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On Numerical Modeling of a Space-Charge Limited Beam in the Self-Consistent Field

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Abstract: A new way estimated space charge limited beam is considered. Based on the integral equation method approximate algorithm for solving stationary axially symmetric self-consistent problem of electronic optics is proposed. Numerical experiments are presented.

Key Words: Self-consistent Problem, Space-charge Limited Beam, Integral Equation Method

Introduction

At the planning of various electronic devices using heavy-current space charge beams, for example, accelerators, injectors for thermonuclear plants, gas discharge lasers, it is necessary to investigate a motion of the beam in the external electromagnetic field (Ellyn, 1985; Muller, 1984 and Rudakov, 1990). In practice the case of intensive flows when the beam's space charge has to be taken into account is particularly important. Estimation of non-relativistic beam consists in the stationary self-consistent problem of electronic optics (Ellyn, 1985 and Kirstein *et al.*, 1970), namely, in satisfaction Poisson's equation, the equation of motion and the equation of charge continuity under the boundary and initial conditions.

To solve this problem it is necessary to define the potential distribution, the densities of current and space charges, to calculate charge's trajectories into inertelectrode space. This process is accomplished by the successive approximations of space charge (Ellyn, 1985), modeled by the current tube method (Ellyn, 1985 and Roshal, 1979).

Up to this point the basic process used was to solve Poisson's equation by finite difference methods (Ellyn, 1985 ., Hamza, 1966 and Hamza, 1967). The effective algorithms solving this problem can be obtained based on the Integral Equation Method (IEM) (Kress, 1989), as since its application associates with the finding of unknown values only on the domain boundary. This means that sought function depend on one argument in two-dimensional boundary value problem and two arguments – in three-dimensional problem. In contrast to the net point method, which needs a closed domain, IEM admits open domain. The accuracy of the potential derivative calculation by the net point method is not so high inasmuch as derivatives are determined on differentiation of the table of potential value in the net points. The IEM guarantees the same accuracy of potential calculation and its derivatives that raises the computational accuracy of trajectories. Taking into account singularities on the boundary of open surface and in the kernel of integral equation the high computational accuracy is obtained near the boundary. In addition the IEM is eminently suited to solving problem of this type.

The plan of the paper is as follows. In section "Formulation of the Problem" the detailed formulation of the problem is described. Section "Numerical Solution of Self-consistent Problem" clarifies each stage to solve self- consistent problem. Boundary value Dirichlet problem for Poisson's equation in space with slits is solved based on IEM. The sought solution is represented according to the potential theory. Obtained integral equation with weak singularity in the kernel is solved by the quadrature method using Ermit formula. The predictor-corrector-type method realizes the numerical integration of the equation of motion. As a result the iterative procedure including Nachamkin-Hancock convergence acceleration (Nachamkin and Hancock, 1981) is built. Finally in Section "Numerical Experiments" some results of numerical experiments are presented. The potential and current density of plane parallel diode with unlimited emissive capability of cathode (Kirstein *et al.*, 1970) have been computed. One five-electrode optoelectronic system has been estimated. The self-consistent problem of the plasma boundary formation of high voltage glowing discharge (Ellyn, 1985) has been solved (Ostudin and Romanenko, 2002). Obtained results are shown to be in good agreement with theoretical assumption and experimental data.

Formulation of the Problem: Let's consider spaciousness V, filled of the space charge with density ρ, and piecewise smooth open electrodes' surface S in it. Surface S is represented as aggregate of a cylindrical surfaces, made by rotating around axis z in some cylindrical coordinates system (r, z, φ) some generators L_i, which in rOz plane are given by parametric equations

$$\begin{cases} r_i = r_i(\tau), \\ z_i = z_i(\tau), \end{cases} \quad \tau_1^{(i)} \le \tau \le \tau_2^{(i)}, i=1,\ldots,N, \text{ at that } L = \bigcup_{i=1}^{N} L_i$$

Let a potential value be given on S (potential of emitter is equal to zero). The charge particles, supposed to be non-relativistic (the magnetic field arising during the motion is not considered), fly out from the cathode into spaciousness V. Let's calculate a space-charge beam.

The potential distribution u is obtained as a solution of external boundary value Dirichlet problem for Poisson's equation in space with slits

$$\Delta u = -\rho/\varepsilon_0 \quad \text{Outside } S := \bigcup_{i=1}^{N} S_i, \qquad (1)$$

$$u_{|S_i} = \Phi_i(M), \qquad M \in S_i, \qquad (2)$$

Under condition of regularity of solution on infinity, where ε_0 is dielectric constant. Note that function ρ is determined by the pattern of beam's motion, i.e. in turn depend on field.

By the Gauss system let's denote the equation of motion of particle with mass m and charge e in Newton's form

$$\frac{d^2\vec{r}(t)}{dt^2} = \frac{e}{m}E, \qquad (3)$$

Where $\vec{r}(t)$ is a radius-vector of particle; t is a time variable; E is a vector of electrical field tensity, at that $E = -\operatorname{grad} u$. Let's suppose that particles make a start on some distance $\bar{d}_0$ from the cathode with velocity equal to zero

$$\vec{r}(0) = \bar{d}_0, \qquad \vec{r}\,'(0) = 0 \qquad (4)$$

As it was mentioned above the modeling of space charge is accomplished by the current tube method. According to it cathode or emitting surface is divided into some segments and it is supposed that each electron's trajectory from one segment is similar to each other. Those trajectories make up a current tube with a permanent current. Let this current be concentrated in the center of tube, so that its central trajectory may be considered as elementary ray. Current of p-tube is defined as $I_p = S_p J_p$, where S_p is a square of elementary cathode, J_p is a current density of emitting surface, which depends on potential modification near the cathode by the Child-Langmuir law (Kirstein *et al.*, 1970). The equation of charge continuity for discrete flow model has the following form

$$I_p = \text{const}. \qquad (5)$$

Equations (1)–(5) make a closed system. To solve this problem it is necessary to define the potential distribution, the densities of current and space charges, to calculate charge's trajectories into inetrelectrode space.

