Pakistan Journal of Information and Technology 1(2): 193-201, 2002

Design of Prestressed Concrete Girder Bridges Using Optimization Techniques

[1]Samer Barakat, [2]Ali Salem Al Harthy and [3]Aouf R. Thamer
[1]Department of Civil Engineering, University of Sharjah, UAE
[2]Department of Civil Engineering, Sultan Qaboos University, Muscat, Oman
[3]Department Civil Engineering, University of Science and Technology, Jordan - Irbid, Jordan

Abstract: A comprehensive study for deterministic design of simply supported prestressed concrete girder bridges is presented. Using the feasible direction method, a set of optimal geometrical dimensions, girders spacing, amount of prestressing steel, prestressing loses and tendon profile are obtained. The constraints that have been used are based upon flexural stresses at initial and final stages, crack width, initial camber, deflection due to both dead and live loads, total loses, ultimate moment capacity with respect to the factored loads and cracking moment and the ultimate shear strength. This aims at finding the optimum design of prestressed concrete girder bridge system and finding the most economical tendon profile out of three profiles, namely parabolic, L/3 harped, and triangular tendon profiles. The problem is formulated in general form so that introduction of specific regulations following from national codes is possible. The computer program for optimal design of prestressed girder bridges is developed. The numerical examples are computed taking into account the rules of AASHTO or ACI design codes. Numerical examples of prestressed concrete girder bridges with standard and general sections are solved to show the efficiency of the above formulation. The results indicate that the parabolic tendon is the optimum one, especially for long spans. Also, the cost is slightly nonlinear function of span length. Therefore, it can be reasonably approximated as a linear relationship. There are more savings on the cost when the section is allowed to crack. Stress at jacking end controlled the optimum design for long spans, while the shear strength requirement controlled the design for short spans.

Key Words: Deterministic Design, Prestressed Concrete, Girder Bridges, Optimal Design

Introduction

The idea of optimization is becoming more important and attractive to enable more economic structures, in which construction and material costs increase. Reinforced concrete was adopted for the design of girders in bridges up to certain span length. However, since concrete is weak in tension, cracks were unavoidable in an economical design. If these cracks in the tension zone can be arrested up to a certain limit, their responses are accepted Collins and Mitchell, 1991.

This study will investigate the optimum design of prestressed concrete I-girder bridge system (transverse configuration), using The Method of Feasible Directions as the optimization technique. This method finds the minimum of a constrained function by first finding bounds on the solution and then using polynomial interpolation. The design of prestressed concrete (PC) I-girder Bridge system is coded into a program coupled with the optimization program, in which the minimum cost of the bridge is the objective function. The set of requirements by the design methods of the AASHTO code, 1989, are the constraint functions.

The idea of optimum synthesis of prestressed structures was firstly introduced by Rozvany, 1964. One of the earliest works in the optimum design of prestressed concrete bridges was carried out by Torres, et al, 1966. The design of prestressed precast concrete, single span, composite highway bridge of standard sections was considered based on a comprehensive minimum cost. Nonlinear design problem was reduced to linear programming problem. Khaleel and Itani, 1993, presented a comprehensive study on the optimization of simply supported partially prestressed concrete girders, using sequential quadratic programming. A practical approach to the optimal design of precast, prestressed concrete highway bridge girder system was presented by Lounis and Cohn, 1993. This approach aims at standardizing the optimal design of bridge systems, as apposed to standardizing girder sections. An efficient method for evaluating the safety of existing girder bridge as a function of the load and resistance parameters was presented by Khaleel and Itani, 1993. The bridge capacity was determined using a nonlinear finite element program in terms of the truck load which is increased until structural collapse occurs. Johanson, 1972, developed an interactive design and analysis program for prestressed concrete girders. Design of simply supported post- or pre-tensioned girders of composite or noncomposite sections were considered. Standard sections as database were provided for the interactive technique as a design optimization process rather than using mathematical formulation as in linear programming. Aguilar et al, 1973, developed a theory useful in the design of multispan, simply supported highway bridges. Yu, et al, 1986, presented an application of generalized geometric programming to the optimal design of prestressed concrete box bridge girder.

Research Significance: The objective of this paper is to investigate the optimum design of prestressed concrete I-girder bridges based on AASHTO code, 1989, provisions. To achieve this objective, a computer program for the analysis and design of a highway bridge system simple span was coupled with local optimization technique as used in the optimization process.

This optimization formulation has been applied to both standard sections of AASHTO code and general I-shaped sections. In addition, the study presents a comparison among the optimum designs for three tendon profiles, which are parabolic, harped at L/3 of the span and triangular profile.

The prestress losses during the initial and final stages are accounted for. The objective function is the minimization of the concrete volume, formwork area, weight of prestressing tendons, and the weight of mild steel. The design of prestressed concrete girder was subjected to a number of nonlinear constraints as required by AASHTO code, 1989, such as the ultimate flexural and shear capacity, the compressive and tensile stresses at initial and final stages, , the deflection, the crack width, the losses, and the stress at the jacking end.

