exploratory studies of the basic concepts of portfolio theory. Dubova (2005) and Velez (2007) basically found that it is not possible to establish a relationship between the risk premium and expected returns due to inefficient information regarding the Colombian stock market. These results are compatible with the conclusions of Duarte *et al.* (2013) who additionally demonstrated the inexistence of additional risk premium due to the size of companies, rejecting both the size effect and the inverted effect in the Colombian stock market.

However, beyond having an extensive literature that studies and analyzing the market in its entirety, it is not evident in literature specific studies that simulate the behavior of any real portfolios in real conditions through retroactive methodologies, so that the proposed analysis in this research is unprecedented and capable of further development in depth in order to become proactive to the actual investment decisions by the managers.

Although for the purposes of this study, the environment variables that partly justify some behaviors are not taken into account, it is almost inevitable to come to a short halt in the specific situation in the period considered, despite that some of them match their pressure on other similar phenomena in regional markets and around the world.

First, it is necessary to mention that along with internal situations of diplomatic nature with neighboring countries, the so-called crisis of sub-prime mortgages in the United States during the years 2007, 2008 and part of 2009 were regressive in their direct effect on the Colombian stock market, implying a strong stock devaluation as a result of high levels of risk present in the market. There was also another downward pressure (this time not as steep) during 2011 from the debt crisis in the European Union due to monetary cohesion.

By the end of 2012, Colombia experienced one of the most known cases of stock market embezzlement in the country's financial history of this country: The so-called "collapse" of INTERBOLSA, which until then had been consolidated as the largest brokerage firm in Colombia. This occurred due to the systematic speculation upon FABRICATO's share, a textile company, since 2009 and forced the official control corporation to intervene and subsequent liquidation of the brokerage, with negative results thereafter on the stock considered (DINERO., 2012).

In addition, 2013 was also not a good year for Colombia stock market with the explosion of INTERBOLSA's scandal, which cause the economy to experience the volatile rhythms from the monetary policy of the United States and additionally due to a sharp drop (correction, as mentioned by specialists) in ECOPETROL's stock, which makes up a significant part of the stock index.

MATERIALS AND METHODS

Back-testings of different investment strategies based on building optimal portfolios through maximizing the sharpe ratio were performed using an algorithm as the optimal solution, which generates a set of optimal weight investment, maximizing the return relationship expected adjusted to risk.

As mentioned above, this was used in this study the statistical model of asset valuation, Single-Index Model (SIM), because it specifies that all common movements between stocks can be captured by a single index, here the IGBC.

Besides being a simpler model than the portfolio selection model of Markowitz, it reduces the number of data required for use. The model incorporates the α parameter, allowing it to have the advantage of reflecting the additional risk premium attributable to private information.

Selecting assets to build portfolios: For the development of the optimization model mentioned above, it is necessary to consider some defined criteria to get a reliable and properly structured data base: (a) The information used to optimize the portfolio considers a representative index (Benchmark) of the Colombian equity market, 32 stocks belonging to the IGBC valid for 2013 and (b) The sample used for optimization is daily frequency and closing prices are often used (adjusted to reflect splits, exchanges or consolidations of stocks, dividend/incentive in stocks and offers of right), obtained through the Bloomberg International information terminal (Historic formula to download to Excel a stock's data = BDH ("Ticker stock", "PX_LAST", "Start date", "End date"), through which conventional daily returns are obtained.

The optimization algorithm only considers stocks that have at least five years of historical data, thus those with data N/A are removed. In turn, from the stocks of the IGBC those who have not presented any change in price in at least one month (monthly return = 0%) are excluded with the primary goal of not considering stocks that could present liquidity problems due to lack of transactions.

In seeking to simplify this study, in addition to the Markowitz model assumptions, it is considered that: (a) All investors can access the same risk-free rate, (b) Information is free, instant and available to all investors, (c) Short selling is not permitted and (d) Only buying positions are considered for selected shares by the optimization model designed.

And as previously stated, the particular behavior of the phenomena of the environment in terms of critical points is not taken into account (heydays and/or economic crisis) of the values because, in addition to the conventionality of these circumstances and the considerable size of the time frame analyzed, the comparison with the IGBC also suggests that the exposure of all references is the same and therefore, the conclusions of the exercise are not affected by these situations.

RESULTS

The results of the application of the portfolio optimization model by maximizing sharpe's ratio are now presented and analyzed, hereinafter regardless from the optimization model or algorithm.

Investment strategies and rebalancing criteria: In this study the researchers propose and analyze 4 equity investment strategies that share an initial analysis window of 5 years, a rolling window of 5 years and an investment horizon of 7 years. They differ, however, on the periodicity of their respective rebalancing and/or changes in the composition of the portfolio, thereby obtaining 7, 14, 28 and 84 for consecutive annual, biannual, quarterly and monthly rebalancing for their respective reinvestment strategies.

For the purposes of the simulation, a specific amount of money is "Invested" (beginning 31/12/2006) in an optimal portfolio, whose stocks and investment weighting per share is determined by optimizing the SR applied to Historic stock returns in a previous period of 5 years (2002-2006). This investment is maintained during a period of 1 year, semester, quarter or month (depending on the rebalancing strategy applied) after which it was again reinvested for the same period, the amount of investment, plus the profit in a new portfolio that is determined through optimization of SR applied to historical stock returns coming from a new time frame of 5 years. For its constitution, you can buy, sell and/or maintain stocks and investment weighting of the previous portfolio.