Let's proceed from physical model to mathematical by the initiation of those dimensionless values:

$$z' := \frac{z}{L_0},\ r' := \frac{r}{L_0},\ u' := \frac{u}{u_0},\ \rho' := \frac{\rho}{\rho_0},\ \sigma' := \frac{\sigma}{\sigma_0},$$
$$t' := \frac{t}{t_0},\ I' := \frac{I}{I_0},\ \bar{q}' := \frac{\bar{q}}{\rho_0 L_0^3},\ J' := \frac{J}{J_0},\ S' := \frac{S}{L_0^2},$$

Where L_0, t_0, u_0, I_0, ρ_0, σ_0, J_0 are certain typical values of the length, time, potential, current, density of space charge, density of surface charge, current density, at that

$$t_0 := \frac{L_0}{v_0},\ v_0 := \sqrt{2\frac{|e|}{m}u_0},\ I_0 := \frac{\rho_0 L_0^3}{t_0},\ \sigma_0 := \rho_0 L_0,\ J_0 := \rho_0 v_0.$$

Numerical Solution of Self-consistent Problem: The solving process is carried out by the successive approximations of space charge. The field determination under space charge pertaining to zero initiates the iterative procedure. Inetrelectrode spaciousness is divided on $\bar{n}$ elementary cells with extent V_k – cylindrical tubes with height Δz_k and width Δr_k, made by rotating around axis z of the rectangles

$$\Omega_k := \left\{ r, z:\ r_k - \frac{1}{2}\Delta r_k \le r \le r_k + \frac{1}{2}\Delta r_k,\ z_k - \frac{1}{2}\Delta z_k \le z \le z_k + \frac{1}{2}\Delta z_k \right\}$$

with midpoints (r_k, z_k). The extent of tube

$$V_k = \pi((r_k + 0{,}5\Delta r_k)^2 - (r_k - 0{,}5\Delta r_k)^2)\Delta z_k = 2\pi r_k \Delta r_k \Delta z_k.$$

Element modeling the field of space charge $\bar{q}_k$ is located within k-cell. The part of charge $\bar{q}_{pk}$, bringing by p-ray in, is determined as $\bar{q}_{pk} = I_p t_{pk}$, where t_{pk} is a dwell time of p-ray in k-cell. The next iteration total charge $\bar{q}_k^{(n+1)}$ is obtained by the summarizing of all trajectories passing through the V_k

$$\bar{q}_k^{(n+1)} = \sum_p I_p^{(n+1)} t_{pk}, \quad n = 0, 1, 2, \ldots,\ \bar{q}_k^{(0)} = 0 \qquad (6)$$

Let's suppose that charge $\bar{q}_k^{(n)}$ in k-cell after n-iteration can be transformed in distributed density of space charge $\rho_k^{(n)} = \bar{q}_k^{(n)}/V_k$. Having solved the Dirichlet problem (1)–(2) we define the potential, which is used in integration of the equation of motion (3) under conditions (4). Further by virtue of trajectories pattern, currents of each tube the new charge distribution $\bar{q}_k^{(n+1)}$ is defined from the Equation (6). The whole process is then repeated through several iterative cycles until the neighboring current approximations agree with given accuracy ε :

$$\max_p \left| 1 - \frac{I_p^{(n+1)}}{I_p^{(n)}} \right| < \varepsilon \qquad (7)$$

Boundary Value Dirichlet Problem for Poisson's Equation: According to the potential method theory (Ellyn, 1985) solution of the problem (1)-(2) is represented as

$$u(M) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(N)\,dS_N}{R(M,N)} - \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(P)\,dV_P}{R(M,P)} \qquad (8)$$

Here first integral evaluates a potential of surface charges, spaced with density $\sigma(N)$ on S ($R(M, N)$ - distance from point M to point $N \in S$), while second integral evaluates a potential of space charges, spaced with density $\rho(P)$ ($R(M, P)$ - distance from point M to point $P \in V$).
Taking into account a dimensionless pattern of investigated values, Poisson's equation (1) becomes such as

$\Delta u' = -D\rho'$, where $D := \frac{\rho_0 L_0^2}{u_0 \varepsilon_0}$,

and analogue of (8) is

$$u'(M) = \frac{D}{4\pi}\int_{S'} \frac{\sigma'(N)\,dS'_N}{R'(M,N)} - \frac{D}{4\pi}\int_{V'} \frac{\rho'(P)\,dV'_P}{R'(M,P)}$$

In further after initiation of dimensionless analogues dashes are turned down, if not mentioned something else. Without losing generality, let $D := 1$.
Granting axial symmetry of field, the potential distribution we define in plane $\varphi_0 = \mathrm{const}$, which is a half of a meridian section of surface S, as

$$u(\bar r,\bar z) = \frac{1}{\pi}\int_L \frac{\sigma(r_N,z_N)\,r_N\,\mathrm{K}(k_N)}{\sqrt{(r_N+\bar r)^2+(z_N-\bar z)^2}}\,dL_N - \frac{1}{\pi}\int_\Omega \frac{\rho(r_P,z_P)\,r_P\,\mathrm{K}(k_P)}{\sqrt{(r_P+\bar r)^2+(z_P-\bar z)^2}}\,dr_P\,dz_P \qquad (9)$$

Here Ω is a domain in meridian half-plane $\varphi_0 = 0$, filled of space charge; $(\bar r, \bar z)$ are the coordinates of observation point M, in which potential is defined; (r_P, z_P) and (r_N, z_N) - coordinates of points P and N belonging to domain Ω and contour L respectively, at that

$$\mathrm{K}(k) := \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1 - k\sin^2\alpha}}$$

is a full elliptic integral of first kind, in which

$$k := 4r\bar r\left[(r+\bar r)^2 + (z-\bar z)^2\right]^{-1}.$$

Taking into account parametrical representation of L and satisfying boundary conditions on L the integral equation for determination of sought density $\sigma(\tau) = (\sigma_1(\tau),\ldots,\sigma_N(\tau))$ is obtained

$$\sum_{i=1}^N \frac{1}{\pi}\int_{\tau_1^{(i)}}^{\tau_2^{(i)}} \frac{\sigma_i(\tau)F_i(\tau)r_i(\tau)\mathrm{K}(k(r_i(\tau),z_i(\tau),r_j(\bar\tau),z_j(\bar\tau)))}{\sqrt{(r_i(\tau)+r_j(\bar\tau))^2+(z_i(\tau)-z_j(\bar\tau))^2}}\,d\tau = \bar u_j(\bar\tau), \qquad (10)$$

Where

$$\bar u_j(\bar\tau) = \Phi_j(\bar\tau) + \frac{1}{\pi}\int_\Omega \frac{\rho(r_P,z_P)\,r_P\,\mathrm{K}(k_P)}{\sqrt{(r_P+r_j(\bar\tau))^2+(z_P-z_j(\bar\tau))^2}}\,dr_P\,dz_P, \qquad (11)$$

$F_i(\tau) := \sqrt{[r_i'(\tau)]^2 + [z_i'(\tau)]^2}$,

$(r_j(\bar\tau), z_j(\bar\tau)) \in L_j$, $\tau_1^{(j)} < \bar\tau < \tau_2^{(j)}$, $j = 1, \ldots, N$.