Model Description: The problem under investigation deals with the analysis and optimum design of post-tensioned, simply supported partially prestressed concrete girder. The cross section of the girder is a general I-section, which is prismatic along the span length. The general concrete cross section consists of six dimensions representing the width and height of top and bottom flanges, the width of the web and the overall height of the section as shown in Fig. 1. The prestressing tendons or strands, which are post-tensioned, may be laid in

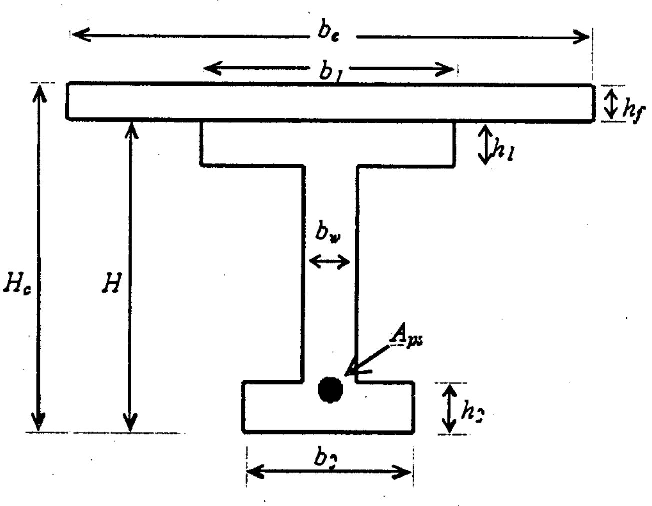

Fig. 1: Prestressed concrete I-girder composite section

parabolic, harped or triangular profile along the girder as shown in Fig. 2. These prestressing tendons consist of one cable (duct) that may be jacked at different stress levels at both ends of the girder. Two bars (#5) of mild steel are used in the top and bottom of the girder, to hold stirrups and to resist the flexural stresses with the prestressed tendons. The eccentricity of the tendons at midspan is the most practical eccentricity (i.e. minimum concrete cover).

In the transverse configuration the number of girders is equal to the number of spaces ranging the deck width. The outer deck overhangs at half of spacing between girders as shown in Fig. 3. The concrete I-section girder is treated as a composite section as shown in Fig. 1.

Problem Formulation

Description of Design Variables: In prestressed concrete girder bridge design, nine variables are considered. Figs. 1 and 3 show the cross-section of the girder section with the design variables. The design variables vector can be written as follows:

$$X = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \end{Bmatrix} = \begin{Bmatrix} A_{ps} \\ qq \\ b_1 \\ b_2 \\ b_w \\ h_1 \\ h_2 \\ H \\ S \end{Bmatrix} \tag{1}$$

Description of Objective Function: The objective function (nonlinear implicit function) is the total cost of concrete, formwork, prestressed steel, and mild steel required for the structure, as follows:

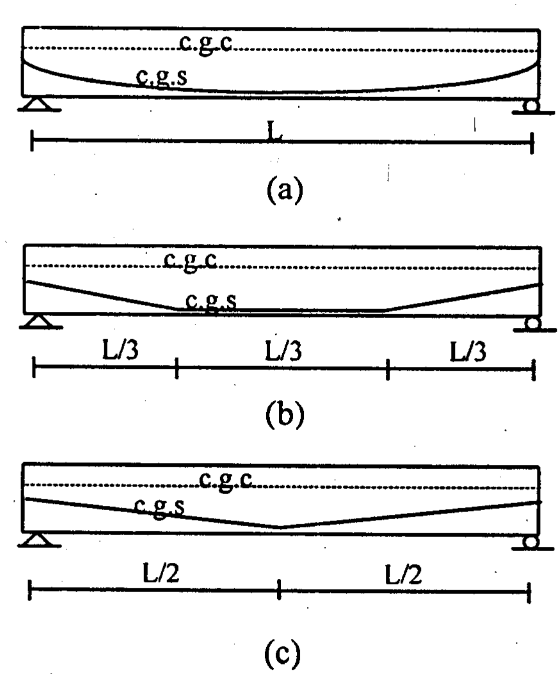

Fig. 2: Possible tendon profiles (a) Parabolic. (b) L/3 Harped. (c) Triangular

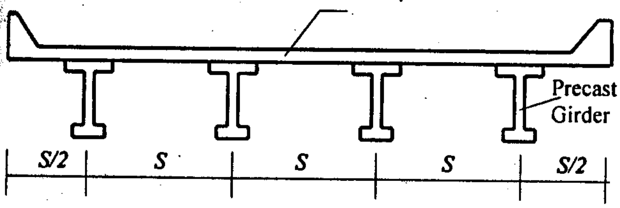

Fig. 3: Girder Bridge (Transverse Configuration)

$$Min.\ F(X)=C_c.V_c(X)+C_f.A_f(X)+C_p.W_p(X)+C_s.W_s(X) \quad (2)$$

Description of Constraints: Two types of constraints are considered; the bounds on the design variables and the nonlinear equations of inequality constraints. The bounds on the design variables are as follows:

$$X_i^{min} \le X_i \le X_i^{max} \quad (3)$$

The design of prestressed concrete girder is subjected to a number of nonlinear constraints as required by the design requirements of the AASHTO code. The various inequality constraints in this study. are summarized in Table 1.

