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A Quasi-newton Acceleration EM Algorithm for OFDM Systems Channel Estimation

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Abstract: Orthogonal Frequency-division Multiplexing (OFDM) has been proposed to combat severe frequency selective fading in high rate wireless communication system, the Pilot assisted symbol modulation Expectation Maximum (EM) channel estimation algorithm has lower bit error rate compared to the traditional interpolation method, however it has higher complexity and convergence slowly. Aim to the deficiency, a Quasi-Newton Acceleration (QNA) EM algorithm has been proposed to do channel estimation, the new method can reduce the complexity and iteration times and simulation result states, it has still lower BER compared to traditional method

Key words: EM, Quasi-Newton, acceleration algorithm Channel estimation, OFDM, mobile communications

INTRODUCTION

Orthogonal Frequency-division Multiplexing (OFDM), which can transform a frequency-selective fading channel into many parallel flat fading subchannels, is an efficient technique to combat multipath delay spread in high-rate wireless systems. A dynamic estimation of channel is necessary before the coherent demodulation of OFDM signals. In wideband mobile channels, the pilot-symbol-aided channel estimation scheme has been proven to be a feasible method for OFDM system (Meng and Che, 1998). Channel estimation techniques based pilot symbol arrangement in OFDM systems have been analyzed in Necker and Stuber (2004), there have many method, including linear, cubic interpolation, matched filter (Mazet *et al.*, 2002; Necker and Stuber, 2004), Radial Basis Function (RBF), these method has high bit error rate and floor effect, the proposed channel estimation method based on EM algorithm has good performance (Xie and Georghiadis, 2003), but this traditional EM algorithm convergences very slow and has high complexity (Todd, 1996).

We proposed a way to reduced the complexity of the EM algorithm and accelerate the convergence in this study, i.e., the Quasi-Newton Acceleration EM algorithm (QNA EM), which has a quadratic rate of convergence (Mortaza and Jehnich, 1997), although a little performance degraded compared to Traditional EM (TEM), but it is much better that the tradition methods are. And it has fast convergence rate than TEM which has only sublinear convergence. It is a good tradeoff between Bit Error Rate (BER) and rapidity.

CHANNEL SYSTEM MODES

In this study, we use block pilot symbols: a typical inserted with Cyclic Prefix (CP) base band OFDM system. We assume that there are N subcarriers in OFDM system and M subcarriers of them are used to transmit pilot symbols vector P of size $M \times 1$. At the transmitter, the binary source data are converted into $N_p = N - M$ parallel data streams, each of them is modulated by Quadrature Phase Shift Keying (QPSK) and QPSK is used in this study. Pilot symbols $P = [P(0), P(1), \dots, P(M - 1)]^T$ of a length M are equally inserted to the modulated data vector $D_n = [D_n(0), D_n(1), \dots, D_n(N_p - 1)]^T$ and we will attain the n th OFDM symbol vector X_n of size $N \times 1$, where the superscript T denotes the transpose. After modulated by N point IFFT, CP is inserted to prevent ISI. We assume that the length of CP is larger or equal to that of the CIR and there is no ISI in the OFDM systems. The transmitted signal is then sent to a frequency selective time varying channel. Removing CP at the receiver and operating IFFT demodulation the n th received symbol vector Y_n of size $N \times 1$ can be represented as follows:

$$Y_n(k) = H_n(k)X_n(k) + W_n(k) \quad k = 0, 1, \dots, N - 1 \quad (1)$$

and represented as matrix form, for simplicity we omit subscript n in the rest of the study and denote:

$$Y = \text{Diag}(X)H + W = \text{Diag}(X)F_L h + W \quad (2)$$

We assume that the channel is constant in one OFDM symbol period in this study, where,

