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An Elegant and Simple Method to Test the Stability of 2-D Recursive Digital Filters

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Abstract: Several methods for testing stability of first quadrant quarter-plane two dimensional (2-D) recursive digital filters have been suggested in 1970's and 80's. Though Jury's row and column algorithms, row and column concatenation stability tests have been considered highly efficient they still fall short of accuracy if the computational time is a major consideration. In this research we present a very simple method which requires very little computation time particularly when applied to the second order 2-D filter.

Key words: Stability of 2-D recursive filters, simplest test

INTRODUCTION

There was a time when the problem of testing 2-D recursive digital filters transfer function, devoid of non-essential singularities of the second kind, for stability received most attention of several researchers all over the world. Even now some researchers are working in this area (Bistritz, 2002; 2004). Many of the important methods considered very efficient for testing the stability of 2-D filters have been presented by Huang (1981). Recently the researchers have suggested a method (Ramesh and Reddy, 2006) based on evaluation of complex integrals. Though this method requires as much, if not more, computational time, the accuracy of the method is very good since it requires finding whether a particular parameter is positive or negative. In this study we further simplify the computational part of the test and this perhaps is the best of all available methods when applied to second order 2-D recursive digital filters. The method presented in this research as applied to the second order 2-D filters gains importance since the second order digital filters form the building blocks for the design of higher order filters in the cascade or parallel connections reducing coefficient sensitivity and quantization errors.

REVIEW OF 2-D INTEGRAL EVALUATION

Here we review the method suggested by Hwang (1978, 1981) to find out the complex integrals to evaluate the variance both in 1-D and 2-D cases.

1-D case : Let

$$H(z) = \frac{\sum_{i=0}^{N} a_i Z^{-i}}{\sum_{i=0}^{N} b_i Z^{-i}} = \frac{A(z)}{B(z)} \qquad (1)$$

be a stable 1-D transfer function of a recursive digital filters, where B(z) has zeros inside the unit circle. If h(n) is the impulse response which is nothing but the inverse Z-transform of H(z) given in (1) then

$$\sum_{n=0}^{\infty} h^2(n) = \frac{1}{2\pi j} \oint_{|z|=1} H(z)\, H(z^{-1}) \frac{dz}{z} \qquad (2)$$

Hwang (1978) has given a decomposition method to evaluate $\sum_{n=0}^{\infty} h^2(n)$ as follows: Let

$$H(z)\,.\,H(z^{-1}) = \frac{A(z).B(z^{-1})}{B(z).B(z^{-1})} = \frac{P(z)}{B(z)} + \frac{P(z^{-1})}{B(z^{-1})} \qquad (3)$$

Where $P(z) = p_0 Z^N + p_1 Z^{N-1}$ .... $+ p_N$
By equating the like coefficients on both sides in (3) the coefficients p_0, p_1, ...p_N can be obtained and P(z) will be known.
if

$$\frac{P(z)}{B(z)} = \frac{p_0 Z^N + p_1 Z^{N-1} + \dots\dots + p_N}{b_0 z^N + b_1 z^{N-1} \dots\dots + b_N} \qquad (4)$$

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Then

$$\sum_{n=0}^{\infty} h^2(n) = 2\ \frac{p_o}{b_o} \qquad (5)$$

This method given by Hwang is the most efficient to determine the variance in the case of 1-D recursive digital filters. But the decomposition of the type (3) fails when we try to extend it to evaluate the variance of 2-D recursive digital filters (Agathoklis *et al.*, 1980). This is because one cannot solve the set of equations one gets in obtaining a 2-D decomposition of the type (3). So in the 2-D case Hwang (1981) has suggested a modification which works satisfactorily at least in evaluating the variance of lower order 2-D recursive digital filters. It is as follows:

2-D case :

If $$H(z_1,z_2) = \frac{\sum_{i=0}^{N}\sum_{j=0}^{N} a_{ij}\, z_1^{-i}\, z_2^{-j}}{\sum_{i=0}^{N}\sum_{j=0}^{N} b_{ij}\, z_1^{-i}\, z_2^{-j}} = \frac{A(z_1,z_2)}{B(z_1,z_2)} \qquad (6)$$

is the transfer function of a stable 2-D recursive digital filter, then the variance $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n)$ is given by the double integral

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = \frac{1}{(2\pi j)^2}\oint_{|z_1|=1}\oint_{|z_2|=1} H(z_1,z_2)\, H(z_1^{-1}, z_2^{-1})\,\frac{dz_1 dz_2}{z_1\ z_2} \qquad (7)$$