Let's insert notations relative to the variables τ and $\bar\tau$:

$$T_{i,j}(\tau,\bar\tau) := \sqrt{(r_i(\tau)+r_j(\bar\tau))^2 + (z_i(\tau)-z_j(\bar\tau))^2},$$

$$k_1(\tau,\bar\tau) := \left[(r_i(\tau)-r_j(\bar\tau))^2 + (z_i(\tau)-z_j(\bar\tau))^2\right] T_{i,j}^{-2}(\tau,\bar\tau),$$

$$\mathrm{K}_{i,j}(\tau,\bar\tau) := \mathrm{K}(k(\tau,\bar\tau)) = \sum_{p=0}^4 a_p k_1^p - \ln k_1 \sum_{p=0}^4 b_p k_1^p + \varepsilon(k_1),\ k_1 := 1-k' -$$

unknown polynomial approximation (Abramowitz and Stegun, 1964) of elliptic integral. Then integral equation (10) assumes the form

$$A\sigma := \sum_{i=1}^N \frac{1}{\pi}\int_{\tau_1^{(i)}}^{\tau_2^{(i)}} \sigma_i(\tau)r_i(\tau)\mathrm{K}_{i,j}(\tau,\bar\tau)T_{i,j}^{-1}(\tau,\bar\tau)F_i(\tau)\,d\tau = \bar u_j(\bar\tau) \qquad (12)$$

Thus Dirichlet problem is reduced to determination of distribution charge density along curve L, since the right part in (10) is supposed to be known. When σ has been carried out, the potential in arbitrary point $(\bar r, \bar z)$ is defined by (9). Components $E_{\bar r}$ and $E_{\bar z}$ of the tensity-vector E are obtained by differentiation by $\bar r$ and $\bar z$ of the stating (9)

$$E_{\bar r} = -\frac{\partial u}{\partial \bar r} = -\frac{1}{\pi}\int_L \sigma(r_N,z_N)\,r_N\left\{\frac{2\mathrm{D}(k_N)\bar r_N - \mathrm{E}(k_N)(r_N-\bar r)\,(1-k_N)}{\left[(r_N+\bar r)^2+(z_N-\bar z)^2\right]^{3/2}}\right\}dL - \qquad (13)$$

$$-\frac{1}{\pi}\int_\Omega \rho(r_P,z_P)\,r_P\left\{\frac{2\mathrm{D}(k_P)\bar r_P - \mathrm{E}(k_P)(r_P-\bar r)\,(1-k_P)}{\left[(r_P+\bar r)^2+(z_P-\bar z)^2\right]^{3/2}}\right\}dr_P\,dz_P,$$

$$E_{\bar z} = -\frac{\partial u}{\partial \bar z} = -\frac{1}{\pi}\int_L \sigma(r_N, z_N) r_N \frac{\mathrm{E}(k_N)(z_N - \bar z)/(1-k_N)}{\left[(r_N+\bar r)^2 + (z_N - \bar z)^2\right]^{3/2}} dL + \qquad (14)$$
$$+\frac{1}{\pi}\int_\Omega \rho(r_P, z_P) r_P \frac{\mathrm{E}(k_P)(z_P - \bar z)/(1-k_P)}{\left[(r_P+\bar r)^2 + (z_P - \bar z)^2\right]^{3/2}} dr_P dz_P,$$

where $\mathrm{E}(k) := \int_0^{\pi/2} \sqrt{1 - k\sin^2\alpha}\, d\alpha$ is a full elliptic integral of second kind, $\mathrm{D}(k) := \frac{\mathrm{K}(k) - \mathrm{E}(k)}{k}$.

The analysis of the equation (12) has been carried out in modified weight Hölder spaces and its approach made by the quadrature method using Ermit formula (in details see (Ostudin and Romanenko, 2000).
After substitution of integrals in (12) for finite sums and selection as points of observation the abscissas of quadrature formula with change of variables and notation

$$\tau(s) := 0{,}5(\tau_1^{(i)} + \tau_2^{(i)}) + 0{,}5(\tau_2^{(i)} - \tau_1^{(i)})\, s\,,$$
$$q_i(s_k^{(i)}) := \frac{1}{P}\sqrt{1 - (s_k^{(i)})^2}\,\sigma_i(s_k^{(i)}) r_i(s_k^{(i)})\, F_i(s_k^{(i)})$$

we obtain the following system of linear algebraic equations relative to the q_m, $m = 1,\ldots,P$

$$\bar u_l = \sum_{i=1}^{N} \sum_{\substack{k=1\\(k\neq n, i=j)}}^{N_i} \mathrm{K}_{i,j}(s_k^{(i)}, s_n^{(j)}) T_{i,j}(s_k^{(i)}, s_n^{(j)}) q_m + \qquad (15)$$
$$+\frac{1}{2r_j(s_n^{(j)})}\left[N_j \ln 2 + \ln\frac{8r_j(s_n^{(j)})}{F_j(s_n^{(j)})} + \sum_{\substack{\gamma=1\\ \gamma\neq n}}^{N_j} \ln\left|s_\gamma^{(j)} - s_n^{(j)}\right|\right] q_l,$$

or

$$\bar u_l = \sum_{\substack{m=1\\ m\neq l}}^{P} A_{l,m}\, q_m + A_{l,l}\, q_l,$$ where

$$q_m := q_i(s_k^{(i)}),\ q_l := q_j(s_n^{(j)}),\ P := \sum_{\gamma=1}^{N} N_\gamma\ ;$$

$$m = k + \sum_{\gamma=1}^{i-1} N_\gamma, \quad k = 1,\ldots, N_i, \quad i = 1,\ldots, N\ ;$$

$$l = n + \sum_{\gamma=1}^{j-1} N_\gamma, \quad n = 1,\ldots, N_j, \quad j = 1,\ldots, N.$$

Let's note, that

$$s_k^{(i)} := \cos\left(\frac{2k-1}{2N_i}\pi\right),\ s_n^{(j)} := \cos\left(\frac{2n-1}{2N_j}\pi\right) \qquad (16)$$

are the abscissas of quadrature Ermit formula with N_i and N_j points respectively.

During the motion charge particles produce a space charge affecting the potential distribution and other field characteristics. Using (11) we define a right part of the system (15)

$$\bar u_l := \tilde\Phi_l + \sum_{k=1}^{\bar n} D_{l,k}\, \bar q_k\ ,$$
$$\tilde\Phi_l = \tilde\Phi_l(\bar r_l, \bar z_l) := \Phi_j(r_j(\tau(s_n^{(j)})), z_j(\tau(s_n^{(j)}))),$$

where $D_{l,k} := \frac{1}{2\pi^2}\cdot\frac{\mathrm{K}(k(r_k, z_k, \bar r_l, \bar z_l))}{\sqrt{(r_k + \bar r_l)^2 + (z_k - \bar z_l)^2}}$, since

$$\rho(r_k, z_k) = \frac{\bar q_k}{V_k}$$

When the point of observation $(\bar r_l, \bar z_l) \in L_j$ is located on the distance d, $|d| << 1$, from the center of k-cell, due to the singularity

$$D_{l,k} := \frac{1}{2\pi^2\sqrt{(r_k+\bar r_l)^2 + (z_k - \bar z_l)^2}}\left\{\ln\left[4\sqrt{(r_k+\bar r_l)^2 + (z_k - \bar z_l)^2}\right] + \frac{3}{4} - \right.$$
$$\left. - \frac{1}{16\Delta r_k \Delta z_k (r_k - \bar r_l)(z_k - \bar z_l)}\left(b_1^2 \ln b_1 - b_2^2 \ln b_2 - b_3^2 \ln b_3 + b_4^2 \ln b_4\right)\right\},$$