Thus, for additional portfolios created for each period in each investment strategy the authors obtain (both for the respective period as annualized way) returns achieved, the calculation of the standard deviation (σ) and the data yielded by the Sharpe's Ratio (SR). These data are compared with their actual behavior equivalent of the IGBC, later used to find Jensen's alpha (α) and statistical significance from the corresponding t-statistic for each respective rebalancing frequency.

Regarding this analysis, it is necessary to mention that according to Orito *et al.* (2010) cases of outstanding operational behavior of ETFs (also called passive management funds), as compared with other types of funds are widely discussed in the literature. Experiments by Malkiel (1995), Gruber (1996) and Elton *et al.* (1996) and more recently Phillips *et al.* (2014) support and validate this statement.

The IGBC constitutes the quintessential Colombian stock market index, created by the Colombian stock exchange (hereinafter BCV), which consists of a weighted average of the most representative stocks in terms of their liquidity and capitalization. Thus their behavior is generally, a direct radiography of the stock market behavior in that country.

Precisely in this regard, although it does not affect the essence of the study, it is interesting to see an obvious poor IGBC performance experienced during the simulated period

(2007-2013). This achievement done in a span of seven years with a total return of only 17% is quite daunting to say the least. However, as it was clarified above, the justifications for this behavior are in response to specific situations and make us think that independently of these, the validity of the exercise performed is not put in risk.

Exercise: After running the simulation in R language (it is characterized for being free and open source) and in general terms, it was remarkable.

The algorithm for the 7 years of investment, a total of 32 common stocks gave optimal recommendations for 9 stocks for the annual investment strategy, 11 stocks for the biannual investment strategy, 11 stocks for the quarterly investment strategy and finally 10 stocks for the monthly investment strategy.

It is noted that with the exception of GRUPOSUR in the annual investment strategy, the algorithm did not recommend investing in stocks that were not part of the biannual, monthly and quarterly investment strategies.

Of all the recommendations thrown by the model, there were only 7 shares that were included in at least one of the portfolios created for all rebalancing strategies: BCOLO, CELSIA, CORFICOL, ECOPETL, EXITO, MINEROS, NUTRESA and PROMIG.

This leads us to the realization that including the four investment strategies only 12 stocks in total were recommended by the model created (AVAL, BCOLO, BOGOTA, CELSIA, CORFICOL, ECOPETL EXITO, GRUPOSUR, MINEROS, NUTRESA, PROMIG and TABLEMA) responding to the behavior described in Table 1.

The stocks belong to only four economic sectors: Five shares for the industrial sector, three shares of each one of the investment and financial sectors and only 1 share belonging to the commercial sector (Table 2).

Despite the recent apparent attractiveness of shares as EEB, DAVVND and CLH the optimization model excluded them of the respective recommendations, largely because of their "Youth" in the Colombian stock market, as they began their formal price between 2009 and 2012, which precluded a deeper retroactive analysis by the algorithm created and thus their "Future reference".

Special mention is due for ECOPETL, whose stock debuted in November, 2007 and in spite of this from the beginning it had a high trading volume and therefore it was able to be taken into account by the portfolios optimization model proposed in the final periods of the four developed rebalancing strategies.

Table 1: Mentioning recommended stocks for each investment strategy regardless of weight or frequency of recommendation

Annual strategy	Biannual strategy	Quarterly strategy	Monthly strategy
	AVAL	AVAL	AVAL
BCOLO	BCOLO	BCOLO	BCOLO
	BOGOTA	BOGOTA	
CELSIA	CELSIA	CELSIA	CELSIA
CORFICOL	CORFICOL	CORFICOL	CORFICOL
ECOPETL	ECOPETL	ECOPETL	ECOPETL
ÉXITO	ÉXITO	EXITO	ÉXITO
GRUPOSUR			
MINEROS	MINEROS	MINEROS	MINEROS
NUTRESA	NUTRESA	NUTRESA	NUTRESA
PROMIG	PROMIG	PROMIG	PROMIG
	TABLEMA	TABLEMA	TABLEMA

Table 2: Names of companies with their corresponding economic sectors

Stock	Name	Sectors
AVAL	Grupo AVAL Acciones y Valores S.A.	Investments
BCOLO	BANCOLOMBIA S.A.	Financial
BOGOTA	Banco de BOGOTÁ S.A.	Financial
CELSIA	CELSIA S.A. E.S.P., previously COLINVERS	Industrial
CORFICOL	Corporación financiera Colombiana CORFICOLOMBIANA	Financial
ECOPETL	ECOPETROL S.A.	Industrial
EXITO	Almacenes ÉXITO S.A.	Commercial
GRUPOSUR	Grupo de Inversiones SURAMERICANA S.A.	Investments
MINEROS	MINEROS S.A.	Industrial
NUTRESA	Grupo NUTRESA S.A.	Investment
PROMIG	PROMIGAS S.A. E.S.P.	Industrial
TABLEMA	TABLEMAC S.A.	Industrial

Source: Website Colombia stock exchange (www.bvc.com.co)

The algorithm recommended portfolios of at least two shares for every biannual rebalancing strategy developed, but not for the annual, quarterly and monthly portfolio strategies where 1 share was recommended at 1, 3 and 13 times, respectively. No portfolio recommendation superior to 3 shares in the annual rebalancing strategy was presented. The maximum number of shares recommended by portfolio was 4 for the other three strategies. Accordingly, it is concluded that the suggested portfolios were not compatible to the basic financial principle of diversification, which states that an investor should not put all his resources into a single asset or a relatively small number of assets, but into a large number of investment instruments, helping to cushion the variability of returns of individual assets. This phenomenon is attributed to the slight deepening of the Colombian stock market and, therefore, the low visibility of some stocks that have not historically consolidated its attractiveness in terms of profitability and/or risk, thus the model did not consider them in its recommendations. The composition of the recommended portfolios of the different strategies applied responds to Table 3 and 4 and Fig. 3-5, respectively (In the area of information, the table format is used to present the composition of the 84 monthly portfolios created by the algorithm in his last run strategy).