The integral above is known as Parseval's integral. For this case also, Hwang (1981) has suggested the decomposition which is slightly different from that of 1-D case as follows:

$$H(z_1,z_2)\ .\ H(z_1^{-1}, z_2^{-1}) = \frac{A(z_1,z_2).A(z_1^{-1}\, z_2^{-1})}{B(z_1,z_2).B(z_1^{-1}\, z_2^{-1})}$$

$$= \frac{p_1(z_1)\, z_2^{N-1} + p_2(z_1)\, z_2^{N-2} \ldots\ldots\ldots + p_N(z_1)}{b_o(z_1)\, z_2^{N} + b_1(z_1)\, z_2^{N-1} + \ldots\ldots\ldots + b_N(z_1)}$$

$$+ \frac{q_o(z_1^{-1})\, z_2^{-N} + q_1(z_1^{-1})\, z_2^{-(N-1)} + \ldots\ldots\ldots + q_N(z_1^{-1})}{b_o(z_1^{-1})\ z_2^{-N} + b_1(z_1^{-1})\, z_2^{-(N-1)} \ldots\ldots\ldots + b_N(z_1^{-1})} \qquad (8)$$

It has been found that the decomposition of (8) is always possible and the unknown coefficient vector $[q_o(z_1^{-1}), q_1(z_1^{-1}) \ldots\ldots\ldots q_N(z_1^{-1}), p_1(z_1), p_2(z_1) \ldots\ldots\ldots p_N(z_1)]^T$ can be obtained by solving a matrix equation (Hwang, 1981). When once we solve the matrix equation (not given here) for $q_o(z^{-1})$, it has been shown that

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = \frac{1}{2\pi j}\oint_{|z_1|=1}\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}\,\frac{dz_1}{z_1} \qquad (9)$$

For stable 2-D filters, the variance as given in (9) by a 1-D complex integral is always positive. It may be noted that when once we arrive at Eq. (9), the same 1-D method of evaluating the variance as discussed earlier can be followed. It is because that it can be shown that $\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}$ will be such that it is a ratio of two self inversive polynomials. So If

$$\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})} = \frac{A_1(z_1)}{B_1(z_1)} \qquad (10)$$

$B_1(Z_1)$ can be decomposed into a product of two polynomials like

$$B_1(z_1) = B_2(z_1)\, B_2(z_1^{-1}) \qquad (11)$$

$B_2(z_1)$ will be a polynomial having zeros inside the unit circle and $B_2(z_1^{-1})$ will have zeros outside the unit circle.

AN ELEGANT AND VERY SIMPLE TEST FOR THE STABILITY OF 2-D FILTERS

Here we propose a very simple and computationally less time consuming method for testing the stability of 2-D recursive digital filters. Since we are not interested in finding the variance $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n)$, we consider 2-D transfer functions of the form

$$H(z_1,z_2) = \frac{1}{\sum_{i=0}^{N}\sum_{j=0}^{N} b_{ij}\ Z_1^{-i}\ Z_2^{-j}} = \frac{Z_1^N\ Z_2^N}{B(Z_1, Z_2)}$$

That is,

$$H(z_1,z_2) = \frac{Z_1^N\ Z_2^N}{\sum_{i=N}^{0}\sum_{j=N}^{0} b_{ij}\ Z_1^{i}\ Z_2^{j}} \qquad (12)$$

Obviously, we are not considering 2-D transfer functions which have non essential singularities of the second kind. Though the method we are now suggesting is valid for any value of N, here we restrict ourselves to only second order transfer functions where N = 2. Thus the transfer function to be tested for stability is of the form.

$$H(z_1, z_2) = \frac{z_1^2 \, z_2^2}{b_{22}\, z_1^2\, z_2^2 + b_{21}\, z_1^2\, z_2 + b_{20}\, z_1^2 + b_{12}\, z_1\, z_2^2 + b_{11}\, z_1 z_2 + b_{10} z_1 + b_{02}\, z_2^2 + b_{01} z_2 + b_{00}} \quad (13)$$

That is,

$$H(z_1,z_2) = \frac{z_1^2 \, z_2^2}{(b_{22}\, z_1^2 + b_{12} z_1 + b_{02})\, z_2^2 + (b_{21} z_1^2 + b_{11}\, z_1 + b_{01})\, z_2 + (b_{20}\, z_1^2 + b_{10} z_1 + b_{00})} \quad (14)$$

