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A Numerically Approximate High-speed Divider for Image Processing Applications

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Abstract: Integer division by nine is a very common operation in image processing applications. The implementation of such division operations is normally performed through standard modules, which are not optimized for this function. In this study, we propose an approximation algorithm that can be used with any pre-existing architecture such as LUT to achieve a speed up. However this is achieved at a cost in accuracy, but this is shown to be acceptable for image processing applications. The performance of the proposed approximation is analyzed by using it in the spatial average filtering of an image corrupted by gaussian noise and by comparing the filtered image with that in which accurate division by nine is used.

Key words: Division algorithm, nonlinear filtering, spatial averaging filter, image processing

INTRODUCTION

In comparison with other basic arithmetic operations, such as addition, subtraction and multiplication - division is considered to be an important operation, which affects the speed and the total performance of digital signal processors. In general, division algorithms are categorized into the following types: digit recurrence, functional iteration, high radix , table lookup and variable latency. An overview of division algorithms is discussed by (Stuart *et al.*, 1997). All these algorithms, except those using lookup tables, rely on recursion with usually complex operations in the loop (Derek *et al.*, 1992). Also the latency of these algorithms is usually high due to the number of iterations required. Therefore, in order to find a faster method, we present an exact formula to perform the division of an integer by nine, via a division by eight. Here, we exploit the nearness of 8 and 9 in the number line to approximate the divide by nine operations via a divide by eight operations. Second, we exploit the fact that eight being a power of 2 has a very simple divide operation in the binary system - right shift by 3.

The basic operation to be done by divide by 9 is probably always an averaging operation on a 3×3 pixel cell. This occurs extensively in 2-D filtering etc. Spatial averaging filters are used in the suppression of gaussian/uniform noise in images (Gonzalez and Woods, 2002). In this technique, a 3×3 square window is centered at the pixel to be processed. The average value of the nine pixels in the window is computed as the filter output. In the average calculation, the maximum value for the dividend may be 2295 in a 3×3 window assuming 8 bits/pixel and the divisor is nine. To demonstrate the quality of our proposed algorithm the spatial filtering of an image corrupted by gaussian noise is performed and compared with the accurate divide by nine operations.

Approximate division by nine through division by eight

Theorem: The quotient Q_9 when a number N is divided by nine is given in terms of the quotient Q_8 and remainder R_8 when N is divided by 8 as:

$$Q_9 = Q_8 - \text{quo}(Q_8, 9) + \text{quo}(9 - (\text{mod}(Q_8,9) + R_8), 9) \quad (1)$$

The remainder R9 is then given as

$$R_9 = \text{mod}(9 - (\text{mod}(Q_8,9) + R_8), 9) \quad (2)$$

Where, we define the operations quo (a,b) and mod (a, b) as the quotient and remainder when we perform the operation a/b.

Proof

Case 1: Let us consider the problem of approximating N, when N is the form of $Q_8 * 8$. The closest approximation of N is $Q_9 * 9$ with,

$$Q_9 = Q_8 - 1 - \text{quo}(Q_8,9) \quad (3)$$

This is obvious, as 9* n - 8*n = n is an arithmetic progression of common difference unity, when n = 1, 2, ….

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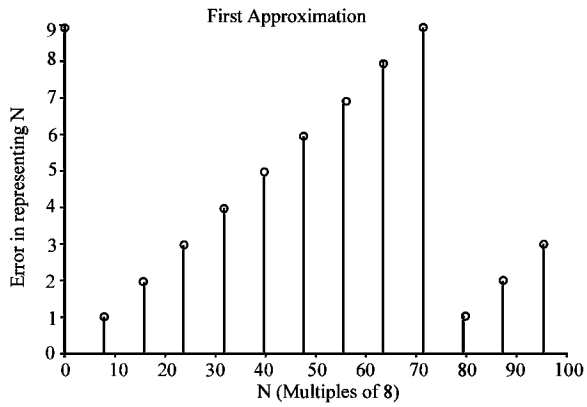