Where

$$\begin{cases} b_1 := d^2 + \Delta r_k(r_k - \bar r_l) + \Delta z_k(z_k - \bar z_l); & b_2 := d^2 - \Delta r_k(r_k - \bar r_l) + \Delta z_k(z_k - \bar z_l); \\ b_3 := d^2 + \Delta r_k(r_k - \bar r_l) - \Delta z_k(z_k - \bar z_l); & b_4 := d^2 - \Delta r_k(r_k - \bar r_l) - \Delta z_k(z_k - \bar z_l). \end{cases}$$

Having determined from the system (15) unknowns q_m, $m = 1,\ldots,P$, we find a potential in arbitrary non-boundary point $(\bar r, \bar z)$

$$u(\bar r, \bar z) = \sum_{m=1}^{P} q_m \frac{\mathrm{K}(k(r_m, z_m, \bar r, \bar z))}{\sqrt{(r_m + \bar r)^2 + (z_m - \bar z)^2}} - \frac{1}{2\pi^2}\sum_{k=1}^{\bar n} \bar q_k \frac{\mathrm{K}(k(r_k, z_k, \bar r, \bar z))}{\sqrt{(r_k + \bar r)^2 + (z_k - \bar z)^2}}, \qquad (17)$$

where $r_m := r_i(\tau(s_k^{(i)}))$, $z_m := z_i(\tau(s_k^{(i)}))$. If the point of observation is located near the j-electrode on the distance $F_j(\tau(\bar s_0))\left|\bar d\right|$, $\left|\bar d\right| << 1$, then, having supposed that $\bar s_0 := s_n^{(j)}$, we assume

$$u(\bar r, \bar z) = \sum_{\substack{m=1\\ m\neq l}}^{P} A_{l,m} q_m + A_{l,l} q_l - \sum_{k=1}^{\bar n} D_k \bar q_k, \quad P = \sum_{\gamma=1}^{N} N_\gamma\ ,$$

where

$$A_{l,m} := \frac{\mathrm{K}(k(r_i(s_k^{(i)}), z_i(s_k^{(i)}), \bar r, \bar z))}{\sqrt{(r_i(s_k^{(i)}) + \bar r)^2 + (z_i(s_k^{(i)}) - \bar z)^2}}\ ,$$

$$D_k := \frac{1}{2\pi^2}\frac{\mathrm{K}(k(r_k, z_k, \bar r, \bar z))}{\sqrt{(r_k + \bar r)^2 + (z_k - \bar z)^2}}\ ,$$

$$A_{l,l} := \frac{1}{4r_j(s_n^{(j)})}\left(\sum_{\substack{\gamma=1\\ \gamma\neq n}}^{N_j} \ln\left[\bar{d}^2 + (s_\gamma^{(j)} - s_n^{(j)})^2\right] + 2\ln\frac{8r_j(s_n^{(j)})}{F_j(s_n^{(j)})} - \frac{N_j}{\pi}\mathrm{i}_1\right),$$

number“ $k = 1,\ldots, N_i$ ”, right is

... number $l = n + N_1 + \ldots + N_{j-1}$ is known already, $q_l := q_j(\bar{s}_0)$, $m = k + N_1 + \ldots + N_{i-1}$, $k = 1,\ldots,N_i$, $i = 1,\ldots,N$, and integral ...

$$\mathrm{i}_1 = \int_{-1}^{1}\frac{1}{\sqrt{1-s^2}}\ln\left[\bar{d}^2 + (s-\bar{s}_0)^2\right]ds = \pi\ln\left[\xi_1 + \sqrt{2(\xi_1 + \xi_1\xi_2 + \xi_2^2 - 1)} + \xi_2\right] - 2\pi\ln 2,$$
(18)

where

$$\xi_1 := \bar{d}^2 + \bar{s}_0^2,\quad \xi_2 := \sqrt{(\bar{d}^2 + (1-\bar{s}_0)^2)(\bar{d}^2 + (1+\bar{s}_0)^2)},\ |\bar{d}| \ll 1,\ -1 \le \bar{s}_0 \le 1.$$

If the point of observation $(\bar{r}, \bar{z})$ is located in the k-cell,
then

$$D_k := \frac{1}{4\pi^2 r_k}\left\{\ln\frac{16 r_k}{\sqrt{\Delta r_k^2 + \Delta z_k^2}} + \frac{3}{2} - \frac{\Delta z_k}{2\Delta r_k}\operatorname{arctg}\frac{\Delta r_k}{\Delta z_k} - \frac{\Delta r_k}{2\Delta z_k}\operatorname{arctg}\frac{\Delta z_k}{\Delta r_k}\right\}.$$

Axis potential is determined by the formula

$$u(0,\bar{z}) = \frac{\pi}{2}\sum_{m=1}^{P}\frac{1}{\sqrt{r_m^2 + (z_m - \bar{z})^2}}q_m - \frac{1}{4\pi}\sum_{k=1}^{\bar{n}}\frac{1}{\sqrt{r_k^2 + (z_k - \bar{z})^2}}\bar{q}_k.$$

Cauchy Problem for the Equation of Motion: To integrate the equation of motion (3) it is necessary to define components of the tensity-vector by (13) and (14). From those equations at $\bar{r} = 0$ we assume

$$E_{\bar{r}}(0,\bar{z}) = 0,$$

$$E_{\bar{z}}(0,\bar{z}) = \frac{\pi}{2}\sum_{m=1}^{P}\frac{\bar{z} - z_m}{\left[r_m^2 + (z_m - \bar{z})^2\right]^{3/2}}q_m - \frac{1}{4\pi}\sum_{k=1}^{\bar{n}}\frac{\bar{z} - z_k}{\left[r_k^2 + (z_k - \bar{z})^2\right]^{3/2}}\bar{q}_k.$$

If point of observation tends to electrode L_j, then

$$E_{\bar{r}}(\bar{r},\bar{z}) = \sum_{\substack{m=1\\ m\neq l}}^{P} A_m q_m + A_l q_l - \sum_{k=1}^{\bar{n}} B_k \bar{q}_k.$$