If

$$\begin{aligned}
a &\triangleq (b_{22}\, z_1^2 + b_{12}\, z_1 + b_{02}) \\
d &\triangleq (b_{21}\, z_1^2 + b_{11}\, z_1 + b_{01}) \\
g &\triangleq (b_{20}\, z_1^2 + b_{10}\, z_1 + b_{00}) \\
a^{-1} &\triangleq (b_{22}\, z_1^{-2} + b_{12}\, z_1^{-1} + b_{02}) \\
d^{-1} &\triangleq (b_{21}\, z_1^{-2} + b_{11}\, z_1^{-1} + b_{01}) \\
g^{-1} &\triangleq (b_{20}\, z_1^{-2} + b_{10}\, z_1^{-1} + b_{00})
\end{aligned}$$

Then we can decompose $H(z_1, z_2)\, H(z_1^{-1}, z_2^{-1})$ as

$H(z_1, z_2)\; H(z_1^{-1}, z_2^{-1}) =$

$$\frac{p_1(z_1) z_2 + p_2(z_1)}{a z_2^2 + d z_2 + g} + \frac{q_o(z_1^{-1})\, z_2^{-2} + q_1(z_1^{-1})\, z^{-1} + q_2(z_1^{-1})}{a^{-1}\, z_2^{-2} + d^{-1}\, z_2^{-1} + g^{-1}} \quad (15)$$

like in Eq. (8). Equating the like coefficeints on both sides of (15) we get the following matrix equation (Agathoklis *et al*., 1980; Hwang, 1981).

$$\begin{bmatrix} a & d & g & d^{-1} & g^{-1} \\ 0 & a & d & g^{-1} & 0 \\ 0 & 0 & a & 0 & 0 \\ d & g & 0 & a^{-1} & d^{-1} \\ g & 0 & 0 & 0 & a^{-1} \end{bmatrix} \begin{bmatrix} q_o(z_1^{-1}) \\ q_1(z_1^{-1}) \\ q_2(z_1^{-1}) \\ p_1(z_1) \\ p_2(z_1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

Solving the matrix Eq. (16), we get $q_o(z_1^{-1})$. Then we form $\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}$ as (Ramesh and Reddy, 2006)

$$\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})} = \frac{(aa^{-1} - gg^{-1})}{aa^{-1}(aa^{-1} - gg^{-1}) - d(-dg^{-1}a^{-1} + ad^{-1}a^{-1}) + g(-g^{-1}dd^{-1} + ad^{-2} - aa^{-1}g^{-1} + gg^{-2})} \quad (17)$$

$$= \frac{A_1(z_1)}{B_1(z_1)} \quad (18)$$

where $A_1(z_1)$ and $B(z)$ are self inversive polynomials. $B_1(z_1)$ will be of degree 8 in positive powers of z_1. In general it will be of degree N^2 in both negative and positve powers of z_1 for any N. If the 2-D transfer function $H(z_1,z_2)$ is stable as mentioned in Section II, $B_1(z_1)$ will be decomposible as the product of $B_2(z_1)\, B_2(z_1^{-1})$ with $B_2(z_1)$ having all its zeros inside the unit circle and $B_2(z_1^{-1})$ having its zeros all out side the unit circle. In this case the integral $\frac{1}{2\pi j}\oint_{|z_1|=1} \frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}\frac{dz_1}{z_1}$ can be evaluated and this gives $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n)$.

On the other hand if the transfer function $H(z_1, z_2)$ is not stable we will not be able to decompose $B_1(z_1)$ as $B_2(z_1)\, B_2(z_1^{-1})$ as discussed earlier though $B_1(z_1)$ is a self-inversive polynomial. What will happen is that $B_1(z_1)$, though a self inversive polynomial, will have some zeros on the unit circle and the decomposition will not yield stable $B_2(z_1)$. This makes it impossible to use the 1-D method of Hwang (1978) and obtain the variance of the 2-D filter. It has been shown in (Ramesh and Reddy, 2006) that if we evaluate the integral

$$\frac{1}{2\pi j}\oint_{|z_1|=1} \frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}\frac{dz_1}{z_1}$$

by the residue method we will endup with a negative value for variance thus indicating that the given transfer function $H(z_1,z_2)$ is unstable. We do agree that evaluating the integral by the residue method is very time consuming. Though we programmed it using Matlab the method is computationally very time consuming.

Instead, we now suggest that after getting $B_1(z_1)$ of (18) to test the stability of $H(z_1,z_2)$, simply find out the root distribution of $B_1(z_1)$. If $H(z_1,z_2)$ is unstable some zeros of the self inversive polynomial $B_1(z_1)$ will be on the unit circle or else $B_1(z_1)$ will have negative sign prefixed . Finding the root distribution by any method is very simple and least time consuming. Thus this method is more accurate and less time consuming when compared to all the existing methods of testing $H(z_1,z_2)$ for stability.

SOME EXAMPLES

Here we give four examples Three transfer functions belong to unstable 2-D filters and the other belongs to a stable filter.

