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New Results with Blind Time Domain Equalization for OFDM System

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Abstract: The Multicarrier Equalization by Restoration of RedundancY (MERRY) algorithm has been shown to blindly and adaptively shorten a channel to the length of the guard interval in an OFDM system. Most published works on blind Time Domain Equalization (TEQ) convergence analysis are confined to T_s-spaced equalizers. The common belief is that analysis of fractionally-spaced TEQ (FSTEQ's) is a straightforward extension with similar results. This belief is, in fact, untrue. In this study, we present a convergence analysis of MERRY fractionally-spaced TEQ's that proves the important advantages provided by the FSTEQ structure. We show that the FSTEQ MERRY algorithm converges significantly faster than the non-fractional TEQ MERRY algorithm. The main reason is that a fractionally-spaced blind adaptive TEQ admits infinitely many realizations of perfect channel shortening for a specific delay whereas a non-fractionally-spaced TEQ admits only one realization. Computer simulation demonstrates the performance improvement provided by the blind adaptive fractionally-spaced TEQ using MERRY algorithm for OFDM system.

Key words: Orthogonal Frequency Division Multiplexing (OFDM), blind, Fractionally-spaced Time-domain Equalizer (FSTEQ), multicarrier modulation

INTRODUCTION

Multicarrier (MC) modulation, such as Orthogonal Frequency Division Multiplexing (OFDM) and discrete multitone (DMT) is commonly adopted in broadband communications as very effective tools to compensate for the time dispersion encountered in wireless channels affected by multipath or in subscriber lines (Wang and Giannakis, 2000). MC systems can easily combat channel dispersion when the channel delay spread is not great than the length of the Cyclic Prefix (CP). However, when the CP is not long enough, the orthogonality of the subcarriers is lost, causing intercarrier and intersymbol interference (ICI and ISI) and a prefilter is needed at the receiver to shorten the effective channel to appropriate length. This prefilter is called a time-domain equalizer (TEQ) (Martin *et al*., 2005; Chow *et al*., 1993; Al-Dhahir and Cioffi, 1996; Melsa *et al*., 1996; Farhang-Boroujeny and Ding, 2001; Arslan *et al*., 2001; Martin *et al*., 2002).

Most channel shortening (or TEQs) schemes in the literature have been designed in the context of ADSL, which runs over twisted pair telephone lines (Al-Dhahir and Cioffi, 1996; Melsa *et al*., 1996; Farhang-Boroujeny and Ding, 2001; Arslan *et al*., 2001). As a consequence, most of the TEQ designs in the literature are trained and nonadaptive, have high complexity.

Recently, blind and adaptive TEQ design has received increasing attention. The Multicarrier Equalization by Restoration of RedundancY (MERRY) algorithm (Martin *et al*., 2002), induces channel shortening by restoring the redundancy in the received data that is due to the CP. The algorithm is low-complexity and converges to the minimum MSE solution (for a white input).

A fractionally-spaced TEQ is a transversal filter whose tap spacing is less than the symbol interval T_s. It was shown in (Ye and Ding, 1996; Chen and Nikias, 1992; Gitlin and Weinstein, 1981; Vaidyanathan and Vrcelj, 2002) that when the tap spacing is less than the reciprocal of twice the highest frequency of the analog channel, a fractionally-spaced equalizer can realize any analog filter, including the best linear receiver and its steady state performance Mean Square Error (MSE) - is insensitive to the timing phase. In this paper, we apply the MEERY algorithm to the blind adaptive fractionally-spaced TEQ and we will examine its converging speed.

SYSTEM MODEL

For simplicity, we will consider the $T_s/2$-spaced TEQ, where T_s denotes the symbol period. The (baseband) discrete-time model of OFDM system with a fractionally-

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spaced TEQ is as follows. Each of the N frequency bins is modulated with a Quadrature Amplitude Modulated (QAM) signal. Modulation is performed via an Inverse Fast Fourier Transform (IFFT) and demodulation is accomplished via an FFT. Channel shortening is performed by a fractionally-spaced TEQ of length $2L_w$ (where L_w is the length of T_s-spaced TEQ) and the FSTEQ output is decimated by a factor of two to create the T_s-spaced output sequence $\{y(k)\}$. Decimation is accomplished by disregarding alternate samples thus producing the T_s-spaced y(k) and the resulting shortened effective channel is equalized by a frequency-domain equalizer (FEQ), a bank of complex scalars. After the cyclic prefix (CP) add, the last ν samples are identical to the first ν samples in the symbol, i.e.,

$$x\ (Mk + l) = x\ (Mk + l + N),\quad l \in \{1, ..., \nu\} \tag{1}$$

where $M = N + \nu$ is the total symbol duration and k is the symbol index. Let the sampling interval be

$$\Delta = \frac{T_s}{2} \tag{2}$$

Then, the sampled channel output becomes

$$r(k\Delta) = \sum_{n=0}^{2L_h-1} x_n \cdot h(k\Delta - n2\Delta) + e(k\Delta) \tag{3}$$