Here

$$A_m := \frac{2\,\mathrm{D}(k_m) r_m - (r_m - \bar{r})\mathrm{E}(k_m)/k_{1,m}}{\left[(r_m + \bar{r})^2 + (z_m - \bar{z})^2\right]^{3/2}},$$

$$k_{1,m} := 1 - k_m = \frac{(r_m - \bar{r})^2 + (z_m - \bar{z})^2}{(r_m + \bar{r})^2 + (z_m - \bar{z})^2},$$

$$r_m := r_i(\tau(s_k^{(i)})),\ z_m := z_i(\tau(s_k^{(i)})),\ \left\{m = k + \sum_{1}^{i-1} N_\gamma,\ k = 1,\ldots,N_i,\ i = 1,\ldots,N\right\};$$

$$B_k := \frac{1}{2\pi^2}\cdot\frac{2\,\mathrm{D}(k_k) r_k - (r_k - \bar{r})\mathrm{E}(k_k)/k_{1,k}}{\left[(r_k + \bar{r})^2 + (z_k - \bar{z})^2\right]^{3/2}},$$

$$k_{1,k} := 1 - k_k = \frac{(r_k - \bar{r})^2 + (z_k - \bar{z})^2}{(r_k + \bar{r})^2 + (z_k - \bar{z})^2},$$

where (r_k, z_k) are the coordinates of the center of k-cell, coefficient A_l (under condition that $l := n + N_1 + \ldots + N_{j-1}$ is known, since point $(\bar{r}, \bar{z})$ is located on the distance $\bar{d}$ from the point $\{r_j(\tau(s_n^{(j)})), z_j(\tau(s_n^{(j)}))\})$ is determined by formula

$$A_l := \frac{F_j(\bar{s}_0)\bar{d}\cos\alpha}{32 r_j^3(\bar{s}_0)}\left(2\ln\frac{8r_j(\bar{s}_0)}{F_j(\bar{s}_0)} - 1\right) + \frac{1}{4r_j^2(\bar{s}_0)}\left(\ln\frac{8r_j(\bar{s}_0)}{F_j(\bar{s}_0)} - 1\right) +$$
$$+\frac{N_j}{\pi}\left\{C_1\mathrm{i}_1 + C_2\mathrm{i}_2\right\} - C_1\sum_{\substack{\gamma=1\\ \gamma\neq n}}^{N_j}\ln\left[\bar{d}^2 + (s_\gamma^{(j)} - \bar{s}_0)^2\right] - C_2\sum_{\substack{\gamma=1\\ \gamma\neq n}}^{N_j}\frac{1}{\bar{d}^2 + (s_\gamma^{(j)} - \bar{s}_0)^2}.$$

Let's note that $C_1 := -\frac{1}{r_j^2(\bar{s}_0)}\left[\frac{F_j(\bar{s}_0)\bar{d}\cos\alpha}{32 r_j(\bar{s}_0)} + \frac{1}{8}\right]$,

$$C_2 := \frac{\bar{d}\cos\alpha}{2r_j(\bar{s}_0)F_j(\bar{s}_0)},$$ i

integral i_1 is defined by (18), while

$$\mathrm{i}_2 = \int_{-1}^{1}\frac{1}{\sqrt{1-s^2}}\cdot\frac{1}{\bar{d}^2 + (s - \bar{s}_0)^2}ds = \frac{\sqrt{\xi_3 + \sqrt{\xi_3^2 + \xi_4^2}}}{|\bar{d}|\sqrt{2}\sqrt{\xi_3^2 + \xi_4^2}}\pi, \qquad (19)$$

where

$$\xi_3 := \bar{d}^2 - \bar{s}_0^2,\ \xi_4 := 2\bar{d}\,\bar{s}_0,\ |\bar{d}| \ll 1,\ -1 \le \bar{s}_0 \le 1$$

Angle α is obtained from the equations

$r'_{js}(\tau(\bar{s}_0)) = F_j(\tau(\bar{s}_0))\sin\alpha$ and
$z'_{js}(\tau(\bar{s}_0)) = F_j(\tau(\bar{s}_0))\cos\alpha$

If point of observation $(\bar{r}, \bar{z})$ belongs to k-cell, then

$$B_k := \frac{1}{16\pi^2 r_k^2}\left\{\ln\frac{128 r_k}{\Delta r_k^2 + \Delta z_k^2} + 1 - \frac{\Delta r_k}{\Delta z_k}\operatorname{arctg}\frac{\Delta z_k}{\Delta r_k} - \frac{\Delta z_k}{\Delta r_k}\operatorname{arctg}\frac{\Delta r_k}{\Delta z_k} + \frac{1}{8r_k}\right\}.$$

Let's define the $E_{\bar{z}}(\bar{r}, \bar{z})$, when $\bar{r} \neq 0$. We assume

$$E_{\bar{z}}(\bar{r},\bar{z}) = \sum_{\substack{m=1\\ m\neq l}}^{P} A_m q_m + A_l q_l - \sum_{k=1}^{\bar{n}} B_k \bar{q}_k,$$

where

$$A_l := \frac{F_j(\bar{s}_0)\bar{d}\sin\alpha}{32 r_j^3(\bar{s}_0)}\left(1 - 2\ln\frac{8r_j(\bar{s}_0)}{F_j(\bar{s}_0)}\right) + \frac{N_j}{\pi}\left\{C_3\mathrm{i}_1 + C_4\mathrm{i}_2\right\} -$$
$$-C_3\sum_{\substack{\gamma=1\\ \gamma\neq n}}^{N_j}\ln\left[\bar{d}^2 + (s_\gamma^{(j)} - \bar{s}_0)^2\right] - C_4\sum_{\substack{\gamma=1\\ \gamma\neq n}}^{N_j}\frac{1}{\bar{d}^2 + (s_\gamma^{(j)} - \bar{s}_0)^2},$$

$$A_m := \frac{(\bar{z} - z_m)\,\mathrm{E}(k_m)/k_{1,m}}{\left[(r_m + \bar{r})^2 + (z_m - \bar{z})^2\right]^{3/2}},$$

$$B_k := \frac{1}{2\pi^2} \cdot \frac{(\bar{z} - z_k)\,\mathrm{E}(k_k)/k_{1,k}}{\left[(r_k + \bar{r})^2 + (z_k - \bar{z})^2\right]^{3/2}},$$

$$C_3 := \frac{F_j(\bar{s}_0)\bar{d}\sin\alpha}{32 r_j^3(\bar{s}_0)},\ C_4 := -\frac{\bar{d}\sin\alpha}{2 r_j(\bar{s}_0) F_j(\bar{s}_0)}.$$

Integrals $\dot{\mathrm{I}}_1$ and $\dot{\mathrm{I}}_2$ are determined from (18) and (19) respectively.
When $(\bar{r},\bar{z}) \in V_k$, coefficient

$$B_k := \left[128\ \pi^2\ r_k^3\right]^{-1}$$

Granting the notation relative to the dimensional values we obtain turned down dashes the equation of motion in dimensionless form

$$\begin{cases} \dfrac{d^2\bar{r}}{dt^2} = \dfrac{1}{2} \cdot \dfrac{\partial u(\bar{r},\bar{z})}{\partial \bar{r}} = -\dfrac{1}{2} \cdot E_{\bar{r}}(\bar{r},\bar{z}), \\ \dfrac{d^2\bar{z}}{dt^2} = \dfrac{1}{2} \cdot \dfrac{\partial u(\bar{r},\bar{z})}{\partial \bar{z}} = -\dfrac{1}{2} \cdot E_{\bar{z}}(\bar{r},\bar{z}). \end{cases}$$