Example 1 :

Consider

$$H(z_1,z_2)=\frac{z_1 z_2}{0.5z_1z_2+0.2z_2+0.5z_1+1}=\frac{z_1 z_2}{(0.5z_1+0.2)z_2+(0.5z_1+1)}$$

The matrix equation to be solved for $q_o(z_1^{-1})$ is

$$\begin{bmatrix}(0.5z_1+0.2) & (0.5z_1+1) & (0.5z_1^{-1}+1)\\ 0 & (0.5z_1+0.2) & 0\\ (0.5z_1+1) & 0 & (0.5z_1^{-1}+0.2)\end{bmatrix}\begin{bmatrix}q_o(z_1^{-1})\\ q_1(z_1^{-1})\\ p_1(z_1)\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

Solving for $q_o(z_1^{-1})$, we get

$$\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}=\frac{-2.5z_1}{(z_1+1.8633)\ (z_1+0.5367)}$$

This is the case where $B_1(z)$, though does not have zeros on the unit circle, it is prefixed with negative sign. So $H(z_1,z_2)$ is unstable.

Example 2 : Consider [Huang, 1981, 1, p. 129, b_3]

$$H(z_1,z_2)=\frac{z_1^2\ z_2^2}{z_1^2 z_2^2-1.2z_1^2 z_2+0.5z_1^2-1.5z_1 z_2^2+1.8z_1 z_2-0.75z_1+0.6z_2^2-0.72z_2+0.2718}$$

We identify

$$a = z_1^2-1.5z_1+0.6$$
$$d = -1.2z_1^2+1.8z_1-0.72$$
$$g = 0.5z_1^2-0.75z_1+0.2718$$

and a^{-1}, d^{-1}, g^{-1} are obtained by replacing z_1 by (z_1^{-1}) in a,d and g, respectively. Using a Matlab program for (17)

$$\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}=\frac{0.464z_1^2-1.821z_1+2.7237-1.821z_1^{-1}+0.464z_1^{-2}}{0.0737z_1^4-0.5805z_1^3+2.048z_1^2-4.0903z_1+5.106-4.0903z_1^{-1}+2.048z_1^{-2}-0.5805z_1^{-3}+0.0737z_1^{-4}}$$

So

$$B_1(z_1)=0.0737z_1^8-0.5805z_1^7+2.048z_1^6-4.0903z_1^5+5.106z_1^4-4.0903z_1^3+2.048z_1^2-0.5805z_1+0.0737.$$

It has be found that $B_1(z_1)$ has four zeros on the unit circle. These are

$$0.66250259550876\pm j\ 0.74905961775027$$

and

$$0.92245865178430\pm j\ 0.3860958893180$$

So the transfer function $H(z_1, z_2)$ is unstable.

Example 3:

Consider [Huang, 1981, p.124]

$$H(z_1,z_2)=\frac{z_1^2\ z_2^2}{z_1^2 z_2^2-1.2z_1^2 z_2+0.5z_1^2-1.5z_1 z_2^2+1.8z_1 z_2-0.75z_1+0.6z_2^2-0.72z_2+0.29}$$

The $B_1(z_1)$ for is found to have no zeros on the unit circle. So the filter is stable.

Example 4 :

Consider [Huang, 1981, p. 129 b_2]

$$H(z_1,z_2)=\frac{z_1^2\ z_2^2}{z_1^2 z_2^2-0.75z_1^2 z_2+0.9z_1^2-1.5z_1 z_2^2-1.2z_1 z_2+1.3z_1+1.2z_2^2+0.9z_2+0.5}$$

It has been found by using the matlab program developed by as that

$$\frac{q_o(z_1^{-1})}{b_o(z_1^{-1})}=\frac{A_1(z_1)}{B_1(z_1)}=\frac{A_1(z_1)}{2.7428z^8+8.5549z^7+9.8173z^6+1.5159z^5-5.4042z^4+1.5159z^3+9.8173z^2+8.5549z+2.7428}$$

The root distribution of $B_1(z_1)$ shows that it has two zeros on the unit circle. They are

$$0.84496738998511 \pm j\, 0.53481782866855$$

So the filter transfer function is unstable.

CONCLUSIONS

In this research we have followed the double integral evaluation (Parseval's Integral) approach suggested by Hwang (1981) and presented a simple method which is computationally very efficient to test the stability of second order 2-D recursive digital filter transfer functions. The method boils down to finding zeros of an 8th degree polynomial. It is very accurate unlike the algebraic and mapping methods existing in the literature (Huang, 1981). This method can be used to higher order filters provided we give a simple method to evaluate the determinant of a matrix of symbols.

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