Optimization Method: The method of feasible directions is used to deal directly with the nonlinearity of the problem. This method was employed in an optimization package, Manual of ADS (Manual of ADC). This package includes many optimization methods. However, after examining these methods on our problem, the modified feasible directions method was found to be the most proper. The advantage of this modified method is that, on designs where many side constraints are active, the one-dimensional search can continue without the necessity of calculating new gradient information, usually a time-consuming process. For cases where this modification applies, it often considerably improves the efficiency of the optimization process and is recommended when using this and other direct search algorithms.

Results and Discussion

Different spans of concrete girder bridges are considered; these spans range between (40 and 100ft). The design of concrete girder bridge is performed for three different numbers of lanes (two, three, and four). These concrete girder bridges are designed to satisfy serviceability requirements as well as strength requirements.

Effect Of the Tendon Profile: In this study, three tendon profiles have been introduced .The results show that the selection of the tendon profile affects the cost of girder bridge. Fig 4 shows the optimum cost, span length, number of lanes interactions for parabolic, L/3 harped, and triangular tendon profile, respectively. It is clear for the studied cases, that parabolic tendon profile is more economical than the other two profiles .It is obvious that the relation between the cost and span length is becoming non-linear when the number of lanes and length of spans increases. Therefore, the relation between the cost and

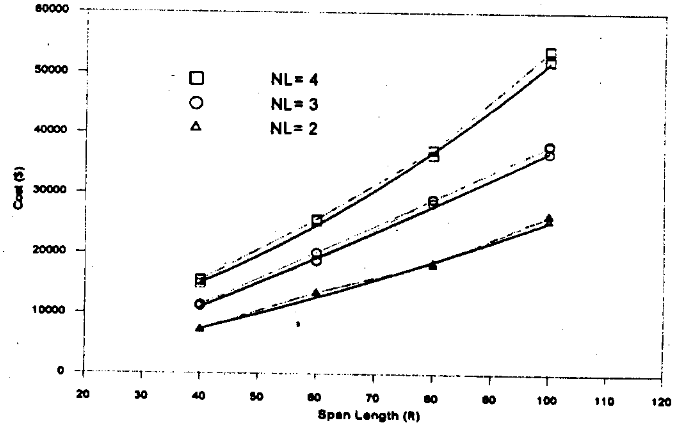

Fig (4) Cost vs. span length for parapolic, L/3 Harpped and Triangular tendon profiles

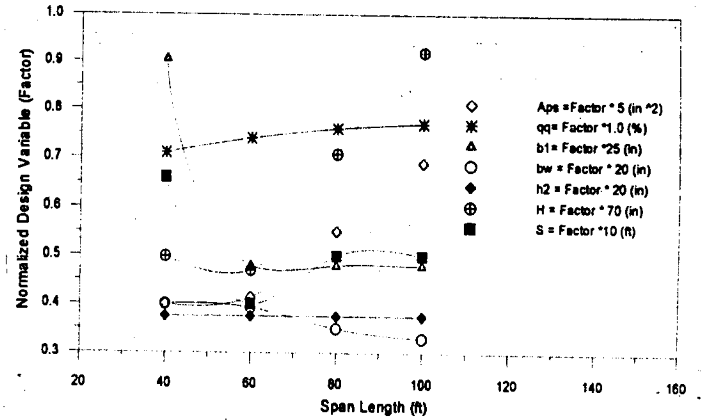

Fig (5) Normalized design variables for a 2-lane concrete girder bridge with a parabolic tendon profile.

span length can be fitted into a linear relationship, which is given, as the following:

* For parabolic tendon profile:

- Cost = 612.86*L-10759.7 (4-lanes)
- Cost = 431.43*L-6455.6 (3-lanes)
- Cost = 298.58*L-4853.7 (2-lanes)

* For L/3 harped tendon profile:

- Cost = 632.68*L-11245.1 (4-lanes)
- Cost = 451.40*L-7206.5 (3-lanes)
- Cost = 311.28*L-5339.2 (2-lanes)

* For triangular tendon profile:

- Cost = 632.46*L-11180.7 (4-lanes)
- Cost = 441.17*L-6286.5 (3-lanes)
- Cost = 368.16*L-5192.2 (2-lanes)

The material costs considered here are 85 $\$/yd^3$ for concrete, 2 $\$/ft^2$ for formwork area, 1.28 \$/Ib for prestressing steel, and 0.37 \$/Ib for mild steel.

