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# Performance Measure of Shewhart $\bar{X}$ Control Chart for Armature Resistance Process

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**Abstract:** Most of the research was done for the process control only on theoretical basis and those results are beyond the approach to understand at the operator level in the manufacturing environment. In order to fill the gap between the academia and practice, a structured format of methodology were explained in this research work with the application of different software tools. In this regard a case study of armature resistance process was taken in this research with the availability of MINITAB and EXCEL software. It was found that the in-control Average Run Length (ARL) performance of armature resistance process for sample size 4 and 5 has the almost same values of the theoretical results of three- sigma limits. As the methods and tools were explained in this research work were applied in the armature resistance process first time, it is expected that the performance of armature resistance process would be improved continuously by follow the structured methodology as explained in this study.

**Key words:** Shewhart $\bar{X}$ control chart, average run length, armature resistance process, process capability, manufacturing process, normal distribution

## INTRODUCTION

Reducing process variability is presently an area of much interest in manufacturing organizations. It is the responsibility of all individuals from top to bottom (including operators, engineers and management) must continuously seek to improve manufacturing process output and reduce the variability. Statistical Process Control (SPC) charts are one of the very effective instruments of reducing the variability in the manufacturing process. The primary purposes of SPC charts are to achieve process stability and reduce variability. The Shewhart $\bar{X}$ control chart is used to monitor the mean of a quality characteristic for a given process. Observations are taken periodically and the sample mean is plotted. The Shewhart $\bar{X}$ control chart utilizes three-sigma control limits and indicates an out-of-control (OOC) signal of a single point falls beyond the control limits. The Shewhart $\bar{X}$ charts are efficient in detecting quickly medium to large shifts in the process mean. The developments of the equations for computing the control limits on the $\bar{X}$ -control chart can be found out from Duncan (1986) and Montgomery (1997).

The performance measure of control chart techniques is usually evaluated by the Average Run Length (ARL) in literature. ARL has the function of determining how many samples are necessary so that control chart presents the indicative signal to detect the change of out of control in the probability distribution. When a process is out-of-control, the users want the control chart to signal quickly, i.e., to have a small out-of-control average run length. Conversely when the process is in control, the users want the chart to produce fewer false alarms, i.e., to have a large in-control average run length.

Several techniques were used to calculate the Average Run Length (ARL) performance of a Shewhart $\bar{X}$ control chart. For example Thijs Vermaat and Roxana (2003), James (2002, 2003), Chen (2001), He *et al*. (2002), Yang-Kwang and Liao (2004) and Saniga (1991) was evaluated in their research the performance of the control charts either Markov chain or Simulation methods. These methods are some how difficult and complicated in calculations. Also it is difficult to understand their methods in practice at the operator level who are carefully watching their process. Most of the research carried by above and other researcher's only theoretical basis and those are beyond their applications in real manufacturing process. As Woodall and Montgomery (1999) point out that many researchers have generally considered their work ended with the publication of their results. There is usually little effort to see the work implemented in the practice. In most of academia, research publications lead to promotion, tenure and pay raises. Therefore, keeping in mind the issue raised by the Montgomery (1999), an effort was made to fill the gap between the practice and theoretical research. The main objective of this study to

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check the performance of Shewhart control chart for the armature resistance process through EXCEL spread sheet beyond the complicated approach as done by the above researchers. There are many type of software are available for example MINTTAB, MATLAB, SAS and STATISTICA to analysis the control charts data. However, the average run length performance feature is unavailable in these software and it needs lot of complicated programming. Therefore, our aim is also provide an EXCEL, a very simple tool to monitor the performance of any manufacturing process without any programming.