Table 3: Monthly portfolio 2007-2009 (Monthly rebalancing strategy)

Years	Months	BCOLO (%)	NUTRESA (%)	CELSIA (%)	PROMIG (%)	AVAL (%)	CORFICOL (%)
2007	M1	45.03	46.76	8.22			
	M2	6.86	24.71	68.44			
	M3		31.45	68.55			
	M4		28.28	71.72			
	M5		30.37	69.63			
	M6			85.73	14.27		
	M7			81.95	18.05		
	M8		31.90	68.10			
	M9		28.90	71.10			
	M10	6.11	18.97	65.11	9.81		
	M11	19.42	14.40	52.15	14.04		
	M12		10.46	66.58	22.96		
2008	M1		29.05	48.14	22.81		
	M2		36.44	25.15	38.41		
	M3		24.70	39.68	35.62		
	M4	3.72	27.83	33.15	35.30		
	M5	3.89	40.03	15.97	40.11		
	M6		54.59		45.41		
	M7		50.07	0.85	49.08		
	M8		46.52	2.49	50.99		
	M9		42.86	11.41	45.74		
	M10		45.68	7.62	46.70		
	M11		30.75	9.63	59.62		
	M12		16.60	20.26	63.15		
2009	M1		17.13	19.06	63.81		
	M2		21.69	7.30	71.01		
	M3		8.71	11.44	79.86		
	M4			19.23	80.77		
	M5			24.19	75.81		
	M6	27.26		6.04	66.70		
	M7	10.44		20.88	63.59	5.08	
	M8			32.36	62.92	4.72	
	M9			28.47	62.54	8.99	
	M10			37.88	57.53	4.58	
	M11			47.99	51.33	0.69	
	M12						100.00

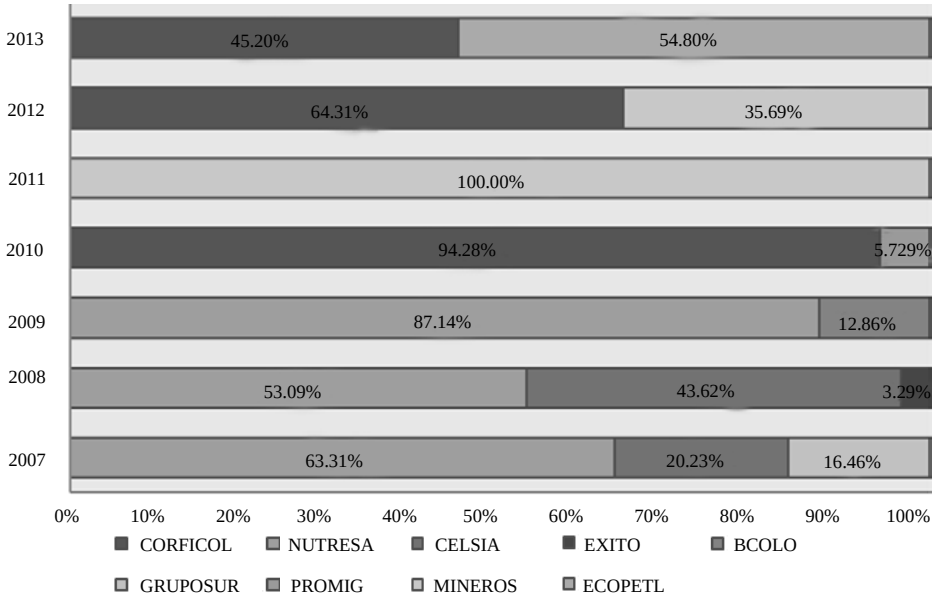


Fig. 3: Seven annual portfolios (annual rebalancing strategy)

Table 4: Monthly portfolio 2010-2013 (Monthly rebalancing strategy)

Years	Months	PROMIG (%)	ÉXITO (%)	AVAL (%)	CORFICOL (%)	MINEROS (%)	TABLEMA (%)	ECOPETL (%)
2010	M1				100.00			
	M2	3. 45			96.55			
	M3	4.31			95.69			
	M4	3.87			96.13			
	M5				53.77	46.23		
	M6				44.12	55.88		
	M7				39.76	60.24		
	M8				27.37	72.63		
	M9				15.49	84.51		
	M10				8.59	91.41		
	M11					100.00		
	M12					100.00		
2011	M1				9.05	90.95		
	M2				5.48	94.52		
	M3				14.76	85.24		
	M4				20.84	79.16		
	M5				39.43	60.57		
	M6				74.67	25.33		
	M7				72.59	26.44	0.97	
	M8				58.31	40.91	0.78	
	M9				47.26	52.74		
	M10				58.85	41.15		
	M11				48.87	51.13		
	M12				41.55	58.45		
2012	M1			16.91	83.09			
	M2				68.81	31.19		
	M3				65.44	34.56		
	M4			14.12	69.61	16.27		
	M5				81.30	18.70		
	M6				87.35	12.65		
	M7				69.94	30.06		
	M8				80.68	19.32		
	M9				77.64	22.36		
	M10				100.00			
	M11				98.95	1.05		
	M12				99.88	0.12		
2013	M1				47.09			52.91
	M2				57.69			42.31
	M3				87.94			12.06
	M4				100.00			
	M5				100.00			
	M6				100.00			
	M7				100.00			
	M8				100.00			
	M9				100.00			
	M10				100.00			
	M11				100.00			
	M12		21.61		78.39			