The results show that the optimum values of the design variables (b_2, h_1) have small variations for all cases considered here and they almost converged to their lower bounds. Figs. 5 to 7 show the normalized design variables for parabolic tendon profile with three numbers of lanes. They display that A_{ps} slightly increases up to 60 ft span length. After this point it highly increases. The qq increases as the length increases, whereas, the design variable b_1 greatly decreases up to 60 ft span length. Beyond this point it will be constant. Dominantly b_w decreases as the span

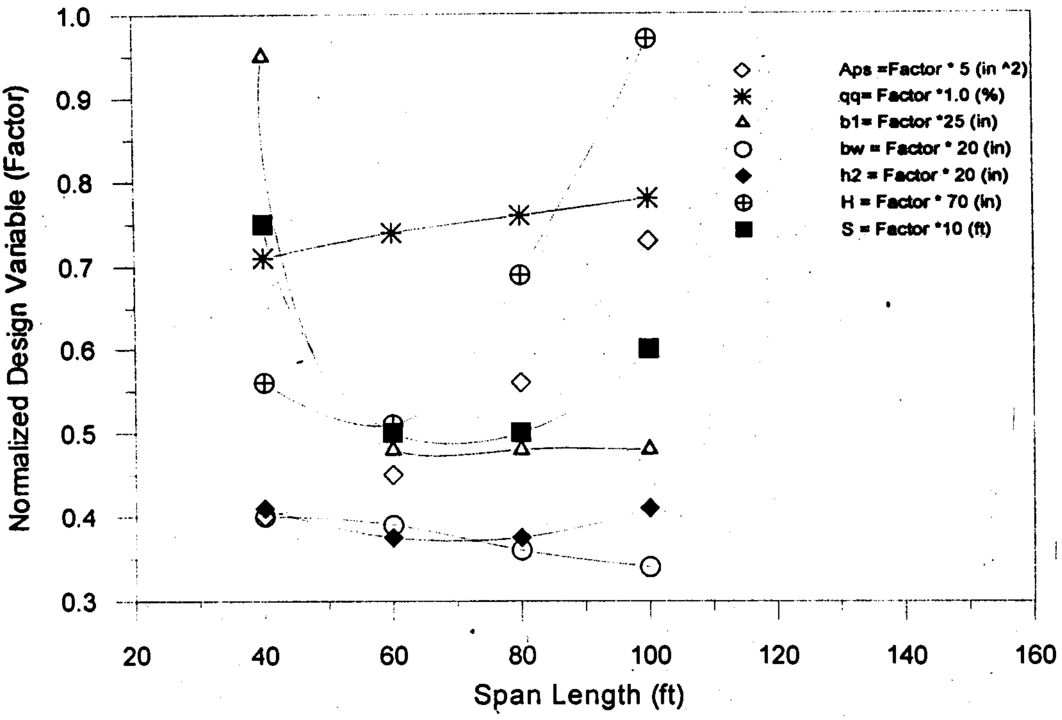

Fig (6) Normalized design variables for a 3-Lane concrete girder bridge with a parabolic tendon profile.