Fig. 1: Approximation case

This may be viewed as an error which may be restricted to 9 by subtracting a term c = 1+ quo(n,9), which is a monotonically increasing, discontinuous function, which increases by one when n is multiple of nine. To get a precise value of the error $e = Q_9 *9 - Q_8 *8$, consider that mod (n, 9) + quo (n, 9) *9 is the overshoot when c is not included. The addition of c*9gives:

$$e = \text{mod}(n, 9) + \text{quo}(n, 9) *9 - c* 9$$
$$= \text{mod}(n, 9) + \text{quo}(n, 9) *9 - (1+ \text{quo}(n, 9)) *9$$
$$e = \text{mod}(n, 9) - 9 \quad (4)$$

The maximum error is $e_{max} = 8$ as shown in Fig. 1.
This is nothing but the remainder $R_9 = 9 - \text{mod}(N,9)$

Case 2: Next is the more general case when N is of the form $Q_8 * 8 + R_8$. We can see that the above formula is no longer the closest approximation as the maximum error by 8 is now $\max(e_{max}) + R_8 = 9 + 7 = 16$ which is greater than 8. We can correct for this by seeing that the undershoot for a set of numbers N having a particular Q_8 is,

$$R_8 - \text{mod}(Q_8, 9)$$

This can be used to correct the above expressions to get

$$Q_9 = Q_8 - \text{quo}(N,9) + \text{quo}(9- (\text{mod}(n,9))+R_8), 9) \quad (5)$$

$$R_9 = \text{mod}(9- (\text{mod}(n,9)+R_8), 9) \quad (6)$$

Which completes the proof. The error graph for the general case is presented in Fig. 2

Performance of the algorithm for image processing: Let Q_a and Q_p denote the exact and the approximated quotients respectively, in the division of integer numbers ranging from 0 to 2259 (for image processing applications) by the divisor nine. The error between the exact and the approximated values is defined as in Eq. 7.