The value of the portfolio in Fig. 6-9 can appreciate the simulated behavior of an investment for each one of the respective rebalancing strategies established during the 2007-2013 period, assuming an investment of $1 on 31 December, 2006.

For the annual rebalancing strategy, a final portfolio worth 43.60% above the initial portfolio value is obtained. In simple terms, this means that if on 30 December, 2006 $1 was invested for a period of with years with a rebalancing every year according to the cast model, it would have received $1.44 on 30 December, 2013.

For the biannual rebalancing strategy, a final portfolio value of a 89.60% above the initial portfolio value is obtained. In simple terms, this means that if on 30 December, 2006, $1 had been invested for a period of 7 years with a rebalancing every 6 months according to the model cast, it would have received $1.90 on 30 December, 2013.

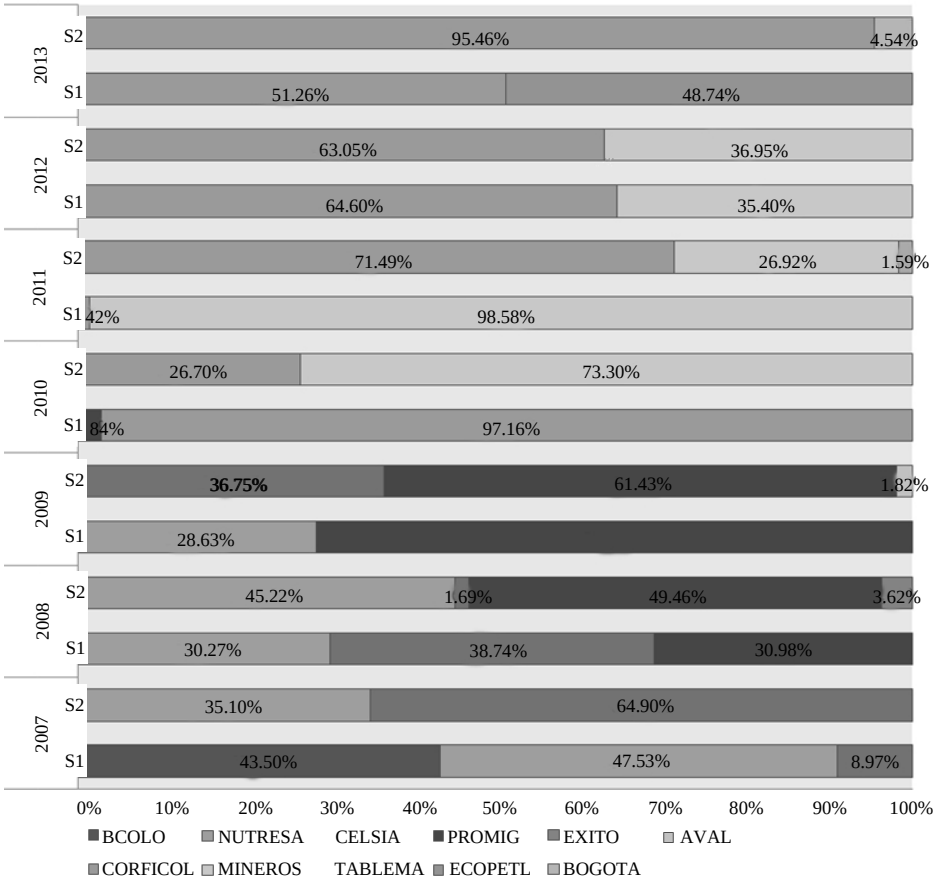


Fig. 4: Fourteen biannual portfolios (biannual rebalancing strategy)

For the quarterly rebalancing strategy, a final portfolio value of a 104.77% above the initial portfolio value is obtained. In simple terms, this means that if on 30 December, 2006, $1 had been invested for a period of 7 years with a rebalancing every three months according to the cast model, it would have received $2.05 on 30 December, 2013.

Finally, for the monthly rebalancing strategy, a 96.59% final portfolio value above the initial portfolio value is obtained. In simple terms, this means that if on 30 December, 2006, $1 had been invested for a period of 7 years with a rebalancing each month according to the cast model, it would have received $1.97 on 30 December, 2013.

Compared to the IGBC, it can be clearly seen in the above figures that the performance of portfolios proposed for the 4 strategies is substantially better than the one cast by the ratio mentioned, since from its registered value on 30 December, 2006 (COP $11,161.14), the IGBC rose to a value of COP $13,071.27 on 30 December, 2013. This means that the actual recovery was 17.11% during the years 2007-2013 or in simpler terms, if $1 had been invested for a period of 7 years in a hypothetical asset called IGBC (with the actual behavior of the index in question, ETF type (Due to the extension on the information, it was decided to graphic without punctual data)) by a strategy in which one buys from the start and hold without addition or subtraction to the end of the period (Buy-and-hold) it would have received $1.17 at the end of the day on 30 December, 2013.

