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BCH Coding Performance Evaluation on a Land Mobile Channel Based OFDM System

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Abstract: In this study, we evaluate the performance of BCH (Bose-Chaudhuri-Hocquenghem) correcting codes when used to protect data over a land mobile channel using OFDM (Orthogonal Frequency Division Multiplexing) modulation. To deal with memory channels, the Gilbert-Elliott (GE) model was considered to simulate a Rayleigh fading channel and BCH codes to analyse the error process. Relating GE parameters to the physical quantities determining the fading statistics, we simulated the effect of introducing OFDM parameters with respect to the parameters of the channel error probability function (e.g., mobile speed, modulation type, delay constraint and parameters of error correcting codes). Simulation results using OFDM-BPSK modulation shows, for different BCH codes, significant performance.

Key words: OFDM, block coding, rayleigh fading, gilbert-elliott channel, interleaving, land mobile channel

INTRODUCTION

Studies of the performance of error correcting codes are most often concerned with situations where the channel is assumed to be memoryless allowing to simplified theoretical analysis (Gilbert, 1960; Elliott, 1963; Ahlin, 1985). In situation, where memory is accounted for, analytical and result studies are few and obtained via simulation (Gilbert, 1960; Elliott, 1963). The received signal in a mobile digital system is known to display Rayleigh statistics. This Rayleigh fading is characterized in the digital domain by having burst errors. To deal with such a complicated channel model, it is possible to use a less complex one that reflects the essential properties of the complicated one. For a channel with memory, the Gilbert-Elliott (GE) channel is one of the simplest models (Gilbert, 1960; Wang and Moayeri, 1995). GE model provides a useful discrete model where its parameters can be readily related to the statistics of the fade. In this model for a slowly varying channel, the channel is assumed to either be in a good state, where the probability of error is small, or in bad state, where the probability of error is larger. The dynamic of the channel are modeled as a first-order Markov chain, where in Wang and Moayeri (1995) and Krishnamurthi (1997) showed its accuracy for a Rayleigh fading channel and in (Sharma *et al*., 1996) presented a way to match the parameters of GE model to the land mobile channel. However, all studies dealing with GE channel was considered in the case of single carrier modulation and multiple carriers modulation was not considered (Yee and Weldon, 1995; Sharma *et al*., 1996; Wilhelmson and Milstein, 1999). In this study, we consider the performance of different binary BCH codes (Lin and Costello, 1983) using OFDM system (Zou and Wu, 1995; Tarokh and Jafarkhani, 2000; Li and Moon, 2001). We use an interleaved GE channel to evaluate performance for land mobile channel using probability of error as a function of channel parameters, interleaver parameters, error correcting code and type of modulation. We focus in our model on 3 parameters. First to cover a large range of mobile communication, we considered the impact of different values of Doppler frequencies (which reflects the mobile speed) for the choice of a critical threshold SNR in which the channel is in the good state. Second we analysed correlation between block codes length and interleaving depth to show how to keep an acceptable delay constraint in a real situation. The third parameters was the number of carriers used in the OFDM modulator to see how performance is improved with respect to mobile speed, interleaver depth and block codes length.

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INTERLEAVED GE CHANNEL

The GE channel is a first-order, discrete-time, stationary, Markov chain with two states, one good and one bad, denoted G and B. the probability that the channel state changes form G to B and form B to G are denoted by b and g, respectively (Fig. 1).

The probability that the channel is in the good and the bad state at the kth instant of time are denoted by $P^k(G)$ and $P^k(B)$, respectively, with matrix notation $P^k = [P^k(G), P^k(B)]$.

The probability of being in state G at time k, given that the channel is in state B at time 0, will be denoted $P^k(G\backslash B)$.