The channel is specified by $T_s/2$-spaced complex valued Channel Impulse Response (CIR) given by

$$h = [\underbrace{h_0\ \ h_1\ \ .....\ \ h_{L_h-1}}_{h_1}\underbrace{h_{L_h}\ ....\ h_{2L_h-1}}_{h_2}]^t \tag{4}$$

with L_h corresponding to the T_s-spaced CIR length and $T_s/2$-spaced sample $e(k\Delta) = e_R(k\Delta) + je_I(k\Delta)$ is an complex Gaussian white noise with, $E\,[e_R^2(k\Delta)] = E\,[e_I^2(k\Delta)] = \sigma_e^2$ where E[.] denotes the expectation operator.

The oversampled channel output $r(k\Delta)$ (Ye and Ding, 1996) can be divided into two subsequences

$$r_k^{(i)} \triangleq r\left[(2k+i)\Delta\right] = r(kT_s + i\Delta), \qquad i = 1,2. \tag{5}$$

By defining subchannel impulse responses as

$$h_k^{(i)} \triangleq h\left[(2k+i)\Delta\right] = r(kT_s + i\Delta), \qquad i = 1,2 \tag{6}$$

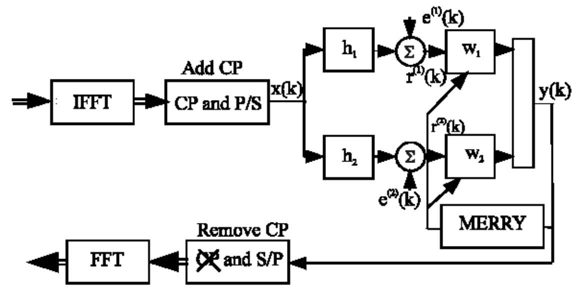

Fig. 1: Multichannel model of OFDM system with a fractionally-spaced time domain equalizer

the two subsequences can be written as

$$r_k^{(i)} = \sum_{n=0}^{2L_h-1} x_n \cdot h_{k-n}^{(i)} + e_k^{(i)} \qquad i = 1,2 \tag{7}$$

These two subsequences can viewed as stationary outputs of two discrete FIR channels

$$H_i(z) = \sum_{k=0}^{L_h} h_k^{(i)} z^{-k}$$

with a common input sequence x_k.

The vector representation of the FSTEQ is shown in Fig. 1.

One adjustable filter is provider for each subsequence $x^{(i)}_k$. Thus, the actual TEQ is a vector of filters

$$W_i(z) = \sum_{k=0}^{L_w} w_k^{(i)} z^{-k} \qquad i = 1,\ 2 \tag{8}$$

The two stationary filter output $\{y^{(i)}_k\}$ are summed to form the stationary TEQ output

$$y_k = \sum_{i=0}^{2} y_k^{(i)} \tag{9}$$

Define the FSTEQ parameter vector as

$$w = [w_0^{(1)}\ \ .....\ \ w_{L_w}^{(1)}\ w_0^{(2)}\ \ ....\ \ w_{L_w}^{(2)}]^t \tag{10}$$

The T_s-spaced TEQ output y(k) is given by

$$y(k) = \sum_{n=0}^{2L_w-1} w_n \cdot r(k-n) = w^t r(k) \tag{11}$$

The TEQ model Eq. 11 forms the basis for discussion of the blind adaptive FSTEQ-MERRY algorithm.

BLIND FRACTIONALLY-SPACED MERRY ALGORITHM

The idea behind MERRY (Martin *et al*., 2002) is that if the channel length $L_h+1 \le \nu$, then the last sample in the CP should match the last sample in the symbol. The MERRY cost function reflects this goal:

$$J_\delta = E[\,|y(\nu+\delta) - y(\nu+N+\delta)|^2\,] \tag{12}$$

where $\nu \in \{0,....,M\text{-}1\}$ and the symbol synchronization parameter δ represents the desired delay. Minimizing Eq. 12 minimizes the energy of the channel outside of a length-ν window.