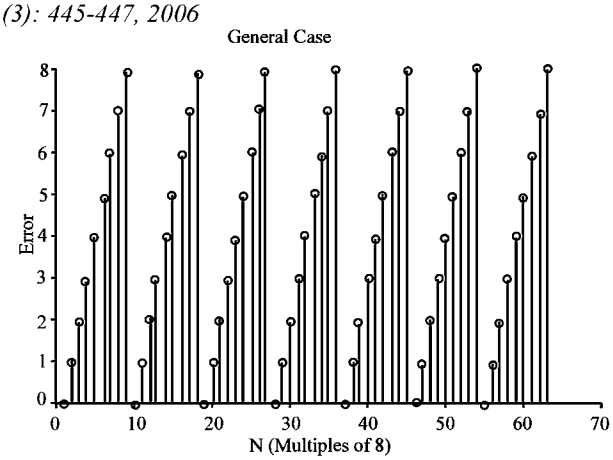


Fig. 2: The error graph for the general case

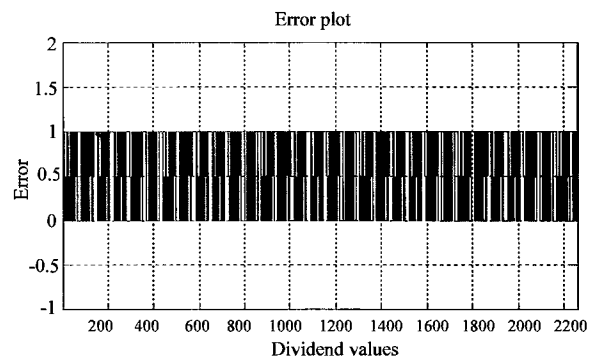
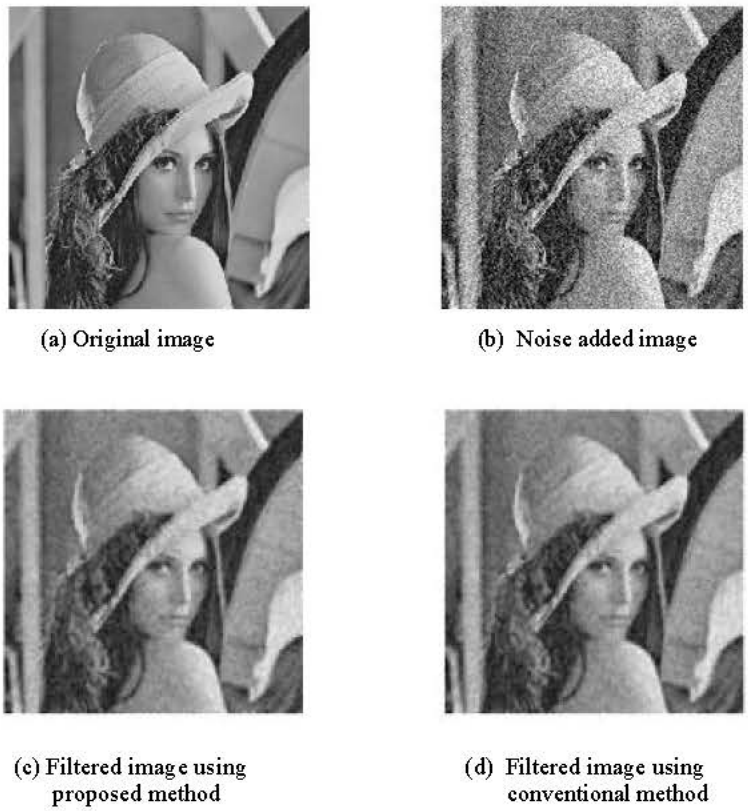


Fig. 3: Error plot

$$E = Q_a - Q_p \quad (7)$$

Figure 3 shows the plot of 'E' w.r.t. Dividend values from 0 to 2259. From the plot it can be seen that the magnitude of the deviation of the approximated quotient from its exact value is 1 of the possible dividend values. This amount of error is insignificant in the averaging technique that is used for suppressing gaussian/uniform noise in images.

The performance of the proposed division by nine approximations for image processing applications is analyzed in MATLAB by considering the spatial averaging (Gonzalez and Woods, 2002) of a 'Lena' image that is corrupted with gaussian noise and impulsive noise separately. A 3×3 square window is used at each pixel position. The gray level sum in each window is calculated. The average value of the pixels within each window is computed using the proposed approximation for division by nine. The center pixel within each window is replaced by the corresponding average value. Figure 4a shows the original 'Lena' image of size 256x256 pixels with 8-bit/pixel. Figure 4b shows the noisy image obtained by adding gaussian noise with zero mean and 1.5% noise variance. Figure 4c shows the filtered image obtained using the proposed approximation in the averaging technique. Figure 4d shows the filtered image obtained



(a) Original image

(b) Noise added image

(c) Filtered image using proposed method

(d) Filtered image using conventional method

Fig. 4: (a) Original 'LENA' image (b) Gaussian Noise added with original image (c) Filtered image using Proposed method (d) Filtered image using Conventional method

Table 1: Comparison of the effectiveness of filtering using the approximation algorithm

	Gaussian noise		Impulsive noise	
Images	PSNR (indB)	MSE (indB)	PSNR (indB)	MSE (indB)
Filtered using Proposed algorithm	25.0485	22.9823	27.0184	21.3124
Filtered using Conventional method	25.4591	22.6717	27.3210	21.1098

using exact division by nine. Table 1 shows the Peak-Signal-Noise Ratio (PSNR) and Mean Square Error (MSE) values for Fig. 4c and d. From the subjective quality of the images shown in Fig. 4c and d and the PSNR and MSE values, it can be seen that the proposed approximation gives almost similar performance but with considerably reduced complexity in computation.

CONCLUSIONS

In this study, a simple algorithm for approximate division by nine via division by eight for image processing applications is proposed. Since eight being a power of 2 has a very simple divide operation in the binary system that is right shift by 3, has simpler implementation. The effectiveness of the proposed approximation is analyzed by using it in the spatial average filtering for suppressing gaussian noise and impulse noise in images. It is found that the subjective quality of the filtered image obtained using the proposed algorithm is almost the same as that obtained with accurate division by nine.

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