Let T denote the transition matrix for the channel,

$$T = \begin{bmatrix} 1-b & b \\ g & 1\text{-}g \end{bmatrix} \quad (1)$$

So that

$$P^{k+1} = P^k T \quad (2)$$

From (1) and (2), we can see that how fast the channel is changing from one state to the other depends on b and g. For the channels that we are interested in, the channel is slowly changing compared to the symbol rate and hence b+g<<1.
The stationary distribution is denoted

$$P^{\infty} = [P^{\infty}(G), P^{\infty}(B)]$$

and is found to be

$$P^{\infty} = [g/(b+g), b/(b+g)] \quad (3)$$

Finally, the probabilities of error for the good and bad states are denoted by $P_e(G)$, $P_e(B)$, respectively.

Because of the large degradation of the performance caused by the memory of the channel, a way to improve the performance is to use an interleaver in order to make code symbols less independent (Yee and Weldon, 1995; Wilhelmson and Milstein, 1999).

The interleaver used is a block with m rows and n columns (for block coding, n is equal to block length to avoid 'wrap-around' effect), where the bits that are to be transmitted are fed in row-wise and fed out column-wise.

Then, the corresponding transition probabilities b' and g' for an interleaved GE channel if observed m moments of time later (Wilhelmson and Milstein, 1999), are:

$$b' = P^{\infty}(B)(1-(1-b-g)^m) \quad (4)$$

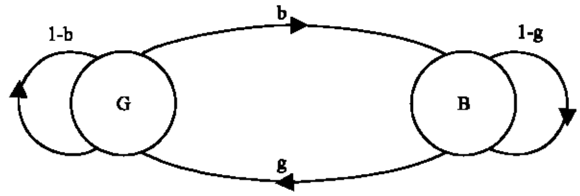

Fig. 1: The Gilbert-Elliott channel model

$$g' = P^{\infty}(G)(1-(1-b-g)^m) \quad (5)$$

From (4) and (5), the effect of the interleaving depth can be clearly seen. The larger value of m, the better the interleaver can be expected to work and if m is infinite, the performance would be the same as for a memoryless channel ($m \rightarrow \infty$, we obtain $b' = P^{\infty}(B)$ and $g' = P^{\infty}(G)$).

If the GE channel is observed at n consecutive instants of time, the probability that the channel is in the bad state d times, $0 \le d \le n$, is given by

$$P_n(d) = \begin{cases} P^{\infty}(G)(P_n(d\backslash GG) + P_n(d\backslash GB)) & \\ + P^{\infty}(B)(P_n(d\backslash BG) + P_n(d\backslash BB)), & 1 \le d < n \\ P^{\infty}(G)(1-b)^{n-1}, & d = 0 \\ P^{\infty}(B)(1-g)^{n-1}, & d = n \end{cases} \quad (6)$$

Here $P_n(d\backslash GG)$ is the conditional probability of being d times in the bad state, conditioned on being in the good state both the first and the last instants of times and the other conditional probabilities are defined accordingly in appendix.

If a t-error correcting block code of length n is used where the interleaving depth is m, the probability of a codeword error is,

$$P_{cw} = \sum_{d=0}^{n} P_n(d) \left[\sum_{i=0}^{d} \left(\binom{d}{i} P_e(B)^i (1-P_e(B))^{d-i} \cdot \sum_{j=\max(0,t+1-i)}^{n-d} \binom{n-d}{j} \cdot P_e(G)^j (1-P_e(G))^{n-d-j} \right) \right] \quad (7)$$

We assume here that d symbols are received when GE channel is in bad state and n-d symbols received when GE channel is in good state.

Indexes i and j denote the number of symbols in error when the GE channel is in the state B and G, respectively (Wilhelmson and Milstein, 1999).

In (7), the parameters b and g in $P_n(d)$ are replaced by using Eq. 3, 4 and 5. In the following section, we describe the way to calculate the GE parameters in function of the statistic characteristics of Rayleigh fading channel (Yee and Weldone, 1995).

MATCHING GE CHANNEL TO LAND MOBILE CHANNEL

There are several works (Ahlin, 1985; Sharma *et al.*, 1996) which give a constructive way to match the GE channel model to a flat Rayleigh fading channel by choosing different matching parameters (level for signal to noise ration, level-crossing rate and arbitrary thresholds).

