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Genetic Design of Fuzzy Mapped PID Controllers for Non-linear Plants

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Abstract: The technique of genetic algorithms is proposed as a means of designing fuzzy gain-scheduled PI control schemes for a class of non-linear plants where the non-linearity is a function of the plant output. It is shown that the use of genetic algorithms for this purpose results in highly effective fuzzy gain-scheduled control systems. These results are illustrated by genetically designing a fuzzy gain-scheduled controller for a highly non-linear discrete-time single-input single-output model.

Key words: Genetic algorithms, fuzzy gain-scheduled control, non-linear plants

INTRODUCTION

Historically, the control engineer has designed a control system with the aim of achieving a desired performance at some nominal operating point about which the plant has been locally linearised. Indeed, the use of this locally linearised model has been one of the central themes in the development of control systems theory over the last half century. This design problem has led to the almost universal development of the fixed-gain PID controller, for which numerous tuning techniques exist[1-3]. However, a much more daunting task is to design a control scheme for a non-linear plant such that the controller performs satisfactorily throughout the operating envelope. Since this task is generally unachievable with a fixed set of controller gains, the problem is often transformed into tuning the plant at a number of operating points throughout the operating envelope and then scheduling these gains against variables which correlate to the plant non-linearity. The design of the gain-schedule becomes even more difficult in the case where the non-linearity is dependent on more than one variable. Furthermore, in such cases the design process is often lengthy as a large number of operating points have to be explored. This problem motivates the consideration of deploying automatic techniques for both searching the operating envelope and designing the appropriate locally-linearised controller for incorporating into the fuzzy gain-scheduled control scheme.

One such technique for the automatic tuning of locally-linearised plants has been proposed by Ajlouni[3]. The technique involves the use of genetic algorithms[4-7]. In fact, it was demonstrated[3] that genetic algorithms provide a much simpler approach to the tuning of PID controllers than the rather complicated non-genetic optimisation algorithms proposed by Polak and Mayne[8], Gensing and Davidson[9]. Moreover, the genetic technique has been extended to embrace unmodelled plants by Jones and Tatnall[10] and multivariable plants by Porter *et al.*[11] and non-linear plants by Jones and Ajlouni[12]. Indeed, Jones and Ajlouni[12] describe how to design gain-scheduled controllers using mathematical functions to describe the non-linear gain-scheduled profile. This technique does require the formal definition of the function prior to using the genetic algorithm to find the parameters of the function. One way of avoiding this problem is to define the gain-schedule profiles by fuzzy rules.

In this paper, the results of Jones and Ajlouni[12] are extended to embrace the tuning of fuzzy gain-scheduled controllers for non-linear plants. The technique is shown to be totally autonomous, other than choosing the number of fuzzy sets to be used in the fuzzy gain schedule. Furthermore, the resulting fuzzy gain-scheduled controller can be tuned to any performance measure, such as minimum rise time or minimum integral of square error. Indeed, the use of genetic algorithms for the design of such fuzzy gain-scheduled controllers can be viewed as replacing the trial and error techniques of the control engineer, with an evolutionary design approach which leads naturally to optimal results. In addition, the practical implementation of such genetically designed fuzzy gain-scheduled controllers is discussed by considering the suitability of both incremental and absolute forms of the PID controllers for inclusion in fuzzy gain-scheduled controllers. In order to demonstrate the performance of

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genetically designed fuzzy gain-scheduled controllers, simulation results are presented of a non-linear system in which the plant output effects plant dynamics. The transient responses of the fuzzy gain-scheduled controller is shown as the plant moves through its operating envelope. Finally, these results are contrasted with the results of an optimal fixed gain controller.

Genetic algorithms: This section outlines the operation of a basic genetic algorithm (GA) and represents the GA adopted in this study. A basic GA consists of five components. These are a random number generator, a "fitness" evaluation unit and genetic operators for "reproduction". "crossover" and "mutation" operation. The algorithm is summarized in Fig. 1a.