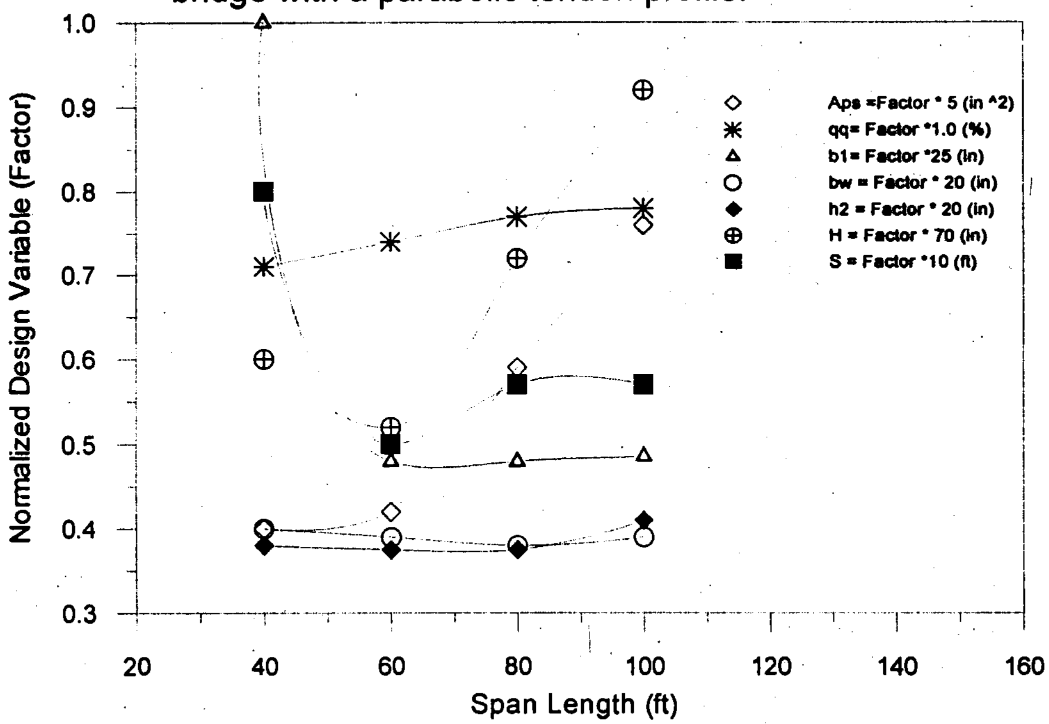

Fig (7) Normalized design variables for a 4-Lane concrete girder bridge with a parabolic tendon profile.

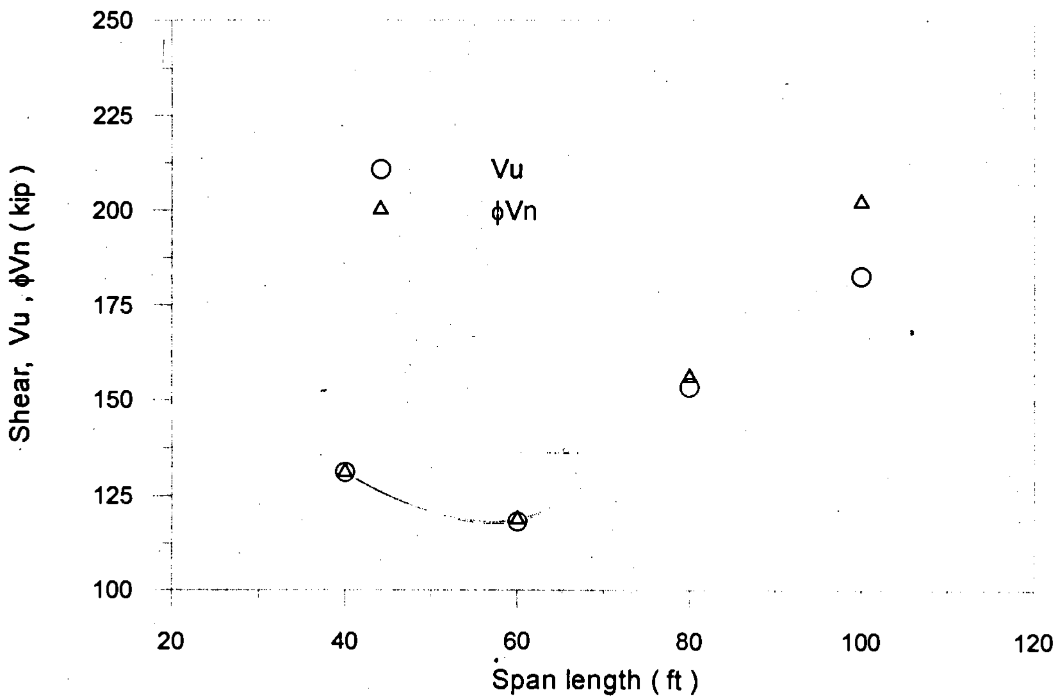

Fig (8) Ultimate Shear capacity and applied shear vs. span length for 4-lane concret girder bridges with parabolic tendon profiles.