However, the way that threshold is chosen affect more the accuracy of the model and for this reason the parameters of the GE model are carefully treated.

Assuming that the channel fades slowly with respect to a bit interval the parameters of the model can be related to various physical quantities. The Rayleigh fading results in an exponentially distributed multiplicative distortion of the signal.

Hence, the probability density function of the SNR, λ, is given by (Sharma *et al.*, 1996).

$$f(\lambda) = \frac{1}{\lambda} e^{-\lambda/\lambda_0}, \qquad \lambda \geq 0 \tag{8}$$

Where λ_0 is the average SNR.

The channel is said to be in the good state while the SNR is above a threshold λ_T and once the SNR falls below λ_T the channel goes into the bad state. The stationary probabilities of finding the GE channel in respective states with respect to λ_T are,

$$P^{\infty}(G) = \int_{\lambda_T}^{\infty} f(\lambda)d\lambda = e^{-\rho^2} \tag{9}$$

Where $\rho^2 = -\lambda_T/\lambda_0$. and

$$P^{\infty}(B) = \int_{0}^{\lambda_T} f(\lambda)d\lambda = 1 - e^{-\rho^2} \tag{10}$$

Using the level crossing rate and the SNR density function, the transition probabilities can be found as follows (Sharma *et al.*, 1996),

$$g = \frac{\rho f_d T \sqrt{2\pi}}{e^{-\rho^2} - 1} \tag{11}$$

$$b = \rho f_d T \sqrt{2\pi} \tag{12}$$

Where T is the symbol interval (specified in terms of symbol rate $R_s = 1/T$) and $f_d = vf_c/c$ is the Doppler frequency (maximum Doppler speed), with v the vehicle speed, f_c the carrier frequency and c the light speed.

The error probabilities in respective states in the GE channel are taken to be the conditional error probabilities of Rayleigh fading channel, conditioned on being in the respective state,

$$P_e(G) = \frac{1}{P^{\infty}(G)} \int_{\lambda_T}^{\infty} f(\lambda)P_e(\lambda)d\lambda \tag{13}$$

$$P_e(B) = \frac{1}{P^{\infty}(B)} \int_{0}^{\lambda_T} f(\lambda)P_e(\lambda)d\lambda \tag{14}$$

Where $P_e(\lambda)$ is the symbol error probability for a given value of λ, which depends on the modulation scheme used. We shall concentrate on using OFDM modulation technique.

OFDM MODULATION

Orthogonal frequency Division Multiplexing (OFDM) is a very attractive modulation scheme for data transmission in multipath fading. OFDM can effectively randomise burst errors caused by Rayleigh fading, which comes from interleaving due to the parallelisation. The FFT-based OFDM system is represented in Fig. 2.

Because of dividing an entire channel bandwidth into many narrow subbands (Louet and Glaunec, 2000; Li and Moon, 2001; Tertois and Glaunec, 2002), the frequency response over each individual subband is relatively flat and the distribution of data over many carriers means that the selective fading causes some bits to be in errors. The implementation of an error correcting code make possible to avoid errors by using a forward error correction. Let N be the number of carriers, C_i, i = {0,.., N-1}, the complex information symbols vector and T the OFDM symbol length. The transmitted signal over a symbol duration T is (Louet and Glaunec, 2000; Tarokh and Jafarkhani, 2000; Li and Moon, 2004),

$$S(t) = \mathrm{Re}\left(\sum_{i=0}^{N-1} C_i \exp\left(j2\pi(f_0 + if\ )t\right) \right) \qquad 0 \leq t \leq T \tag{15}$$