$h = [h(0), h(1), \dots, h(L - 1)]^T$ is the CIR vector and L is the length of CIR, $H = [H(0), H(1), \dots, H(N - 1)]^T$ is channel frequency response, $H(k)$ is fading factor and phase offset at the k th subcarrier and

$$H(k) = \sum_{l=0}^{L-1} h(l)e^{-j\frac{2\pi}{N}kl} \quad k = 0, 1, \dots, N - 1 \quad (3)$$

F_L is the sub-matrix which is the first L column of Fourier transform matrix of size $N \times N$ and

$$[F_L]_{k,l} = e^{-j\frac{2\pi}{N}kl}$$

$k = 0, 1, \dots, N - 1, l = 0, 1, \dots, L - 1, W = [W(0), W(1), \dots, W(N - 1)]^T$ and $W(k), k = 0, 1, \dots, N - 1$, is additive white Gaussian noise with zero-mean and variance σ^2 .

THE QUASI-NEWTON ACCELERATION METHOD OF EM ALGORITHM

We assume that all subcarriers are used to transmit data symbols. Let \mathfrak{R} denote the set of data symbols, i.e., $D(k) \in \mathfrak{R}, \mathfrak{R} = \{\pm 1, \pm j\}$ for QPSK and 4QAM modulation etc., and let C denote the size of the set L , q_l denote the l th symbol in the constellation, $l = 1, 2, \dots, C$.

Given the CIR vector h and noise variance σ^2 , we let $f(Y, d|H)$ denote the joint conditional probability density function (pdf) of the complete data $\{Y, d\}$. We assume that channel is constant during one OFDM symbol period in this study, so data received at N parallel subcarriers are independent. We have

$$f(Y, d|H) = \prod_{k=0}^{N-1} f_Y(Y(k), X(k)|H(k)) \quad (4)$$

where,

$$f_Y(Y(k), X(k)|H(k)) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{|Y(k) - X(k)H(k)|^2}{2\sigma^2}\right]$$

use the (3), we estimate h , so there are two steps in EM-based iteration algorithm

E-step: $Q(h|h^{(p)}) = E \{\log f(Y, d|h)|h^{(p)}, Y\}$ (5)

M-step: $\{h^{(p+1)}\} = \arg \max (Q(h|h^{(p)}, Y))$ (6)

From (5) and (6), given the received data vector Y , the channel parameters of the p th times iteration can be obtained from the following equation:

$$E \{\log (f(Y, D|h))|h, Y\} = E \{\log (f(Y|h, D) f(D|h))|h, Y\}$$

$$= E \left\{ \log \left(\prod_{k=0}^{N-1} f(Y(k)|X(k), H(k)) \right) |h, Y \right\} + A = -N \log \left(\sqrt{2\pi\sigma^2} \right) - \frac{1}{\sqrt{2\pi\sigma^2}} E \{\xi(h, Y, D) | h^{(p)}, Y\} + A \quad (7)$$

where, A is a constant and

$$\xi(h, Y, D) = \sum_{k=0}^{N-1} |Y(k) - X(k)H(k)|^2 = \|Y - \text{diag}(X)F_L h\|^2$$

Given $h^{(p)}$ and Y , we have

$$E \{\xi(h, Y, D) | h^{(p)}, Y\} = E \{\|Y - \text{Diag}(X)F_L h\|^2 | h^{(p)}, Y\} = Y^H Y - 2R(Y^H Qh) + h^H Ph \quad (8)$$

Let $Q = \text{diag} \{E\{X|h^{(p)}, Y\}\} F_L$
 $P = F_L^H \text{diag} (E \{X^H|h^{(p)}, Y\}) F_L$ (9)

where, $R(\bullet)$ takes the real part of complex in (9) and (X) denotes a diagonal matrix which takes all diagonal elements from X in (10). We must mention that the expectation $E\{\}$ is carried out over d in E-step and maximization is carried over h in M-step, so the function $Q'(h|h^{(p)})$ can be simplified as