The initial population required at the start of the algorithm, is a set of number strings generated by the random generator.

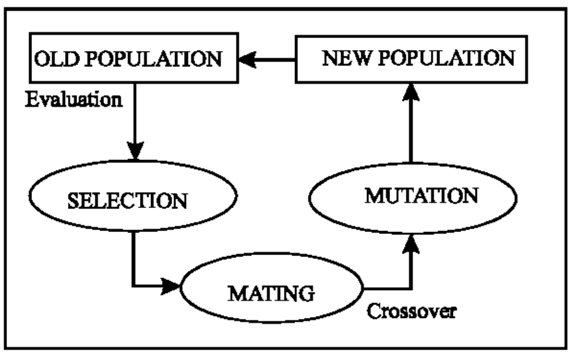


Fig. 1a: The basic genetic algorithm cycle

Each string is a representation of a solution to the optimization problem being addressed. Binary strings are commonly employed. Associated with each string is a fitness value as computed by the evaluation unit. A fitness value is a measure of the goodness of the solution that it represents. The aim of the genetic operators is to transform this set of strings into sets with higher fitness values.

The reproduction operator performs a natural selection function known as "seeded selection". Individual strings are copied from one set (representing a generation only solutions) to the next according to their fitness values, the higher the fitness value? the greater the probability of a string being selected for the next generation.

The crossover operator chooses pairs of strings at random and produces new pairs.

The simplest crossover operation is to cut the original "parent" strings at a randomly selected point and exchange their tails. The number of crossover operations is governed by a crossover rate. This operation is shown in Fig. 1b.

The number of mutation operations is determined by a mutation rate. A phase of the algorithm consists of applying the evaluation, reproduction, crossover and mutation operations. A new generation of solutions is produced with each phase of the algorithm.

Parent 1	1 0 0 0 1 1	0 1 1 1 1 0	tail
Parent 2	0 1 1 0 1 1	0 0 0 1 1 0	tail

New string 2 0 1 1 0 1 1 0 1 1 1 1 0
New string I 1 0 0 0 1 1 0 0 0 1 1 0

Fig. 1b: Simple crossover operation

Old string 1 1 0 0 0 1 0 1 1 1 0 1
New string 1 1 0 0 1 1 0 1 1 1 0 1

Fig. 1c: Simple mutation operation

Analysis: In the case of digital fuzzy gain-scheduled PI controllers, the plants under consideration are assumed to be governed on the discrete-time set $T_T = \{O,T,2T,....,kT,..\}$ by non-linear state and output equations of the form

$$X_{(k+1)}T = \Phi(T, y_{(k)}\,T)\,X_{(k)}T+\Psi(T,y_{(k)}T)\,u_{(k)}\,T \quad 1$$

and

$$y_{(k)}T = \Gamma X_{(k)}T \quad 2$$

where the state vector $X_{(k)}T\ \varepsilon\ R^n$, the input $u\ \varepsilon\ R^T$, the output $y\ \varepsilon\ R^+$, and n is the number of plant states.

The fuzzy gain-scheduled PI Controllers proposed are governed by control-law equations of the form