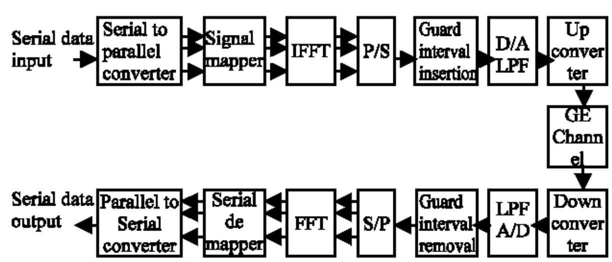

Fig. 2: FFT-based OFDM system

The codeword C_i consists of N symbols chosen from an M-ary modulation method. For MPSK,

$$C_i = e^{j\frac{2\pi}{M}(a_i)} \qquad a_i \in Z_M \tag{16}$$

The duration of an OFDM symbol T is N times the duration of the symbols C_i plus the duration of the cyclic prefix or guard band. The complex envelope of the transmitted signal, sampled at 1/T is,

$$\widehat{S}(n) = \sum_{i=0}^{N-1} C_i \exp(j2\pi ni / N) \tag{17}$$

SIMULATION RESULTS

In order to cover a wide range of mobile communication environments and give different model of fading channel with different degree of correlation, we consider the product f_dT as an independent parameter performed with the values f_dT = 0.001, 0.01, 0.05 and 0.1.

The OFDM scheme based on BPSK type with parameters fixed for ifftsize = 2048, guardtime = 25% of ifftsize (number of samples to use for total guardtime), guardperiod type using half cyclic extension of the symbol and number of carriers N = 512 and N = 800.

Having seen that the GE model can be used to, in an accurate way, to estimate the code error probability for block coded transmission over the land mobile channel using a single carrier BPSK modulation (Wilhelmson and Milstein, 1999), we will now evaluate the effect performance of using multiple carriers BPSK modulation (OFDM) and the effect of the choice of error correcting code over an interleaved GE channel. Figure 3 presents the scheme used.

We consider the case with perfect interleaving (memoryless channel: $m \rightarrow \infty$, $b' = P^{\infty}(B)$, $g' = P^{\infty}(G)$).

Figure 4 and 5 shows the BER plots vs. threshold SNR λ_T for BCH code (7,4,1) for two values of average SNR λ_0, 10 and 20 dB, with different Doppler frequencies. From the figures, it is clear that an increase in f_dT (which reflects the mobile speed) improves the