Let's consider a four-component vector

$Y(t) = \{Y_1(t), Y_2(t), Y_3(t), Y_4(t)\}$. Here
$Y_1(t) := \bar{r}(t),\ Y_2(t) := V_{\bar{r}}(t),\ Y_3(t) := \bar{z}(t),\ Y_4(t) := V_{\bar{z}}(t).$

Where $V_{\bar{r}}$ and $V_{\bar{z}}$ are the components of velocity vector. Then the equation of motion assumes the form

$$\frac{dY(t)}{dt} = g(Y), \qquad Y(0) = Y^{(0)}, \qquad (20)$$

at that components of vector-function $g(Y)$ are determined as

$$g_1(Y) := Y_2, \quad g_2(Y) := -0{,}5 \cdot E_{\bar{r}}, \quad g_3(Y) := Y_4, \quad g_4(Y) := -0{,}5 \cdot E_{\bar{z}},$$

while $Y^{(0)} = \{\bar{r}_{0p}, 0, \bar{z}_{0p}, 0\}$. Functions $E_{\bar{r}}$, $E_{\bar{z}}$ are defined by (13) and (14) respectively.
The equation (20) we solve by the predictor-corrector-type method of Adams-Bashfort-Multon of fourth order, formulas of which for *j*-equation of system ($j = 1, \ldots, 4$) hold

$$\tilde{y}_{i+1}^{(j)} = y_i^{(j)} + \frac{\Delta t}{24}\left[-9g_{i-3}^{(j)} + 37g_{i-2}^{(j)} - 59g_{i-1}^{(j)} + 55g_i^{(j)}\right],$$

$$y_{i+1}^{(j)} = y_i^{(j)} + \frac{\Delta t}{24}\left[g_{i-2}^{(j)} - 5g_{i-1}^{(j)} + 19g_i^{(j)} + 9g_{i+1}^{(j)}\right],$$

$i = 4, 5, \ldots$

a predictor and corrector respectively, where

$g_{i+1}^{(j)} := g^{(j)}(t_{i+1}, \tilde{y}_{i+1}^{(\gamma)}, \gamma = 1,\ldots,4)$, while others

$$g_i^{(j)} := g^{(j)}(t_i, y_i^{(\gamma)}, \gamma = 1,\ldots,4), \quad t_{i+1} = t_i + \Delta t,\ t_1 = 0.$$

Let's note that

$$g_i^{(1)} := y_i^{(2)}, \quad g_i^{(2)} := -0{,}5 \cdot E_{\bar{r}}(y_i^{(1)}, y_i^{(3)}),$$
$$g_i^{(3)} := y_i^{(4)}, \quad g_i^{(4)} := -0{,}5 \cdot E_{\bar{z}}(y_i^{(1)}, y_i^{(3)}).$$

In addition, to define $y_i^{(j)}$ it is necessary to solve all equations for arbitrary *i*. Antecedent fourth values $y_1^{(j)}, y_2^{(j)}, y_3^{(j)}, y_4^{(j)}$, $j = 1,\ldots,4$ we determine by Runge-Kutta method of fourth order.

Determination of the Current Density and Space Charge Density: Now let's define the initial data for current tubes. Since the beam goes in spaciousness *V* through the emitting surface, we divide it into n^e segments with square S_p, $p = 1,\ldots,n^e$. The coordinates of certain point of each segment we take as a start of trajectory, which is identified with current tube. To this end cathode assumes the form

$$r^e := r_1(\tau),\ \ z^e := z_1(\tau),\ \ \tau_1^{(1)} \le \tau \le \tau_2^{(1)},\ \ 0 \le \varphi \le 2\pi.$$

Solving the equation (12) integration by the cathode was accomplishing by quadrature formula with $N^e := N_1$ points (let N^e be odd). Let's divide the set of abscissas $\{s_p^e := -s_k^{(1)}\}, k = 1,\ldots,N_1$, which is determined by (16), into two subsets:

a. subset $\{s_{2p-1}^e\}, p = 1,\ldots,\overline{N^e}$, $\overline{N^e} := [N^e/2] + 1$, from which the coordinates of start points for charge particles is defined as

$$r_p^e := r_1(\tau(s_{2p-1}^e)), \quad z_p^e := z_1(\tau(s_{2p-1}^e));$$

b. subset $\{s_{2p}^e\}, p = 1,\ldots,\overline{N^e} - 1$, with help of which we divide the cathode into elementary segments ΔS_p^e, square S_p of which is determined by formula

$$S_p = \pi F_1(\tau(s_{2p-1}^e))\left\{r_1^2(\tau(s_{2p}^e)) - r_1^2(\tau(s_{2(p-1)}^e))\right\},$$

where $\quad s_0^e := -1,\ s_{2\overline{N^e}}^e := 1.$

Thus the quantity of current tubes is $\overline{N^e}$, current of *p*-tube assumes the dimensionless form

$$I_p = J_p S_p = \frac{4}{9} D^{-1} \frac{u_p^{3/2}}{\bar{d}_0^2} S_p, \qquad (21)$$

where u_p is a potential in the point $(\bar{r}_{0p}, \bar{z}_{0p})$, which is taken as a start of trajectory and located on the distance $\bar{d}_0$ from the cathode (the point (r_p^e, z_p^e)) Although ρ is singular onto cathode surface, total charge per unit of area between emitter and the point $(\bar{r}_{0p}, \bar{z}_{0p})$ is finite and equals (in dimensionless form) to

$$\bar{q}_p = \frac{1}{3} E_{0p} = \frac{4}{9} \cdot \frac{u_p}{\bar{d}_0}, \quad p = 1, \dots, \overline{N^e} \tag{22}$$

Let's calculate total charge in cells V_k. Let $\vec{r}_\xi$, $\vec{r}_{\xi+1}$ be two points of certain trajectory, which are obtained by solving the equation of motion during the time Δt_ξ. In the interval of tube between those points there is a charge $\bar{q}_{p\xi} = I_p \Delta t_\xi$. Therefore total cell charge

$$\bar{q}_k = \sum_p I_p \Delta t_{pk},$$

where the summarizing is accomplished by the all trajectories passing through V_k; Δt_{pk} is a dwell time of p-ray in this cell. In detail, segment $[\vec{r}_\xi, \vec{r}_{\xi+1}]$ is divided into $\bar{N}$ sections

$$[\vec{r}_{\xi,\upsilon}, \vec{r}_{\xi,\upsilon+1}],\ \upsilon = 0, \dots, \bar{N}-1,\ \ \vec{r}_{\xi,0} := \vec{r}_\xi,\ \ \vec{r}_{\xi,\bar{N}} := \vec{r}_{\xi+1},$$

such as having supposed the straight-line uniformly accelerated motion between them charge particle passes during the same time $\Delta t_\xi / \bar{N}$. Then a charge $q_{p\xi_\upsilon} = I_p \Delta t_\xi / \bar{N}$ belonging to one of them is referred to the cell, central point (r_k, z_k) of that is the nearest to the midpoint $\vec{r}_{\xi,\upsilon+1/2}$ with coordinates determined as

$$\vec{r}_{\xi,\upsilon+1/2} := \vec{r}_\xi + \frac{\dot{\vec{r}}_\xi \Delta t_\xi}{\bar{N}} \left(\upsilon + \frac{1}{2}\right) + \frac{\vec{r}_{\xi+1} - \vec{r}_\xi - \dot{\vec{r}}_\xi \Delta t_\xi}{2 \bar{N}^2} \left(\upsilon + \frac{1}{2}\right)^2.$$

At the end it is necessary to add a charge existing between a start point of trajectory and cathode segment to the charge of cells located near the emitting surface (22).