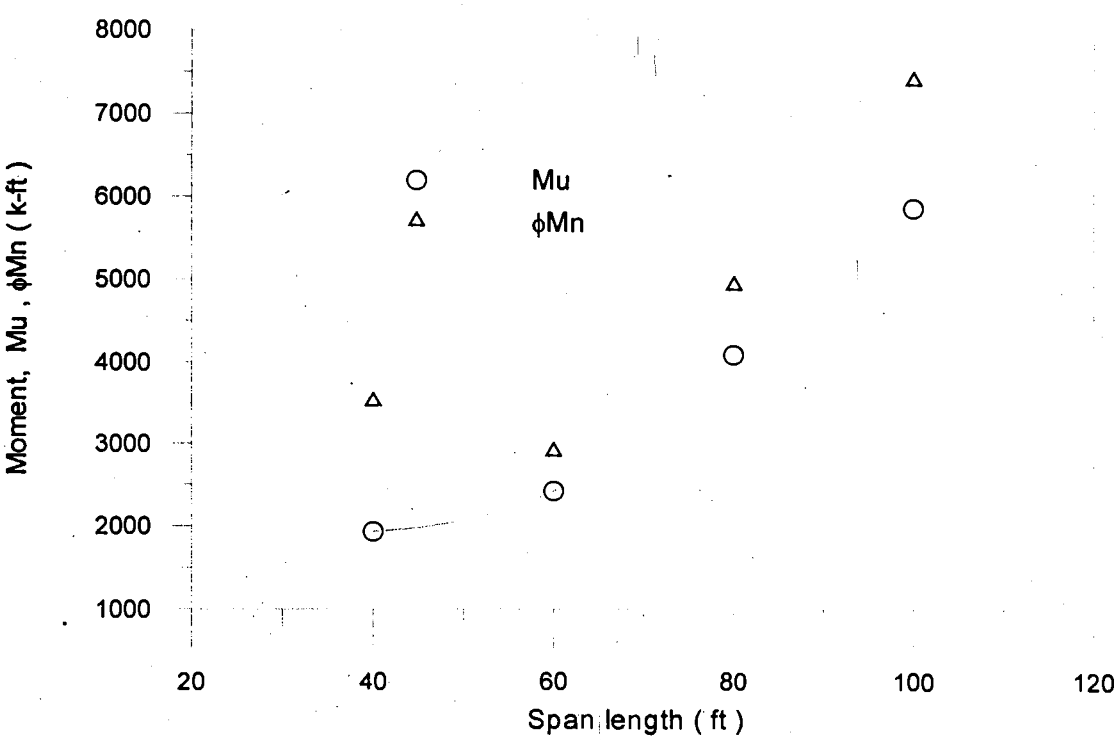

Fig (9) Ultimate Moment capacity and applied moment vs. span length of 4-Lane concrete girder bridges with parabolic tendon profiles.

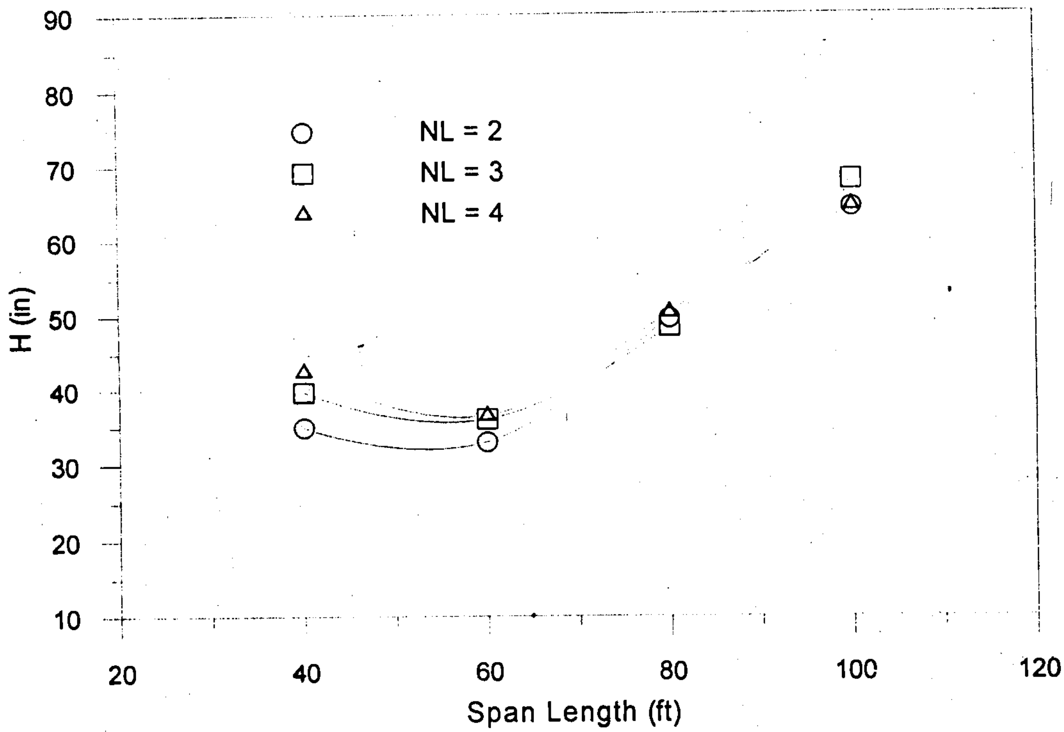

Fig (10) Height of section vs. span length of Concrete girder bridges with parabolic tendon profiles.