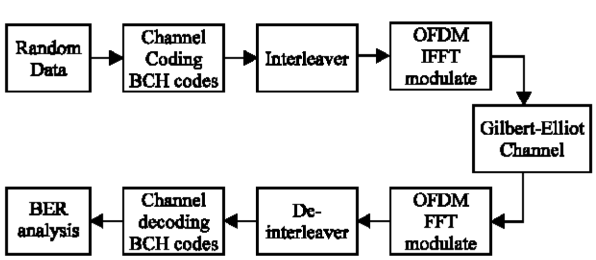

Fig. 3: Simulation system scheme

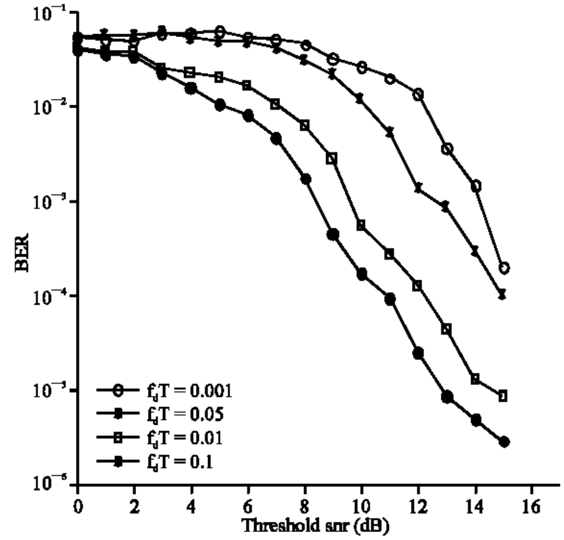

Fig. 4: BER for (7,4,1) BCH code vs. λ_T, λ_0 = 10 dB, N = 512

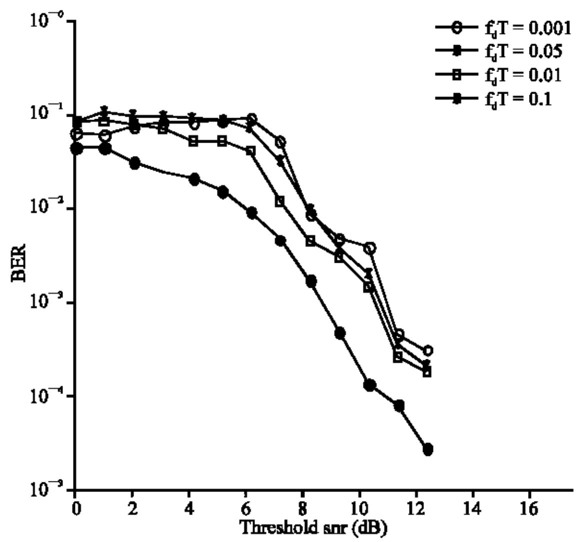

Fig. 5: BER for (7,4,1) BCH code vs. λ_T, λ_0 = 20 dB, N = 512

performance of the BCH code and for large values of f_dT, the errors tend to be more random (independent) as the transition probabilities b and g increase leading to good performance since BCH code is capable to correct such random errors.

In comparison with Fig. 6 (Sharma et *al*., 1996), the use of an OFDM-BPSK system gives a significant BER improvement than a single BPSK modulation.

One can observe from the Fig. 4 and 5, is that the BER is less sensible to small variation of λ_T (which is a critical parameter for GE channel), hence the choice of the exact value of this parameter is not critic for performance. Figure 7 shows the impact of the variation of the average

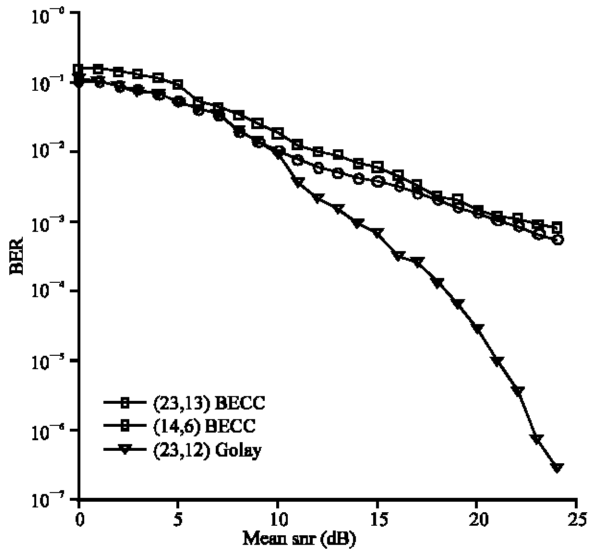

Fig. 6: BER for three codes with BPSK, f_dT = 0.01, λ_T = 10 dB

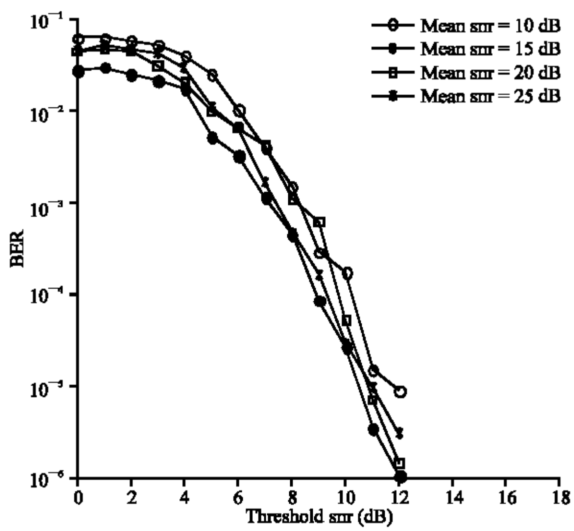

Fig. 7: BER for (15,7,2) BCH code vs. λ_T, f_dT = 0.01, N = 512

SNR λ_0, where we use a BCH (15,7,2) code with approximately the same rate and error capability 2.

