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Analysis of the Classical Equations of Induction Motor by the Program Maple

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Abstract: Electric machines theory starts in the nineteenth century. And many of their studies have some approximation and simplification. Therefore, in electrical machines theory there are approximative equations, but they are in use until now. So it is important to be insure that such classical equations are enough accurate. In this paper with use of the program Maple, conclusion of equations of the induction motor torque, the maximum torque, and the critical slip and comparison of obtained outcomes with classical equations are carried out.

Key words: Electrical machines, induction motor, classical torque equations, equivalent circuit

Introduction

The continued development of large, high-speed digital computers has brought about a change in the relative importance in various techniques in the solution of electrical machines. Digital computer solution depends upon system equations. So it is important to understand the formulation of the equations from which, in obtaining a solution, the program that followed by the computer is derived.

Today many different programs are widely used. The program packets, which are the most widely used, are Mathematica, Mathcad, Matlab, Maple, Macsyma, and Dirive [Diaconov, 2001, Diaconov, 2002, C. D'Aapice, 2000]. While in the past such programs or computers were not available. So some of theory studies of many disciplines have many approximation and simplification. Therefore, in electrical machines theory there are such approximative equations.

Many of electrical machines studies are done on the basis of their equivalent circuits. One of the most common equivalent circuits for the induction motor is illustrated in Fig. 1, the IEEE-recommended equivalent circuit [Sen P.C., 1997], where

r_1 ! The one-phase resistance of the stator windings;
x_1 ! The one-phase leakage reactance of the stator windings;
r_2t ! The rotor circuit resistance (for one phase), referred to the stator side;
X_2t ! The leakage reactance of the rotor circuit (for one phase), referred to the stator side;
x_m! The magnetizing reactance caused by mutual inductance between stator and rotor windings;
S ! The slip.

The classical formula for the maximum electromagnetic torque of induction motor has the form [Tokareov, 1989, Stephen, 2001]:

$$J_{max} = \frac{3V^2}{2T_s c_1\left(r_1 \pm \sqrt{r_1^2 + (x_1 + c_1 x_2)^2}\right)} \tag{1}$$

where

V − The root mean squire value of the stator phase voltage;
T_s − The angular velocity of the stator magnetic field;

$$c_1 = \frac{\sqrt{r_1^2 + (x_1 + x_m)^2}}{x_m}$$

And the classical formula for the critical slip has the form [Sen P.C., 1997, Tokareov, 1989, Lukhachov, 2002]:

$$S_{max} = \pm \frac{c_1 r_2}{\sqrt{r_1^2 + (x_1 + c_1 x_2)^2}} \tag{2}$$

And for induction motor torque calculations the classical theory of electrical machines recommends the simply formula:

$$J = \frac{2J_{max}(1 + b\,S_{max})}{\frac{S}{S_{max}} + \frac{S_{max}}{S} 2bS_{max}} \tag{3}$$

where $b = \frac{r_1}{r_2 c_1}$

And there is another more simplified formula for this torque [Kluchev, 1985]

$$J = \frac{2J_{max}}{\frac{S}{S_{max}} + \frac{S_{max}}{S}}$$

And for getting more accurate equations, the induction motor theory will be used. The induction motor equations in *x, y, 0* coordinates (Eliseev, 1983).

$$\frac{d8_{x1}}{dt} = v_{x1} - i_{x1} r_1 + 8_{y1} T_s\ ;$$

$$\frac{d8_{y1}}{dt} = v_{y1} - i_{y1} r_1 + 8_{x1} T_s\ ;$$

$$\frac{d8_{x2}}{dt} \; ' \; ! i_{x2} r_2 \% 8_{y2} T_s ! 8_{y2} T_2 ;$$

$$\frac{d8_{y2}}{dt} \; ' \; ! i_{y2} r_2 \% 8_{x2} T_s ! 8_{x2} T_2 ;$$

$$i_{x1} \, ' \, \frac{8_{x1} T_s}{x_1 (1 ! k_s k_r)} \; ! \; \frac{8_{x2} T_s k_r}{x_1 (1 ! k_r k_s)};$$

$$i_{x2} \, ' \, \frac{8_{x2} T_s}{x_2 (1 ! k_s k_r)} \; ! \; \frac{8_{x1} T_s k_s}{x_2 (1 ! k_r k_s)};$$

$$i_{y1} \, ' \, \frac{8_{y1} T_s}{x_1 (1 ! k_s k_r)} \; ! \; \frac{8_{y2} T_s k_r}{x_1 (1 ! k_r k_s)};$$

$$i_{y2} \, ' \, \frac{8_{y2} T_s}{x_2 (1 ! k_s k_r)} \; ! \; \frac{8_{y1} T_s k_s}{x_2 (1 ! k_r k_s)};$$

$$J \, ' \, \frac{3}{2} \; p \; \frac{x_m}{T_s} \; (i_{x2} i_{y1} \; ! \; i_{x1} i_{y2});$$

$$\frac{dT_2}{dt} \, ' \, \frac{J ! J_1}{J} p$$

where 8_{x1}, 8_{y1}, 8_{x2}, 8_{y2} - Flux linkages of the stator and rotor circuits along the *x* and *y* axes respectively;