In T_s-spaced TEQ setting, the MERRY adapts $w^{(Ts)}$ as follows

For symbol k = 1,2,….. and For tap $j = 0,1\ldots..L_w\text{-}1$.

$$\begin{aligned} w_j^{(T_s)}(k+1) = w_j^{(T_s)}(k) - \mu.[y^{(T_s)}(M\cdot k+\nu+\delta) \\ - y^{(T_s)}(M.k+\nu+N+\delta)].[r^*(\nu+\delta-j) \\ - r^*(\nu+N+\delta-j)] \end{aligned} \tag{13}$$

where μ is a small positive adaptive gain and $r^*(.)$ is the complex conjugate of $r(.)$.

Similarly, in a fractionally-spaced ($T_s/2$-spaced) TEQ, by using (11), we have the FSTEQ coefficient updating formula:

For symbol k=1,2,….. and For tap $j=0,1\ldots..2L_w\text{-}1$.

$$\begin{aligned} w_j^{(T_s/2)}(k+1) = w_j^{(T_s/2)}(k) - \mu.[y^{(T_s/2)}(M.k+\nu+\delta) \\ - y^{(T_s/2)}(M.k+\nu+N+\delta)].[(r^{(T_s/2)})^*(\nu+\delta-j) \\ - (r^{(T_s/2)})^*(\nu+N+\delta-j)] \end{aligned} \tag{14}$$

CONVERGENCE OF FSTEQ MERRY ALGORITHM

MERRY minimizes the sum of the energy outside of a length-ν window plus the energy of the filtered noise, subject to a constraint (e.g.,$\|w\| = 1$). Now we proved the global convergence of gradient descent of (12). Define

$$\begin{aligned} r_j^{(T_s/2)} &= [\, r(j) \;\; r(j-1) \;\ldots\ldots\ldots\; r(j-2L_W+1)]^t \\ (r_j^{(T_s/2)})^t &= r_j^{(T_s/2)} - r_{j+N}^{(T_s/2)} \end{aligned} \tag{15}$$

Adding a Lagrangian constraint, the cost function (12) becomes

$$\begin{aligned} J_\delta &= E\left[\,|y(\nu+\delta) - y(\nu+N+\delta)|^2\right] + \lambda(1\text{-}w^h w) \\ &= E\left[\,|w^t r_{\nu+\delta} - w^t r_{\nu+\delta+N}|^2\right] + \lambda(1\text{-}w^h w) \\ &= w^t A\;\; w^* + \lambda(1\text{-}w^h w) \end{aligned} \tag{16}$$

where $A = E\left[\,|r'_{\nu+\delta} - (r'_{\nu+\delta})^h|^2\right]$,

with gradient and Hessian

$$\nabla_w J_\delta = 2(Aw + \lambda w) \tag{17}$$

$$H_w J_\delta = A\text{-}\,\lambda \cdot I \tag{18}$$

The gradient is zero if and only if (λ, w) are an eigenpair of A, hence there are exactly $2L_w + 1$ stationary points. The Hessian is positive definite (corresponding to a local minima) if and only if we choose λ to be the smallest eigenvalue. If the smallest eigenvalue is repeated, then there will be multiple minima, but all will have the same cost (equal to the repeated eigenvalue). This proves global convergence of the gradient descent algorithm

Now, we derive the number of realizations of perfect channel shortening for both the T_s-spaced and fractionally-spaced TEQs.

Denoting with $H_k(z)$ and $W_k(z)$ (k = 1, 2) the z-domain transfer function of the k-th channel and filter, respectively. In the T_s-spaced TEQ, corresponding to k = 1, perfect ISI/ICl cancellation is achieved if the combined channel-TEQ transfer function satisfies the relationship

$$W(z) = \frac{1}{H(z)} \cdot z^{-\delta} \tag{19}$$

that is, the transfer function of the perfect TEQ is the reciprocal of the transfer function of the sampled channel scaled by a linear phase factor resulting from the delay δ. Therefore, there is only one T_s-spaced realization of perfect TEQ for a specific delay.

For a fractionally-spaced TEQ with k >1, however, the perfect channel shortening corresponds to

$$C(z) = \sum_{k=1}^{K} C_k(z) = \sum_{k=1}^{K} H_k(z)\, W_k(z) = z^{-\delta} \tag{20}$$

since all these $C_k(z)$ result in $C(z) = z^{-\delta}$, we conclude that there exists infinitely many fractionally-spaced perfect TEQ for a specific delay.

Now, we establish the relations of the number of realizations of perfect channel shortening to the convergence speed of the blind TEQ MERRY algorithm.

The ultimate objective of a blind TEQ algorithm is to reach perfect channel shortening. It does not matter, however, which realization of perfect channel shortening the TEQ finally converges to. For a T_s-spaced case, TEQ has to converge to the one and only realization of perfect channel shortening (for a specific delay), whatever the initial setting. However, for a fractionally-spaced TEQ, there are infinitely many realizations of perfect channel shortening (for a specific delay) it can converge to. Since a gradient method is used, the fractionally-spaced equalizer will most probably converge to the realization closest to its initial setting and stop. That is why the fractionally-spaced blind equalization algorithm converges much faster than the T_s-spaced one (Chen and Nikias, 1992).