Iterative Procedure Including Nachamkin-Hancock Convergence Acceleration: Up to this point we have determined all components to build iterative algorithm. Let's note the following scheme solving self-consistent problem.

1. Initialization of values:
   $n = 0$ – a number of iteration; $\bar{q}_k^{(0)} = 0$ – a value of space charge in k-cell, $k = 1, \dots, \bar{n}$; $\alpha_0 = 1$ – a solution of the equation (24); $\omega_0 = 1$ – a parameter of relaxation; the net making for potential distribution; initiation of current tubes and calculation of squares of cathode segments S_p;
   $J_p^{(0)} = 0$, $I_p^{(0)} = 0$– a current density and current of p-tube, $p = 1, \dots, \overline{N^e}$.
2. Determination of potential $u_L = u^{(0)}$ in the Laplace field (without space charge under given boundary conditions): specification of coefficients of matrix A; definition of right part of the system (15) as $\bar{u}_l = \Phi_j$; the solving of the system (15) by the Gauss method; retention of result $q_m^{(0)}$; calculation of the potential distribution in the net points by (17); calculation of the potential in the points $(\bar{r}_{0p}, \bar{z}_{0p})$.
3. Calculation of current density $\hat{J}_p^{(n)}$ of each tube according to the Child-Langmuir law (21).
4. Computation of trajectories of charge particles in the field $u^{(n)}$.
5. Evaluation of the stream-handling potential $u^{(n)}$ in the points $(\bar{r}_{0p}, \bar{z}_{0p})$ by (17) using the stream-handling values of $q_m^{(n)}$ and $\bar{q}_k^{(n)}$.
6. Determination of new approaches of current density
   $$\tilde{J}_p^{(n+1)} = \alpha_n \hat{J}_p^{(n)},\ J_p^{(n+1)} = \omega_n \tilde{J}_p^{(n+1)} + (1 - \omega_n) J_p^{(n)},\ \omega_n = \frac{2}{3}$$
   for $n > 0$, (23)

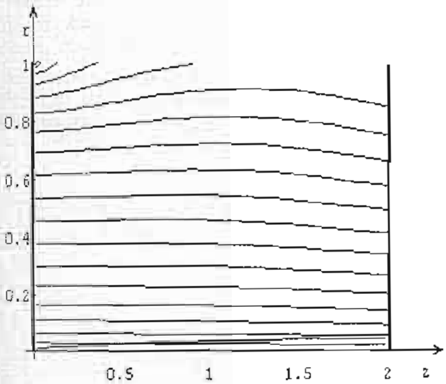
   Current value $I_p^{(n+1)} = J_p^{(n+1)} S_p$ and space charge
   $$\bar{q}_k^{(n+1)} = \sum_p I_p^{(n+1)} \Delta t_{pk}.$$
7. The reckoning of the potential $\hat{u}^{(n+1)}$ under homogeneous boundary conditions and stream-handling space charge value: "restoration" of matrix A; definition of right part of the system (15) under condition that $\tilde{\Phi}_l = 0$; the solving of the system (15) by the Gauss method, let $\hat{q}_m^{(n+1)}$ be a result of it; calculation of the potential $\hat{u}^{(n+1)}$ in the points $(\bar{r}_{0p}, \bar{z}_{0p})$ by (17).
8. Determination of $\hat{J}_{p\max}^{(n)} = \max_p \left| \hat{J}_p^{(n)} \right|$
9. Estimation of positive root β_{n+1} of the equation
   $$\beta_{n+1}^3 \hat{u}^{(n+1)}(\bar{r}_{0p\max}) - \beta_{n+1}^2 u^{(n)}(\bar{r}_{0p\max}) + u_L(\bar{r}_{0p\max}) = 0 \tag{24}$$
   by the secant method with preliminary root localization.

Table 1: Axial potential of the plane parallel Diode (Obtained, Analytical and Laplace Values)

z	0,01	0,02	0,03	0,04	1,97	1,98	1,99
U	0,0820	0,2162	0,3864	0,5832	97,8823	98,6445	99,3838
U_a	0,0855	0,2154	0,3699	0,5429	98,0050	98,6689	99,3339
U_l	0,3164	0,6729	1,0369	1,4048	98,0703	98,7673	99,4422

Table 2: Numerical Results for the Plane Parallel Diode

N	α_n	$\hat{u}^{(n+1)}(\bar{r}_{20})$, V	δI_{20}, %	J, A/m^2
0	1, 00000			
1	0, 64940	− 0, 71354	3680, 78113	118, 49262
2	0, 85068	− 0, 71164	29, 44633	131, 03863
3	0, 92957	− 0, 71502	20, 41091	120, 50894
4	0, 97401	− 0, 70362	11, 66781	112, 86931
5	0, 98641	− 0, 70552	7, 60845	110, 24809
6	0, 99299	− 0, 70629	4, 51973	109, 78882
7	0, 99696	− 0, 70583	2, 77484	109, 66590
8	0, 99856	− 0, 70580	1, 77665	109, 66216
9	0, 99911	− 0,.70610	1, 13185	109, 50877
10	0, 99945	− 0, 70627	0, 70959	109, 46186



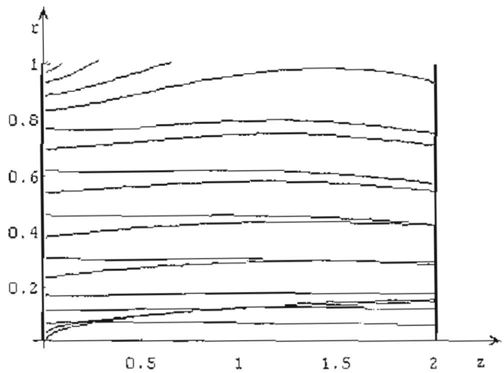


Fig. 1:The Distribution of Trajectories into Plane Parallel Diode Without (left) and Within (right) Space Charge Effect

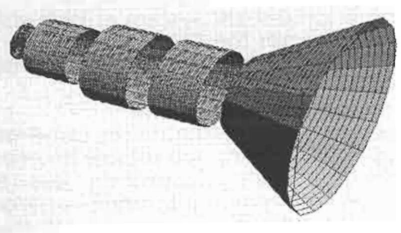


Fig. 2: Five-Electrode Optoelectronic System

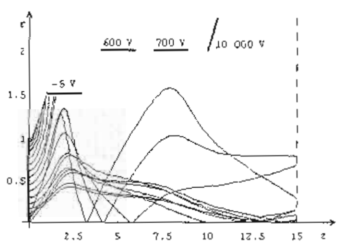


Fig.3: The Distribution of Trajectories into Five-Electrode Optoelectronic System

10. Evaluation of new field $u^{(n+1)}$ by (17) in the net points, assuming

$$q_m^{(n+1)} := q_m^{(0)} + \alpha_{n+1}\, \hat{q}_m^{(n+1)}, \quad \overline{q}_k^{(n+1)} := \alpha_{n+1}\, \overline{q}_k^{(n+1)},$$
$$u^{(n+1)} := u_L + \alpha_{n+1}\, \hat{u}^{(n+1)}, \qquad (25)$$

where $\alpha_{n+1} = \beta_{n+1}^3$.