$$K_p = f_p\{y_{(k)}T\} \quad 3$$

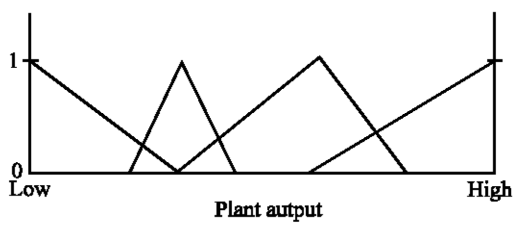
$$K_i = f_i\{y_{(k)}T\} \quad 4$$

$$\Delta u\,(k)T= K_p\,(\Delta e_{(k)}T+Tk_i e_{(k)}T) \quad 5$$

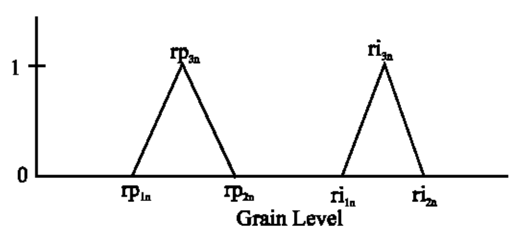
Where $e_{(k)}T = v - y_{(k)}T$ is the error, $v\ \varepsilon\ R^+$ is the set-point. $\Delta e_{(k)}T= e_{(k)}T—e_{(k-1)}T$ is the change in error, $Au_{(k)}T$ is the incremental change in the input, K_p is the proportional gain, K_i is the integral gain, T is the sampling period, and $K_p = f_p\{y_{(k)}T\}$ and $K_i = f_i\{y_{(k)}T\}$ are the fuzzy gain-scheduled profiles for the proportional and integral gains, respectively.

It is interesting to note that by deploying the fuzzy gain-scheduled controller in incremental form, any bumpless transfer techniques associated with the integral state are avoided. This is particularly important in the case of a fuzzy gain-scheduled controller, incorporating integral control because any scheduling of the integral gain would require bumpless transfer of the integral state every time the integral gain was changed.

In such fuzzy gain-scheduled control systems the objective is to design a P1 controller such that good tracking behaviour is obtained across the operating envelope of the plant. In the case of non-linear plants, where the non-linearity is a function of the plant output, one strategy for achieving this goal is to design two sets of fuzzy rules, which schedule the proportional and integral gains, respectively, against the plant output. The fuzzy rules would comprise of fuzzy sets relating the plant output to the proportional and integral gain schedules, respectively. Normally, the fuzzy rules are obtained from operating experience, however, this can prove to be an extremely time-consuming task. This motivates the use of genetic algorithms to obtain the fuzzy rules such that 'optimal' behaviour is exhibited by the overall fuzzy gain-scheduled control system. Indeed, the technique of genetic algorithms in this case provides the only automated solution path. In this case, it is necessary to map the plant output space into a collection of fuzzy sets of the form



and relate each plant output fuzzy set to a fuzzy set representing the proportional and integral gains, respectively of the form



Where rp_{1n}, rp_{2n} and rp_{3n} and ri1n, ri_{2n} and ri_{3n} define the nth fuzzy output sets of the proportional and integral gains, respectively.

To apply the genetic algorithm to this task it is first necessary to encode the fuzzy set of the fuzzy gain-schedule, $\{rp_{11}, rp_{12}, rp_{13}, rp_{21}, rp_{22}, rp_{23,\ldots}, rp_{n1}\, rp_{n2}, rp_{n3}\}$ and $\{ri_{11}, ri_{12}, ri_{13}, ri_{21}, ri_{22}, ri_{23,\ldots}, ri_{n1}, ri_{n2}, ri_{n3}\}$ where n corresponds to the number of fuzzy rules in accordance with a system of concatenated, multi-parameter fixed-point coding[3]. Then each set of fuzzy gain- scheduled controller is represented by a string of binary digits. Then, following initial choice of $\{rp_{11}, rp_{12}, rp_{13}, rp_{21}, rp_{22}, rp_{23,\ldots}, rp_{n1}, rp_{n2}, rp_{n3}\}$ and $\{ri_{11}, ri_{12}, ri_{13}, ri_{21}, ri_{22}, ri_{23,\ldots}, ri_{n1}, ri_{n2}, ri_{n3}\}$, entire generations of such strings can be readily obtained by using the basic genetic operators of selection, crossover, and mutation[3]. In particular, these operators ensure that successive generations of digital fuzzy gain-scheduled controllers thus produced by genetic algorithms exhibit progressively improving behaviour in respect of a fitness measured by a generalised integral square of error (ISE).