$$Q(h|h^{(p)}) = 2R(Y^H Qh) - h^H Ph \quad (10)$$

From (1) and (7) and for $k = 0, 1, L, N - 1$ we have

$$E\{X(k)|h^{(p)}, Y\} = E\{D(k)|h^{(p)}, Y\} = \sum_{l=1}^C q_l \Pr(D(k) = q_l | h^{(p)}, Y) \quad (11)$$

and $E\{|X(k)|^2 | h^{(p)}, Y\} = E\{|D(k)|^2 | h^{(p)}, Y\} = \sum_{l=1}^C |q_l|^2 \Pr(D(k) = q_l | h^{(p)}, Y)$ (12)

where, C is the elements number of QPSK constellation and l is the element index in corresponding constellation. If QPSK modulation is used in OFDM systems and power of data symbols and pilot symbols both are σ_s^2 , we have $E\{|X(k)|^2 | h^{(p)}, Y\} = \sigma_s^2$ and $P = \sigma_s^2 F_L^H F_L$.

From above formula (12) and (13), we need to find the conditional probability $\Pr(D(k) = q_l | h^{(p)}, Y)$. Since $D(k)$ transmitted data symbol at the k th subcarrier is

independent of received data $Y(k')$ and $H(k')$ at the other $k' \neq k$ subcarriers, conditional probability of $D(k)$ can be represented

$$\Pr(D(k) = q_l | h^{(p)}, Y) = \frac{f(Y(k) | h^{(p)}, d(k) = q_l) \Pr(d(k) = q_l)}{f(Y(k) | h^{(p)})} \quad (13)$$

After we find matrix Q , P and $Q(h | h^{(p)})$, the CIR vector and noise variance can be updated in M -step. The CIR vector $h^{(p+1)}$ of the next iteration can be obtained by the following QNA EM, let $g(h^{(p)})$ is the gradient of the (10), so

$$g(h^{(p)}) = \left. \frac{\partial Q(h | h^{(p)})}{\partial h} \right|_{h=h^{(p)}} = 2(Y^H Q - P h^{(p)}) \quad (14)$$

we don't need compute the inverse of P for look for the maximum value, since we the following Quasi-Newton Acceleration Method and the rake scope is much wide the tradition method.

Step 1: Choose initial $h^{(0)}$, constant $\alpha \in (0, 1)$, $B_0 = I$
 $V_0 = \|g(h^{(0)})\|$ $p = 0$

Step 2: If $\|g(h^{(p)})\| = 0$, stop, $h^{(p)}$ is the solution, else goto step 3

Step 3: If $\|g(h^{(p)})\| < \alpha V_p$, choose V_{p+1} satisfy $\|g(h^{(p)})\| \leq V_{p+1} < \alpha V_p$, let $h^{(p+1)} = h^{(p)} - B_p g(h^{(p)})$ goto step 5, else goto step 4.

Step 4: Let $h^{(p+1)} = h^{(p)}$, choose EM step δ_p , let $h^{(p+1)} = h^{(p)} + \delta_p$.

Step 5: For B_p use Broyden symmetry rank one correction formula,

$$B_{p+1} = B_p + \frac{(\Delta_p - B_p \delta_p)(\Delta_p - B_p \delta_p)^T}{(\Delta_p - B_p \delta_p)^T \delta_p}$$

where, $\Delta_p = g(h^{(p+1)}) - g(h^{(p)})$, $\delta_p = h^{(p+1)} - h^{(p)}$

Step 6: Let $p = p + 1$, goto step 2

The convergence of the algorithm is obvious, for $\alpha \in (0, 1)$, $\|g(h^{(p)})\| \leq V_{p+1} < \alpha V_p < \alpha^2 V_{p-1} < \dots < \alpha^p V_0 \rightarrow 0$ and $h^{(0)}$ is obtained by pilot data.