Thus, according to the comparison in which better behavior is clearly observed by the portfolios simulated against the behavior of the IGBC, under the assumed conditions the application of the optimization model could have become an effective tool for active portfolio management during the periods analyzed. And according to the results obtained, it might be useful for the eventual use in determining the future constitution of investment portfolios in the Colombian stock market.

It is now necessary to delve into a more comprehensive analysis of the performance of portfolios since a frequent mistake when deciding on the best investment option is concentrating too much on profitability and not on the levels of risk. For this task it is very important to consider the relationship between the risk it is willing to take compared to

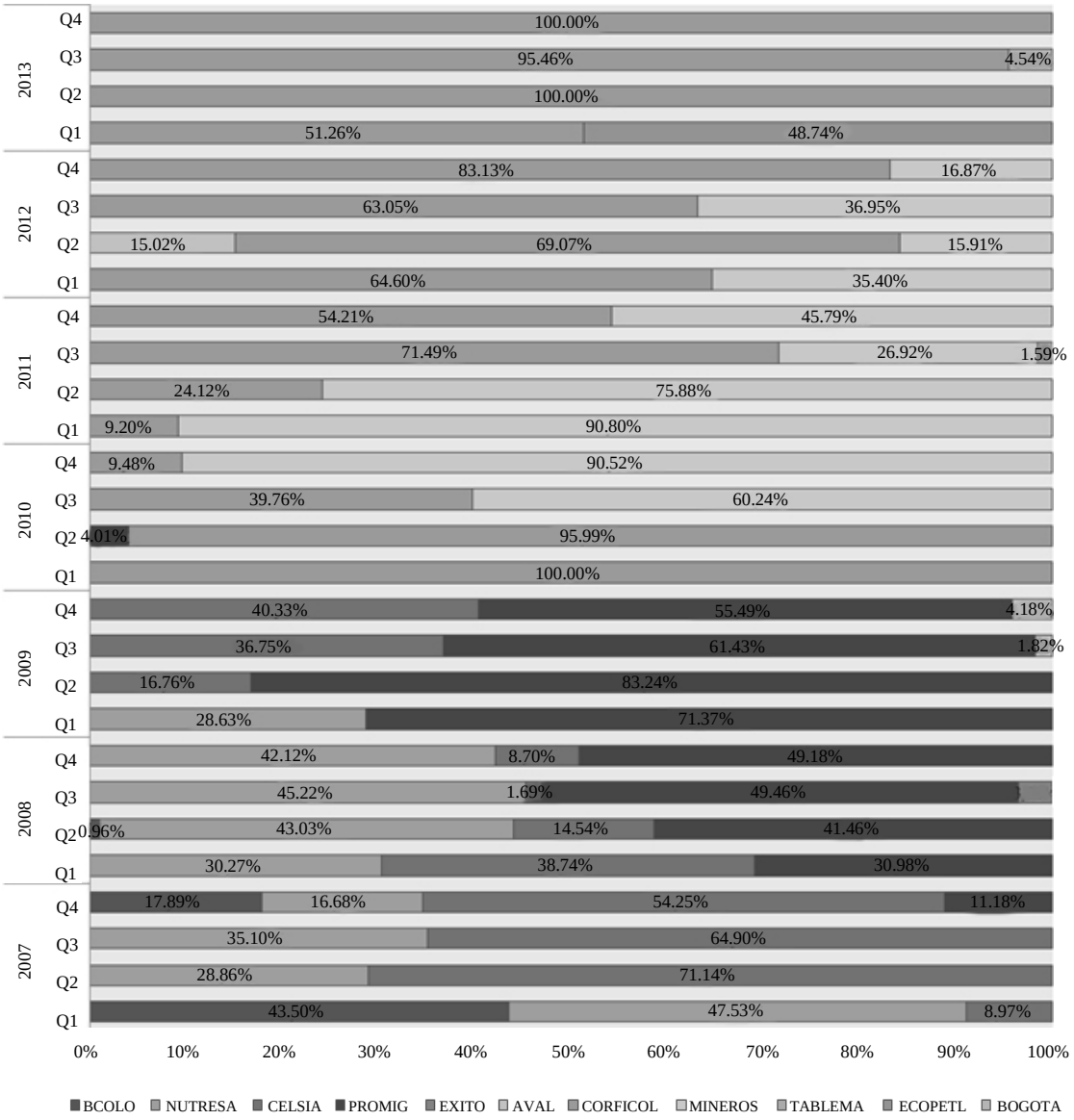
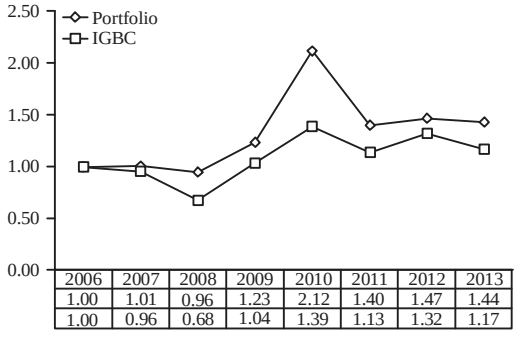