Such a performance index is computed by subjecting the non-linear plant to a succession of set-point changes which span the operating envelope of the plant. The generalised ISE is then obtained by adding the individual performance from each set-point change to obtain

$$ISE = \sum_{l=1}^{M} \lambda_i (ISE)^{(i)} \tag{6}$$

Where m is the number of set-point changes and λ_i (i= 1, 2,..., m) is a weighting parameter which can be chosen to increase or decrease the performance of the plant in certain operating points within the operating envelope.

Illustrative example: This procedure for the synthesis of genetically designed fuzzy gain-scheduled controllers can be illustrated by designing a fuzzy gain-scheduled control system for the single- input single-output non-linear plant governed by the following discrete-time equations

$$a_1 = 0{,}99 - 0.006y_{(k)}T$$
$$b_1 = 0.08 - 0.0004y_{(k)}T$$
$$y_{(k+1)}T = a_1 y_{(k)}T + b_1 u_{(k-10)}T \tag{7}$$

and the incremental fuzzy gain-scheduled PI controller given by

$$e_{(k)}T = v - y_{(k)}T \tag{8}$$
$$\Delta e_{(k)}T = e_{(k)}T - e_{(k-1)}T \tag{9}$$

and from equation 3, 4, and 5.

$$K_p = f_p\{y_{(k)}T\}$$
$$K_i = f_i\{y_{(k)}T\}$$
$$\Delta u_{(k)}T = K_p\,[\Delta e_{(k)}T + 0.\,I\,K_i, e_{(k)}T]$$

Firstly, the results obtained by solving the fuzzy gain-scheduled problem by means of a genetic algorithm, such that the integral of square error to set-point changes across the operating envelope is minimised for the case where there are four input sets considered. In this case, a population of 100, a crossover probability, p_c= 0.65 and