SIMULATION RESULTS

Extensive computer simulations have been conducted to compare the convergence speed of the fractionally-spaced MERRY algorithm against that of non-fractional one using a standard DSL test channel (CSA loop 1) (Farhang-Boroujeny and Ding, 2001). The channel output is contaminated by additive white Gaussian noise with a signal-to-noise ratio (SNR) of 40dB and the CP length is 32. In the FSTEQ case, 12 taps are selected in each of the two filters (w_1 and w_2), whereas the length of the T_s-spaced TEQ is chosen to be 16.

In Fig. 2, we compare the performance of the $T_s/2$-spaced MERRY with conventional T_s-spaced MERRY algorithm in term of intersymbol interference (ISI) defined by

$$\mathrm{ISI} = \frac{\sum_k |c_k|^2 - \max\left(|c_k|^2\right)}{\max\left(|c_k|^2\right)} \tag{21}$$

where c_k is the impulse response of the combined channel-FSTEQ system. It is clearly seen that MERRY algorithm, with fractionally-spaced TEQ, converges faster than the non-fractional MERRY algorithm and can rapidly provide a solution approaching more the optimal MERRY. In order to perform bit allocation at the end of the initialization period, the T_s-spaced MERRY algorithm needs approximately 16, 000 iterations, while the FSTEQ MERRY algorithm needs 9, 000 iterations.

Figure 3 shows the bit rate versus SNR. Here, the bit rate was computed by running for 5000 symbols and gradually decreasing the step size over time. As we can

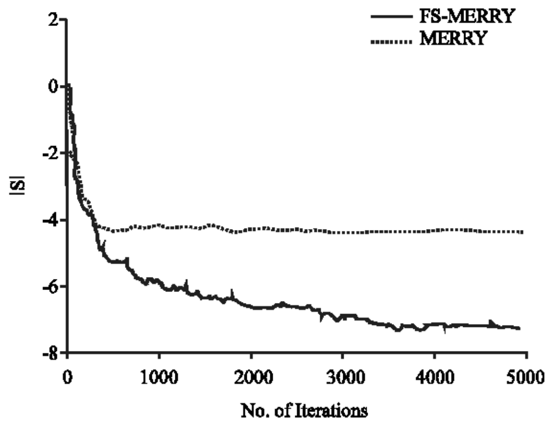

Fig. 2: Performance of the $T_s/2$-spaced and T_s-spaced MERRY algorithm under their respective optimal setting

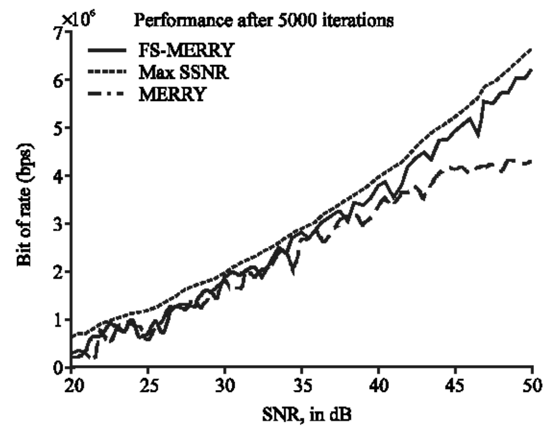

Fig. 3: Bit rate versus SNR at input to receiver

see, the performance of the adaptive blind fractionally-spaced MERRY algorithm approaches much more the maximum shortening SNR (SSNR) solution (Melsa *et al.*, 1996). Therefore, the fractionally-spaced TEQ for OFDM system allows higher data rates to be achieved due to its more powerful channel shortening capability.

We would like to point out that the simulation results are due to the interplay of two factors. One is that a fractionally-spaced TEQ admits infinitely many realizations of perfect channel shortening for a specific delay. The other is that, the fractionally-spaced TEQ has twice as many taps as the T_s-spaced TEQ does and should result in slower convergence speed.

CONCLUSIONS

The MERRY algorithm with fractionally-spaced blind TEQ are formulated and shown to converge significantly faster than the corresponding non-fractionally-spaced ones. The reason for faster convergence of fractionally-spaced blind TEQ is that it retains the infinitely many possible realizations corresponding to perfect equalization. All fractionally-spaced realizations of perfect equalization for a specific delay form a generalized linear subspace in the equalizer coefficient space.

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