Fig. 5: Twenty eight quarterly portfolios (quarterly rebalancing strategy)



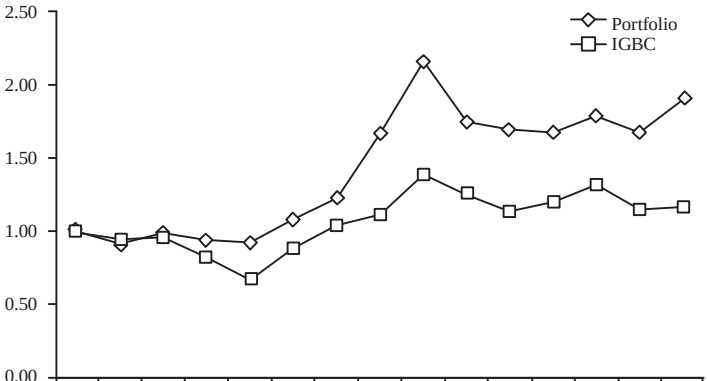
2006	2007	2008	2009	2010	2011	2012	2013
1.00	1.01	0.96	1.23	2.12	1.40	1.47	1.44
1.00	0.96	0.68	1.04	1.39	1.13	1.32	1.17

Fig. 6: Recommended value of portfolio by annual rebalancing strategy vs., indexed fund to the IGBC

the return of the investment for which, as it was mentioned earlier, the SR that shows the point at which the investor is willing to take risk to get a greater return on their investment is used.

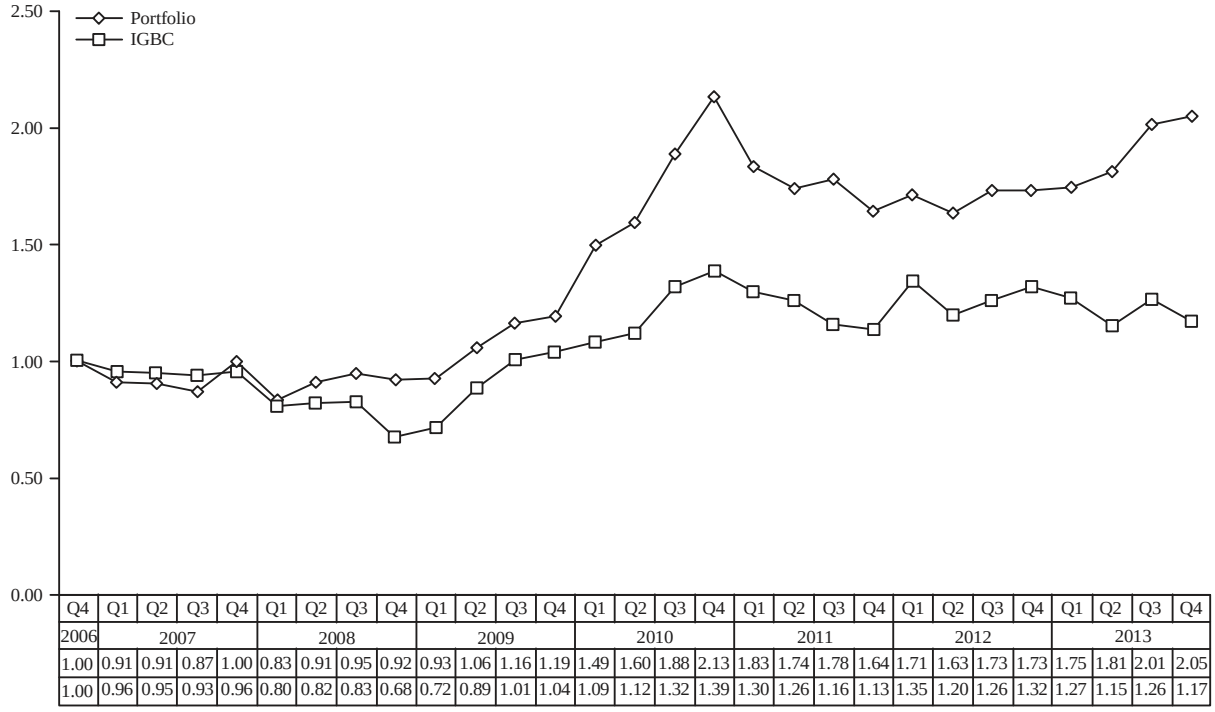
For this last comparison, the profitability results and risks are obtained (by period and annualized) by finding the returns and standard deviations (σ), respectively. The SR is also calculated for each of the periods processed in each rebalancing strategy, also finding their annualized value.