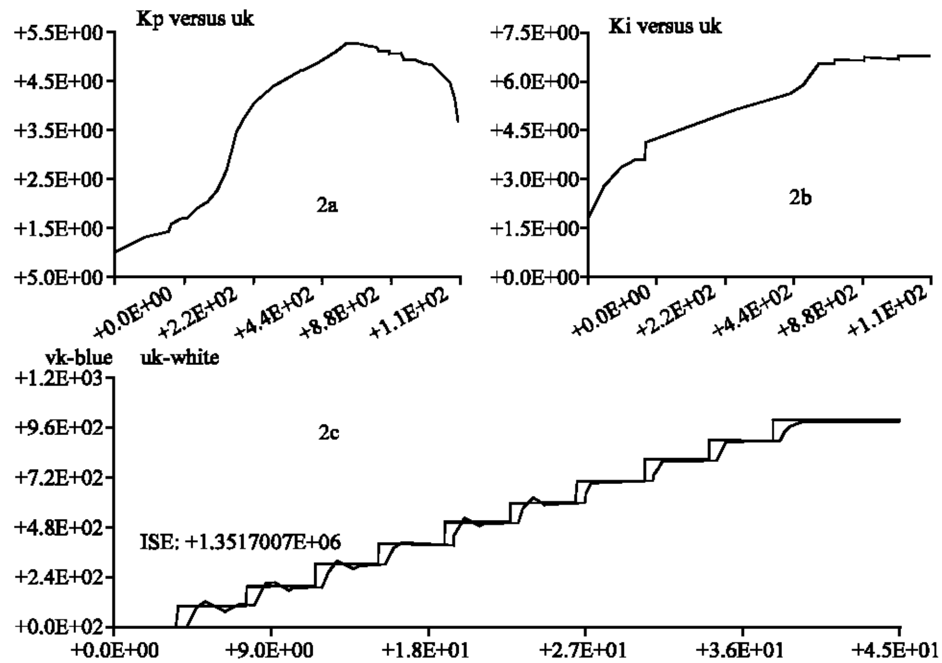


Fig. 2: Genetically designed Fuzzy gain-scheduled controller

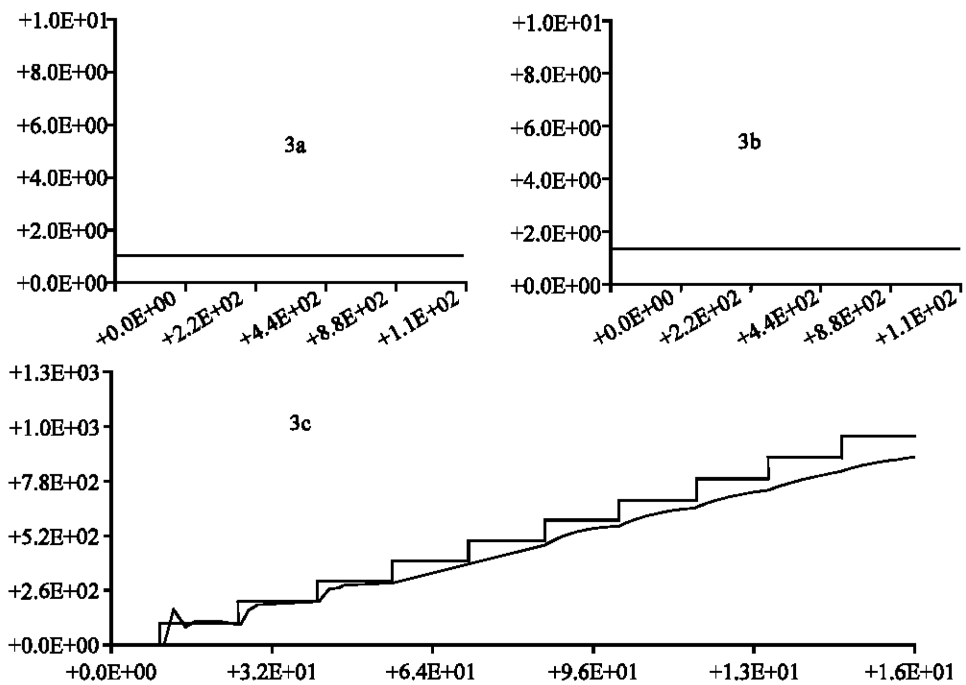


Fig. 3: Genetically designed fixed gain controller

mutation probability of p_m=0.05, was used. Figure 1 shows the resulting genetically designed fuzzy gain-scheduled controller after 500 generations. Figure 2c shows the transient response of the non-linear plant to 100 unit set-point changes in the region 0 —>1000 units, and Fig. 2a and 2b show the resulting fuzzy gain-scheduled profiles for the proportional and integral gains, respectively.

Finally, to contrast the genetically designed fuzzy gain-scheduled controllers, the genetic algorithm was used to design a fixed-gain controller of the form

$K_p = a_0$
$K_i = b_0$

In this case, a population of 100, a crossover probability, $p_c = 0.65$, and a mutation probability, $p_m = 0.005$ was used. Figure 2 shows the genetically designed controller after 500 generations. Figure 3c shows the transient response of the non-linear plant to 100 unit set-point changes in the region 0 —> 1000 units, and Fig. 3a and 3b show the designed fixed-gain profiles for the proportional and integral gains, respectively.

These results clearly indicate the effectiveness of the genetic algorithm to design excellent fuzzy gain-scheduled controllers for non-linear plants, such that optimal behaviour to a train of set-point changes can be obtained. The results also indicate that genetic algorithms can be thus used to design fuzzy gain-scheduled controllers for non-linear plants where hitherto no optimal design techniques existed.

The techniques of genetic algorithms have been proposed as a means of designing fuzzy gain-scheduled controllers for non-linear plants. It has been shown that the use of genetic algorithms for this purpose greatly facilitates the design of such controllers such that an integral square error to set-point changes across the operating envelope of the plant is minimised. These results have been illustrated by genetically designing a fuzzy gain-scheduled controller for a discrete-time non-linear plant, and the results contrasted with that of an optimally-tuned fixed-gain controller.

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