In Fig. 7, the BER decrease with the increase of the level of SNR λ_0, the channel tends to stay more in the good state than the bad state(b>>g), the burst errors length is small in this case and the binary BCH code correcting capability is efficacy. We can see more performance using of dm that the case of Fig. 6.

We now compare the performance of different BCH codes for the situation where the interleaving is not perfect due to the delay constraint.

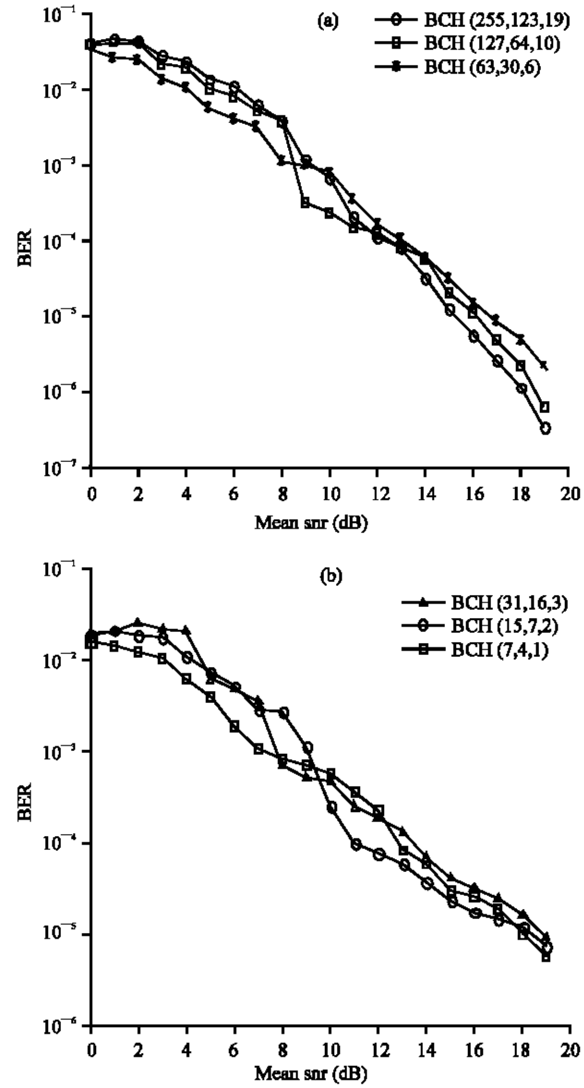

Fig. 8: (a) and (b) BER for BCH code vs. λ_0, λ_T = 10 dB, f_dT = 0.003, N = 512, D = 20 ms

$$\text{Delay} = 2n\varphi\tau \qquad (18)$$

Where φ is the interleaving depth, τ is information rate and n is the block code length. The block codes length with code rate range of 50% used in our simulation are:

Code length	Original code BCH(n,k,t)	Minimal distance	Code rate (%)
7	(7,4,1)	3	57
15	(15,7,2)	5	46
31	(31,16,3)	7	51
63	(63,30,6)	13	47
127	(127,64,10)	21	50
255	(255,123,19)	39	48