They are then are confronted with the data obtained from the application of the algorithm using the returns actually yielded by the behavior of the IGBC index under the particular



S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2
2006	2007		2008		2009		2010		2011		2012		2013	
1.00	0.92	0.99	0.94	0.92	1.08	1.23	1.66	2.16	1.75	1.69	1.68	1.78	1.68	1.99
1.00	0.95	0.96	0.82	0.68	0.89	1.04	1.12	1.39	1.26	1.13	1.20	1.32	1.15	1.17

Fig. 7: Recommended value of portfolio by biannual rebalancing strategy vs., indexed fund to the IGBC



Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
2006	2007				2008				2009				2010				2011				2012				2013			
1.00	0.91	0.91	0.87	1.00	0.83	0.91	0.95	0.92	0.93	1.06	1.16	1.19	1.49	1.60	1.88	2.13	1.83	1.74	1.78	1.64	1.71	1.63	1.73	1.73	1.75	1.81	2.01	2.05
1.00	0.96	0.95	0.93	0.96	0.80	0.82	0.83	0.68	0.72	0.89	1.01	1.04	1.09	1.12	1.32	1.39	1.30	1.26	1.16	1.13	1.35	1.20	1.26	1.32	1.27	1.15	1.26	1.17

Fig. 8: Recommended value of portfolio by quarterly rebalancing strategy vs., indexed fund to the IGBC

conditions of each of the proposed rebalancing strategies. Unlike the previously mentioned IGBC, where a buy and hold strategy is assumed, it is now done by considering each one of the reinvestment frequencies raised so that the data obtained in each periods is accumulated and undergoes the same return of the index considered in the period obtained as if the index fund was also put through reinvestments equivalent to those made by the chosen reinvestment strategies (An ETF (Exchange traded fund, for its acronym in English) is a type of index fund that replicates an index and it is negotiated during the day like a stock. So in this way, an investor could "buy" the IGBC and replicate it). This is the justification for why the data belonging to the IGBC column are different than each other (Table 5). Thus, the annualized data is not the same for all analyzed strategies.

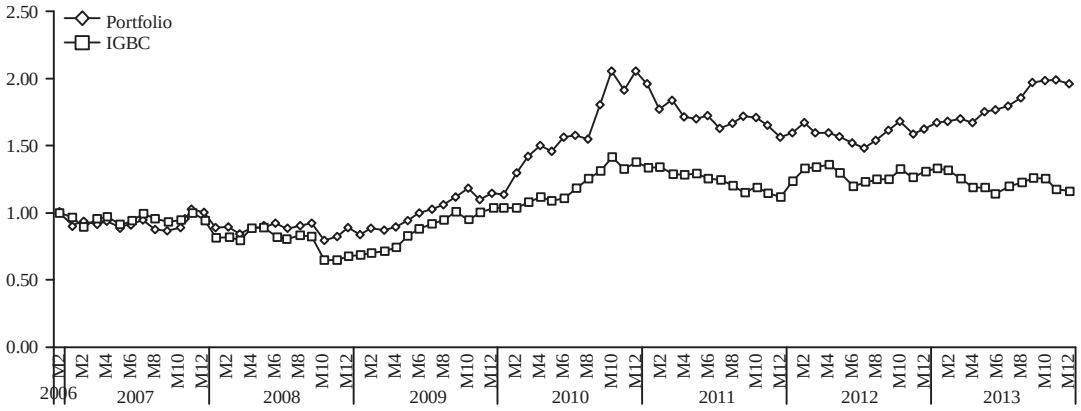


Fig. 9: Recommended value of portfolio by monthly rebalancing strategy vs., indexed fund to the IGBC

Table 5: Results compared on 31/12/2013

	IGBC (Buy and hold)	IGBC (With rebalancing)	Portfolio
Annual F.	$1.17	$1.40	$1.44
Biannual F.	$1.17	$1.29	$1.90
Quarterly F.	$1.17	$1.28	$2.05
Monthly F.	$1.17	$1.27	$1.97

Hypothetical initial investment: $1.00

Table 6: Consolidated results algorithm application (Risk-return)

	Period 2006/12/31-2013/12/31							
	Annual rate		Biannual frequency		Quarterly frequency		Monthly frequency	
	Portfolio (%)	IGBC (%)	Portfolio (%)	IGBC (%)	Portfolio (%)	IGBC (%)	Portfolio (%)	IGBC (%)
Return period	11.55	8.62	5.72	2.16	2.86	1.04	1.01	0.35
Annualized return	11.55	8.62	11.76	4.36	11.95	4.22	12.82	4.24
σ period	35.74	30.50	14.96	14.73	9.21	9.60	5.41	5.53
Annualized σ	35.74	30.50	21.15	20.83	18.43	19.19	18.74	19.16
SR period	0.32	0.28	0.38	0.15	0.31	0.11	0.19	0.06
Annualized SR	0.32	0.28	0.56	0.21	0.65	0.22	0.68	0.22
α	3.04		4.19		2.23		0.77	
t-statistic	0.33		1.39E+13		1.61323E+14		1.83041E+14	

On the other hand, it is reasonable to put the whole analysis in context in order to visualize returns, volatility and portfolio performance, as well as checking how the performance was "In excess" above the market, that is the reason why it was also indicated the corresponding and the t-statistic for each strategy, the statistical significance of their calculation is validated (this usually occurs at 1.96 higher values are obtained) (Table 6).

Thus, once again, the effectiveness of the designed algorithm for the proposed circumstances is validated, to obtain in general better results both in profitability and risk for the 4 applied strategies. Perhaps, it would be important to have a discussion regarding the data of the for annual and biannual rebalancing frequencies that subtly show themselves above for the first case and with a difference a little more pronounced in the second, but that does not generate much influence for purposes of its comprehensive holistic exam from their relationship with the return obtained.

That is why when assessing other values (such as SR (σ)) the difference is much more tangible as their values are decisively more attractive in the 4 stages. Especially in the case of the SR of the respective portfolios, which compared with their counterparts for the behavior of IGBC, you can really see a better performance of the recommendations thrown by the optimization model used.