11. Verification of the criterion (7), if it is not satisfied then repeat stages 3–10, assuming $n := n+1$.

Numerical Experiments: To approve proposed algorithm we define distributions of current density and potential of plane parallel diode with unlimited emissive capability of cathode(Kirstein,1970). As cathode and anode we take disks with radius equal to 1 mm. The distance between them is 2 mm and anode potential is 100 V. Dimensionless values are obtained under $L_0 := 10^{-3}$, $u_0 := 1$, $D := 1$.

Algorithm has been used for twenty current tubes with initial parameter of relaxation $\omega_0 := 0{,}2305$ (next values of parameter were $\omega_n := \omega_0/\alpha_1 = 0{,}3549$), the number of space charge cells equal to 2000, the potential net made by 8000 points. Trajectories have been computed under the start distance $\overline{d}_0 := 0{,}0125$ and time interval $\Delta t := 0{,}01$. Iterative process with $\varepsilon = 0{,}01$ (7) converges in 10 iterations. Maximal relative error of axial potential value is 11 % (see table 1) and error of average current density in tubes, trajectories of which passed fifth part of inetrelectrode space, is 1,4 %.

Table 2 with respect to the number of iteration shows the potential drop $\hat{u}$ in twentieth tube (its current density is maximal) relatively to the Laplacian potential equal to $u_L(\vec{r}_{20}) = 1,85285$ V, relative error of current in this tube and average current density. The space charge affects essentially, Fig. 1 is a graphic evidence of it.

Let's estimate some five-electrode optoelectronic system (Fig. 2), which consist of cathode, controlling electrode (modulator), accelerating and focusing electrodes and anode with potentials in 0, – 5, 600, 700, 10 000 V respectively.

Algorithm has been used for twenty current tubes. Iterative process with parameters of relaxation $\omega_0 := 0.05996$ and $\omega_n := 0.3$ converges in 30 iterations. The potential drop is equal to 27 from 101 V near the cathode in tube with maximal current density. The distribution of trajectories is represented on Fig. 3. Because of large picture size anode is not shown in full. Since the analytical solution of this problem is unknown, obtained results carry out uncritical character, but indicate the effectiveness of the developed approach.

Let's consider the ion generator, in which ion flow is arising at the expense of powerful electric field from plasma gas discharge. Let plasma boundary be clear and not penetrated in for electric field. In this model plasma boundary is an emitter with permanent potential and current density along it. The temperature and density of the corresponding plasma component define the last. During the estimation the value of thermal charge particle velocity is not took into account due to the practical devices using low-temperature plasma. At that in stationary conditions the normal component of total electric field (external and space charge fields) has to be equal to zero.

Let's consider an example of real problem devoted to plasma boundary formation(Ellyn, 1985 and Ostudin and Romanenko, 2002). The modeled electron-ion optical system of high-voltage glowing discharge is shown on Fig. 4. The plasma is located near the anode and has the same potential $u = 0$. Here the plasma boundary is an emitter of singly positive ions of molecular nitrogen, flying to cold cathode, being under potential $u = -20000$ V. The electron beam is emitted from the bombarded cathode segment. It penetrates through the plasma. The electron temperature and ion density of in-anode plasma we take equal to $T_e \approx 3$ eV, $n_i \approx 10^9$ cm^{-3}. By the Beaum's formula

$$J_i = 0{,}43\, n_i\, e \sqrt{\frac{2kT_e}{m_i}},$$

where k is Boltzmann constant and m_i is an ion mass, ion current density of plasma boundary $J_i = 5 \cdot 10^{-5}$ A/cm^2. In addition we assume that current density at sections bombarded by the electron beam is tenfold more ($J_i = 5 \cdot 10^{-4}$ A/cm^2) due to the plasma warming up. The electron space charge density is supposed to be neglect due to the ion velocity is much less than of electron. Formation of plasma boundary is determined by the condition that Child-Langmuir law takes place on the small distance $\overline{d}_0$. This means that initial ion velocities and normal potential derivative onto plasma surface are equal to zero. In contrast to (25), iterative process is built without parameter α_n due to the each iteration plasma boundary changing. As a result Laplace potential is changing too. Because of it stream-handling potential is defined at once. Whole scheme solving this problem is described in (Ostudin and Romanenko, 2002).

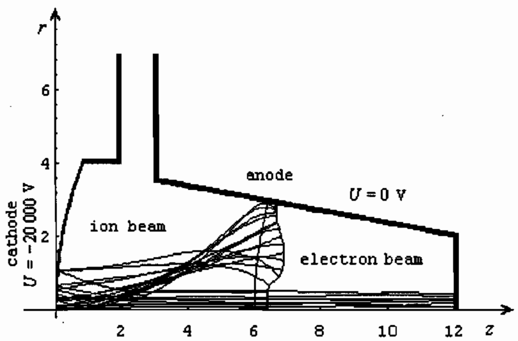


Fig. 4: The Electron-ion Optical System of High-voltage Glowing Discharge: the Distribution of Ion and Electron Beams, the Initial and Obtained Plasma Boundary Formation

Iterative process for twenty current tubes converges in 11 iterations. Fig. 4 shows obtained plasma boundary. The corresponding boundary formation has been determined as an interpolar Lagrange polynomial constructed by the points initial for ion trajectories. Results are shown to be in good agreement with experimental data (Ellyn, 1985). The distribution of ion and electron flow trajectories and the plasma boundary deflection toward the anode are to be evidence of it.

Conclusion

Up to this point the numerical approach of heavy-current self-consistent problems of electronic optics has been staying a complicated task and prepared recommendations for all problems of this kind have not existed. That is why the subject matter of the effective algorithm's development has actuality in this branch due to the principle possibility of its application in microelectronic technology and laser physic problems, in plasma physic problems as for infliction of the high quality covering, in glowing discharge problems. Proposed algorithm together with corresponding mathematical substantiation needs the further development in essential space case and electro-magnetic field. Those tasks have to be carried out next.

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