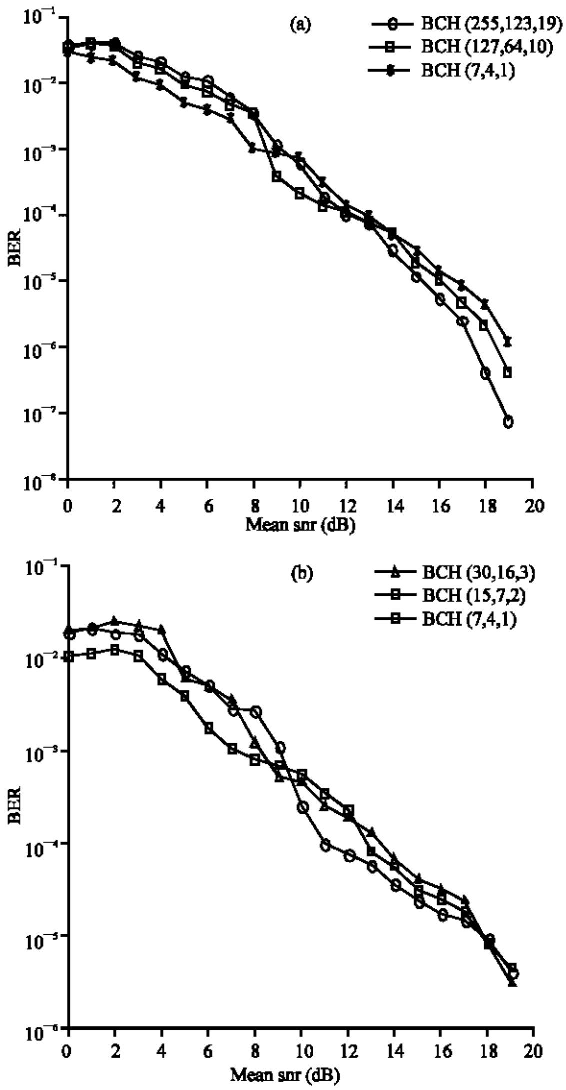

Fig. 9: (a) and (b) BER for BCH code vs. λ_0, λ_T = 10 dB, P_dT= 0.003, N = 800, D = 20 ms

Figure 8 and 9 show that the outstanding performance of the more powerful code is degraded when the threshold SNR λ_T is greater than the average SNR λ_0 and using a more powerful code implies that the block length is increased, therefore the interleaved depth has to be reduced in order to keep an acceptable constraint. As a result, the interleaver is much better for a BCH code with smaller block length. One can observe also from the figures is that the increase of the number of carriers *N* lead to more performance for block code with small length when $\lambda_T \geq \lambda_0$ (burst errors occur when the channel is in the bad state) and once the SNR λ_0 is above the threshold λ_T, the channel is in the good state and the efficacy of block code with large length come evident.

CONCLUSIONS

In this study, we simulated the performance of block code transmission over the land mobile channel (represented by the GE channel model) using OFDM based on BPSK modulation type and considered the impact of several parameters on the performance of BCH codes. The increase in f_dT (which reflects the mobile speed) improves the performance of the BCH code and for large values of f_dT, the errors tend to be more independent as the transition probabilities *b* and *g* increase leading to efficacy correction of random errors.

We analysed the effect of the threshold λ_T on the GE channel and we saw that improving is obtained when λ_0 is large than the threshold λ_T. Using interleaving depth according to the block code length with respect to delay constraint, it was seen that BCH codes with small length give more performance.

APPENDIX

The conditional probabilities are defined as (Wilhelmson *et al.*, 1999),

$$P_n(d/GG) = \sum_{u=2}^{\min(d+1,n-d)} \binom{n-d-1}{u-1}\binom{d-1}{u-2} (1-b)^{n-d-u} b^{u-1} (1-g)^{d-u+1} g^{u-1}$$

$$P_n(d/GB) = \sum_{u=1}^{\min(d,n-d)} \binom{n-d-1}{u-1}\binom{d-1}{u-1} (1-b)^{n-d-u} b^{u} (1-g)^{d-u} g^{u-1}$$

$$P_n(d/BG) = \sum_{u=1}^{\min(d,n-d)} \binom{n-d-1}{u-1}\binom{d-1}{u-1} (1-b)^{n-d-u} b^{u-1} (1-g)^{d-u} g^{u}$$

$$P_n(d/BB) = \sum_{u=2}^{\min(d,n-d+1)} \binom{n-d-1}{u-2}\binom{d-1}{u-1} (1-b)^{n-d-u+1} b^{u-1} (1-g)^{d-u} g^{u-1}$$

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