As a punctual result, a sort of ranking of the SR is obtained, which places the monthly strategy (0.68) in the 1st place, what allows us to conclude that this strategy

achieves the best return compared directly to the amount of risk, it has been assumed in the investment. Assessed against the quarterly strategy (0.65) located on the 2nd place, where it can be seen that despite a relatively similar level of risk, it provides a lower average return.

It follows in its order, semiannual rebalancing strategy (0.56), which does not separate much from the previous value and that it continues for an interesting way of investment effectiveness and finally the annual rebalancing strategy that even if it create a bigger gap compared to previous strategies and its risk is in appearance superior, its SR (0.32) still remains higher than the equivalent value for the behavior of the IGBC, allowing it to remain an attractive option and even valid.

In the case of the evaluation of return in excess compared to the expected return of the portfolio, the researchers turn to the results of (σ), which also suggests a very attractive performance of investment strategies under the influence of the algorithm designed. Positive and important results for this indicator can be seen in the biannual rebalancing strategies (4.19%), quarterly (2.23%) and monthly (0.77%), with the exception of the annual frequency, since in spite of its value (3.04%) high in appearance, it is not possible to admit it as evaluation parameter due to the low result thrown by the corresponding t-statistic.

DISCUSSION

All the proposed investment strategies show a constant superiority in terms of value of portfolio reaching a final value for 7 years of investment superior to the market indexes, represented in the actual values of IGBC during that same time window. This is coincident with previous studyies of Contreras *et al*. (2014, 2015) who evaluated similar decisions in the same context, but with a more limited scope in terms of time window and number of strategies evaluated.

For purposes of annual analysis, the same strategies yield figures above the simulated returns this same values in all rebalancing conditions evaluated and they demonstrate a risk assumed (though it is not always lower) that is close to their equivalent simulated values taking as reference the same behavior of the IGBC.

In terms of performance measured by the sharpe's ratio (annualized total risk-adjusted returns), all strategies showed outstanding values against their reference market, which was complemented by obtaining excess returns above that expected by the market (Alpha Jensen) also for the 4 scenarios analyzed, with the exception of the of the annual rebalancing strategy that does not obtain the statistical validity necessary to consider valid the data obtained.

If a decision has to be made on which strategies would have been the most appropriate for the conditions imposed, determination would focus between quarterly and monthly strategies, as they are the ones that shed major SR annualized data, with 0.65 and 0.68 and more data, α with 2.23 and 0.77%, respectively. This means that if any of the options had generated appropriate results to a possible investor, the center of their decision would be represented in their perception to risk taken, although the former had been substantially more profitable in annualized terms with respect to the second (12.82 vs., 11.95%), the quarterly investment strategy would have been less risky than investing monthly (with an annualized of 18.43 vs., 18.74%).

The relevance of the results obtained and the existence of direct antecedents as is the study of Lizama and Ciudad (2013) for the Chilean case, it is clear the development potential of deeper research where analyzes of developing Latin American economies are parameterized and integrated, starting with the countries of the Latin American Integrated Market (MILA): Chile, Peru and Colombia and then to analyze more in depth the other countries of this region, such as Mexico and some member nations of MERCOSUR.

The alternatives to expand the scope of this study are diverse, as the model can be malleable according to different contexts, time windows and comparisons. This could consolidate also as a versatile instrument that will eventually support managers in their future investment decisions.

Like any optimization algorithm or model focusing on maximizing the sharpe ratio, it has certain limitations, such as assuming the absence of transaction costs and taxes. However, the impact of incorporating them into investment strategies does not greatly affect the results (particularly for considerable amounts of investment) and the aforementioned conclusions as comparison parameters would be acting under the same conditions and in one way or another the effect on the analysis is mutually compensated. However, further analysis could consider this variable using real data or calculating maximum allowable values that support or invalidate the present results.

A contribution that would have been important in the present study is to contrast the performance, risk and performance metric adjusted by risk model optimization strategies versus the major mutual funds (now called collective portfolios in Colombia) of equity of the most prestigious managers in that country. However, to perform

this historical annual returns of all funds with at least 12 years of existence in the market are necessary, which is extremely difficult because of the constant change in Colombian market regulations preventing direct continuity of investment policies on them and that have even changed their denomination over time.

Nor was it possible to tackle the issue of having low levels of diversification of the simulated portfolios. It is believed that this can be overcome by adding a simple restriction on the size of resulting portfolio weights to the optimization problem. Therefore, this can represent an open door for future research studies in this same line.

CONCLUSION

This study intends to prove the viability of the mean-variance portfolio methodology introduced by Harry Markowitz in 1950 as a traditional concept that modern retail investors could use to improve the performance of their investments, over and above that offered by the average actively managed or index equity fund. In order to do so, the performance of portfolios formed by allocating capital to the stocks listed in the Colombian stock market is simulated, while varying the frequency at which these portfolios are rebalanced, simulating an investment made continuously over 7 years. Since the mean-variance optimized portfolios tend to outperform the IGBC, the authors conclude that, in terms of risk-adjusted performance, mean-variance optimized portfolios offer an attractive alternative in terms of their performance in every scenario.

In consequence, it is believed there is sufficient evidence to support the use of traditional concepts and financial tools like the mean-variance optimization as a valid, value-adding mechanism for investors. This is especially true, for those who are unable to do exhaustive security and market analysis because they are not professionally trained and/or lack the access to some of the proprietary data and models used by professional money managers.

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