T_2 ! The angular velocity of the rotor;

J ! The moment of inertia of the motor and the load;

J_1 ! The load torque;

v_{x1}, v_{y1} ! The applied voltages of the stator winding along the *x* and *y* axes respectively;

$i_{x1}, i_{x2}, i_{y1}, i_{y2}$! The stator and rotor currents along the *x* and *y* axes respectively;

p ! The number of pair of poles;

$$k_s \, ' \, \frac{x_m}{x_1} \; ; \quad k_r \, ' \, \frac{x_m}{x_2}$$

These equations can be rewritten as follows:

$$[L].\ D.\ [I]=[\,!A]+[V] \qquad (5)$$

where

D - Differentiation operator;

$$[I]=[i_{x1},\ i_{y1},\ i_{x2},\ i_{y2}];$$

$$[L] = \begin{bmatrix} x_1+x_m & 0 & x_m & 0 \\ 0 & x_1+x_m & 0 & x_m \\ x_m & 0 & x_1+x_m & 0 \\ 0 & x_m & 0 & x_1+x_m \end{bmatrix};$$

$$[V] = [\sqrt{2}\ V\ \ 0\ \ 0\ \ 0];$$

$$[L] = \begin{bmatrix} r_1 & -(x_1+x_m) & 0 & x_m \\ x_1+x_m & r_1 & x_m & 0 \\ 0 & S x_m & r_2 & S(x_2+x_m) \\ x_m & 0 & S(x_2+x_m) & r_2 \end{bmatrix};$$

By using the program packet Maple the equation (5) was solved. And the electromagnetic torque of induction motor is

$$J = \frac{3}{4}\ \frac{V^2.S.x_m^2.r_2}{\left(s.r_1.X_2+r_2.X_1\right)^2 + \left[s.(X_1.X_2-x_m^2)-r_1.r_2\right]^2}$$

where $X_1 = x_1 + x_m$; $X_2=x_2 + x_m$.

And the extreme value of the last function is at the critical slip:

$$S_{max} = \pm \frac{c_1.c_2.r_2}{\sqrt{r_1^2+(x_1+c_2 x_2)^2}} \qquad (6)$$

So the maximum electromagnetic torque of the induction motor will be

$$J_{max} = \frac{3.V^2}{2\,T_s.\left(r_1 \pm \frac{c_1}{c_2}\sqrt{r_1^2+(x_1+c_1 x_2)^2}\right)} \qquad (7)$$

And finally the electromagnetic torque of induction motor may be rewritten in the form

$$J = \frac{2.Jt_{max}\left(1+bt\ St_{max}\right)}{\frac{S}{St_{max}}+\frac{St_{max}}{S}2\,bt\,St_{max}} \qquad (8)$$

where $bt = \frac{r_1}{r_2.c_1}$.

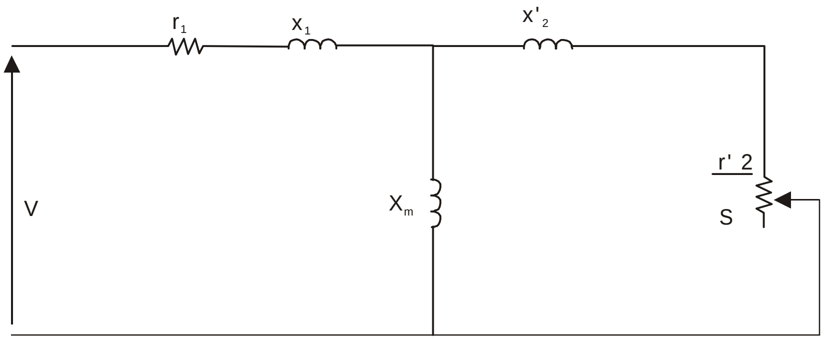

Fig. 1: The equivalent circuit of induction motor

If make a comparison between the classical equations for induction motor torque (3) and (8), which was gotten by using the results of solving the induction motor equations by program Maple, it is clear that both equations have the same form, but there is some difference. The difference is between the critical slip for both cases, and also between the constants b and b'. Motors, which power is from three up to one hundred kW have the following parameters (Sen, P.C., 1997) in unitesG[1].

$x_m=2\div3.5$; $x_1=x_2=0.07\div0.15$; $r_1=r_2=0.02\div0.06$

Substituting these values in the classical and in the obtained equations gives about (3.5÷7.5)% difference in the values of constants b and b' and gives about 1% difference in the value of the maximum torque.

The made comparison study between the classical equations of induction motors and the more exact equations given by solving the differential equations system for induction motors using program Maple proves that the classical equations used in electrical machines theory are enough accurate.

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