In order to avoid local maxima, the selection of $h^{(0)}$ is important. We use L pilots and apply Least Squares (LS) to obtain the initial channel estimate $h^{(0)} = F_L^H X_p Y$, where F_L is the sub matrix of DFT matrix F corresponding to the L pilots and X_p is a diagonal matrix with diagonal entries the pilot symbols.

NUMERICAL RESULTS

The simulation parameters used in the OFDM is DVB-T parameter. and we assume that the guard interval is larger than the maximum delay spread of length of CIR, so the intersymbol interference can be avoided. Simulations have been carried out at Bit Error Rate (BER) over different SNR and Minimum Square Error (MSE) over the iteration times.

In this study, multipath Rayleigh fading channel is considered and the baseband modeled as followings

$$h_n(l) = \sum_{i=0}^{l-1} \alpha_i(n) e^{j[2\pi f_{D_i} T_n]} \delta(1 - \tau_i)$$

where, l is the total number of propagation paths, $\alpha_i(n)$ is the complex impulse response of the i th path which is complex Gaussian, f_{D_i} is the i th path Doppler frequency

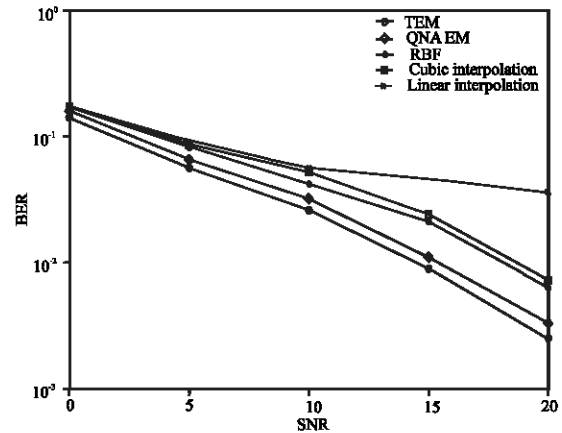


Fig. 1: BER performance with different channel estimation

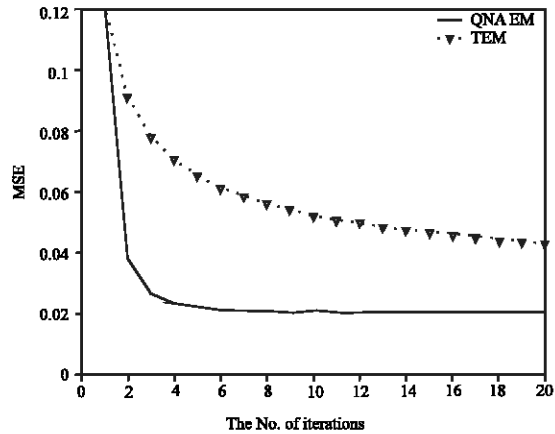


Fig. 2: Iteration number of two EM algorithm

shift, τ_i is delay spread index and l is the i th path delay normalized by the sampling period T , T is the sampling period of time domain for channel. Since we assume that channel is constant during one OFDM symbol period in this study.

From Fig. 1 we can see a QAN EM algorithm has a little performance degrade compared to TEM algorithm, the loss value is about 0.4 dB, but it has still high performance than another tradition method, such linear or cubic interpolation, even RBF channel estimation, From Fig. 2, the, $MSE = \frac{\|h - \hat{h}\|^2}{\|h\|^2}$ we can reach that the iteration time of QAN EM is much less than EM combined TEM algorithm, it convergence faster than the later.

DISCUSSION

In this study, we have applied the quasi-Newton acceleration method of EM algorithm for the channel estimation of OFDM systems with only a few pilots. The proposed acceleration algorithm converges faster than the traditional EM, because the acceleration algorithm has a quadratic rate of convergence, which is much more quick than the traditional EM algorithm and also achieves good performance than the other linear method, just only a little performance loss compared to the traditional EM, It is a good tradeoff between performance and complexity.

The future research should seek a much fast algorithm compared to QAN EM and a good performance is obtained also.

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