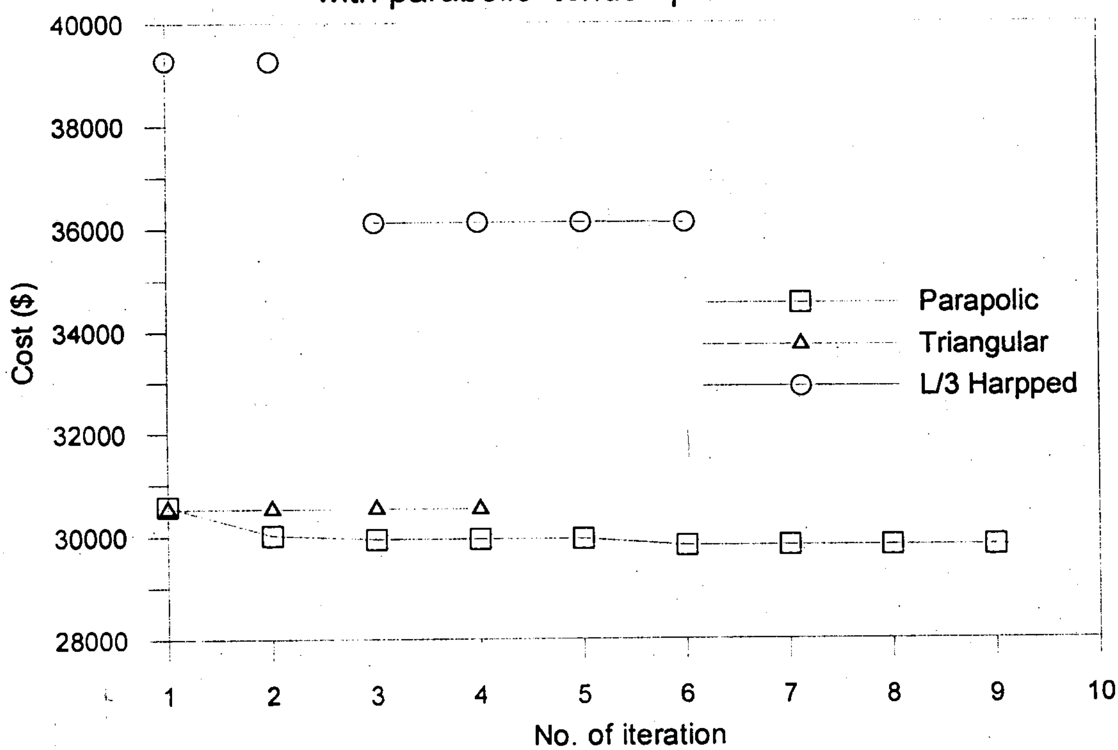

Fig. (11) Cost vs. No. of iteration for 4-Lane Concrete Girder Bridges of 80 ft span lengths.

Table 1: Objective Function and Constraints on Strength and Dimensions of PSC Girder Bridges

Design Variables of the Optimization Problem, X

Variable	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Symbol	A_{ps}	qq	b_1	b_2	b_w	h_1	h_2	H	S

Inequality Constraints

Constraints	Description
$g_1^{strength}(X)=\phi M_n - M_u$	Flexural Strength Capacity
$g_2^{shear}(X)=\phi V_c - V_u$	Shear Strength Capacity
$g_3^{ti}(X)=f_{ti}-\sigma_{ti}$	Stress @ top of girder (initial stage)
$g_4^{bi}(X)=f_{bi}-\sigma_{bi}$	Stress @ bottom of girder (initial stage)
$g_5^{ts}(X)=f_{ts}-\sigma_{ts}$	Stress @ top of girder (final stage)
$g_6^{bs}(X)=f_{bs}-\sigma_{bs}$	Stress @ bottom of girder (final stage)
$g_7^{t.slab}(X)=f_{t.slab}-\sigma_{t.slab}$	Stress @ top of Slab (final stage)
$g_8^{deflc\ 2}(X)=(L/480)-\delta_{final}$	Long term Deflection
$g_9^{cr-width}(X)=0.012-\omega_{max}$	Maximum crack width
$g_{10}^{loss}(X)=-\eta+0.7$	Maximum loss of prestressing force
$g_{11}^{Loss}(X)=\eta-0.85$	Minimum loss of prestressing force
$g_{12}(X)=\left(f_{Pj}+\Delta f_{PA}\right)-0.8f_{Pu}$	The stress at jacking End
$g_{13}^{ratio}(X)=(\phi M_n/M_{cr})-1.2$	Flexure-Cracking Ratio

f_{ti}, f_{bi}, top-fiber and bottom-fiber stress at initial stage, f_{ts}, f_{bs} top-fiber and bottom-fiber stress at final stage, S_{ti},S_{bi}, S_{ts}, S_{bs}, are associated permissible stresses, h is the residual stress factor, V_C, V_u the capacity and applied shears, M_C, M_u the capacity and applied moments, M_{cr}, is the cracking moment, w_{max}, is the maximum crack width, d, is the deflection.

Objective Function: Minimizing the overall cost of the PSC Girder Bridge.

$$Z=F(X)=C_{conc}\ V_{conc}(X)+C_{pres}\ W_{pres}(X)+C_{stel}\ W_{stel}(X)+C_{fmwk}\ A_{fmwk}(X)$$

length increases. It can be seen that h_2 does not change up to 80 ft span length. After this span length, h_2 will increase. Generally the S decreases up to 60 ft span length, then it will increase. The results showed that the shear requirement controls the final design for short spans, while for longer spans the losses requirement controls the final design; this is for parabolic and L/3 harped tendon profiles. In triangular tendon profile case, the stress at initial stage and deflection controls the final design for short spans. The values of the optimum design variables were found reasonable, and economically and practically feasible.

AASHTO Standard Sections: In this study the optimization was carried out using AASHTO standard section to get the minimum cost of the optimized girder bridge. In this case the number of the design variables is reduced to three only, namely A_{ps}, qq, S. The results show that the standard sections are more economical than non-standard section, if and only if, the crack width constraint is allowed to be violated. This violation is due to limited standard section dimensions. Table 2 indicates that the standard sections are not economical when compared with non-standard section if the crack width constraint is celeted from the formulation. It also shows that in this case the ultimate shear capacity requirement controls the final designs.

Shear and Moment Capacity: The ultimate shear

Table 2: Optimum Designs of Concrete Girder Bridges using Type-IV AASHTO Standard Section (Neglecting Crack Width Constraint)

Girder Bridge with span = 80 ft, and 4-lanes			
Tendon Profile	Parabolic	L/3 harped	Triangular
Total Cost ($)	39572.0	43491.0	-
Concrete Volume(yd^3)	162.0	178.0	-
Formwork Area (ft^2)	7997.0	8950.0	-
Tendon Weight (lb)	5812.0	6251.0	-
Mild Steel Weight (lb)	8417.0	8886.0	-
Design Variables			
A_{ps} (in^2)	4.20	3.76	-
qq (%)	0.75	0.77	-
S (ft)	6.66	5.71	-
Active Constraints g's			
Constraint No.	$g_2(X)$	$g_2(X)$	-
Constraint Value	0.98E-5	0.83E-6	-
CPU time	0.073	0.053	-
Function Evaluation	253.000	124.000	-

requirement is found to be an active constraint that controls the optimum designs, while the ultimate moment requirement is not. This result is reasonable for prestressed concrete members due to prestressing forces in the concrete that counteract the applied loads. Fig. 8 shows the ultimate shear capacity and the applied factored shear, while Fig. 9 shows the ultimate moment capacity and the applied factored moment for different span lengths of the bridge. The vertical difference on the two Figs 8, and 9 serves as a factor of safety against shear failure and moment failure, respectively.

Depth of Section: Fig. 10 show the relationship between the section depth and span length of two, three, and four lanes girder bridges with parabolic tendon profiles. It is clear that, up to 60 ft span length, the section depth slightly decreases when the span length increases. Beyond this point, the section depth increases as the span length increases. This can be explained as follows: the spacing between girders at 60 ft span length is less than when the span length is 40 ft. Thus, the number of girders will increase. Consequentially the load on the girder will decrease. Therefore, a shallow depth for the girders will be needed. On the other hand, when the spacing is large, the load on each girder will increase. Thus, a deeper girder depth will be needed to resist the loads on the girders.

Convergence Criteria: It is worth mentioning that the ADS program was sensitive to the initial design that has been provided initially by the user for the first design iteration. A good starting point will reduce the number of design iterations that may be required to reach a satisfactory solution. However, the degree of convergence was different from run to run and from problem to problem depending on the case given during the selection of the initial values of the design variables. The convergence to the optimum values is shown in Fig. 11. It is clear that the parabolic tendon profile solutions have better convergence than L/3 harped tendon profile solutions. In the case of the triangular tendon profile the solutions did not converge. Generally, the longer the span length, the larger the number of iterations is, Khaleel and Itani, 1993.

Conclusion

The following conclusions can be drawn from the study:

- The general optimization approach presented herein is suitable for determining the optimal designs of prestressed concrete girder bridges.
- The optimal girder AASHTO section for total bridge lengths, L=40 ft consists of type II. For L=60 ft the optimal system consists of type III, for L=80 ft, the optimal system consists of type IV, and for L=100 ft the optimal system consists of type VI.
- More savings on the cost occur when the section is allowed to crack.
- Stress at jacking end controlled the optimum design for long spans, while the shear strength requirement controlled the design for short spans.
- Girder spacing is insensitive to bridge width and depends only on the span length. The required girder spacing for short span lengths ($L \le 60$ ft) is found to be greater than that for long spans.
- For both crack controlled and crack uncontrolled sections the optimum web thickness is independent of the span length and the lanes number, while the section depth is directly proportional to span length, especially for long spans. For short spans ($L \le 60$ ft), this ratio is inversely proportional to span length.
- Designing the concrete I-girder section as a composite section keeps the width of the top flange constant, i.e. the width of the top flange converges to the lower bound, especially for long spans.
- The parabolic tendon profile is found to be the optimum one, especially for long spans.
- The cost is a slightly nonlinear function of span length. Therefore, it can be reasonably approximated as a linear relationship.
- Bridge girder with parabolic tendon profile shows better convergence (more optimum) to the optimum values than the other two profiles.

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