## REVIEW OF MATHEMATICAL CALCULATIONS FOR THE TYPE II (β) ERROR AND AVERAGE RUN LENGTH (ARL) OF SHEWHART CONTROL CHART

Let $\bar{X}_i$, I = 1,2,..., be the average of subgroup I. Assume that each subgroup consists of individual measurements $X_1, X_2,..., X_n$, which are independently and identically distributed (i.i.d.) $N(\mu_0, \sigma^2)$ random variables. Thus $\bar{X}_i$, I = 1,2,..., are i.i.d. $N(\mu_0, \sigma^2/n)$ random variables. The Upper Control Limit (UCL) and Lower Control Limit (LCL) for the Shewhart $\bar{X}$ control chart are $\mu_0 + 3\sigma/\sqrt{n}$ and $\mu_0 - 3\sigma/\sqrt{n}$, respectively, where $\mu_0$ is the center line.

The type I error, α; i.e., the probability that a point on the Shewhart $\bar{X}$ control chart falls outside the control (three sigma) limits when process is in control is given by $\alpha = P(\bar{X} > UCL) + P(\bar{X} < LCL) = 0.0027$ and the corresponding in-control Average Run Length (ARL), is $1/\alpha = 1/0.0027 = 370.40$. The type II error, β, is the probability that a given sample will not signal an out-of-control (OOC) when the process average has indeed shift from $\mu_0$ to a new value of average $\mu_1$ and is given by

β = p {Type II error} = P {One $\bar{X}$ falls inside the limits | process is out of control}

$$\beta = P(LCL < \bar{X} < UCL \mid \mu = \mu_1)$$

Since $\bar{X} \sim N(\mu, \sigma^2/n)$ and the upper and lower control limits are given above, therefore equation may be written as

$$\beta = \Phi\left[\frac{UCL - (\mu_0 + k\sigma)}{\sigma/\sqrt{n}}\right] - \Phi\left[\frac{LCL - (\mu_0 - k\sigma)}{\sigma/\sqrt{n}}\right]$$

$$\beta = \Phi\left[\frac{UCL - \mu_1}{\sigma/\sqrt{n}}\right] - \Phi\left[\frac{LCL - \mu_1}{\sigma/\sqrt{n}}\right] \quad (1)$$

Where $\mu_1 = \mu_0 + k\sigma$, shifts in the process mean in terms of standard deviation. Therefore, the in-control (IC) and out-of-control (OOC) ARL is

$$ARL = \frac{1}{1-\beta} \quad (2)$$

## DESCRIPTION OF THE METHODOLOGY

### Case study

**General information:** This case study was taken from real manufacturing process, to investigate the suitability of the statistical methods for the quality improvement.

The company under study with a total area of 168,820 $m^2$, is one of the major manufacturing facilities of the group with current production capacity over 300,000 pieces per day. Small Motors (SM) manufacture and deliver a brand range of micro motor products, well diversified in a variety of consumer and business product applications including automobile components, home appliances, personnel care, power tools, business equipment and multimedia. All the process for the manufacturing of SM motors is continuous process. The total number of process for the manufacturing of micro (SM) motor includes armature resistance process has 17 number of sub-assembly process, end cap has 09 number of sub-assembly process, rear housing has 05 number of sub-assembly process and final assembly process consists of 17 assembly process in order to get final product of micro motor.

**Quality control issues:** Different motor models are manufacturing by the factory. Some models are auto manufacturing and some are not automated. Those models which are not automated, which few processes used by most of the statistical tools to control the quality of the process and products. The armature resistance process was the major issue because they never used SPC tools in this process and also lot of the customer complaints against armature resistance due to of not durable life. In this connection armature resistance process has been taken as a case study for the suitability, performance of the statistical tools and techniques for the process and products control. The application of statistical tools ultimately improves the performance of the armature resistance process.

**Selection of process:** When SPC should be implemented by any manufacturing Industry first time, the quality engineer (practitioners) needs to study and understood all steps of manufacturing process. According to customer (internal and external) requirement the CTQ should be identified. In this case study, armature resistance sub-assembly process was chosen for analysis purpose to implement the statistical methods and its performance measure.

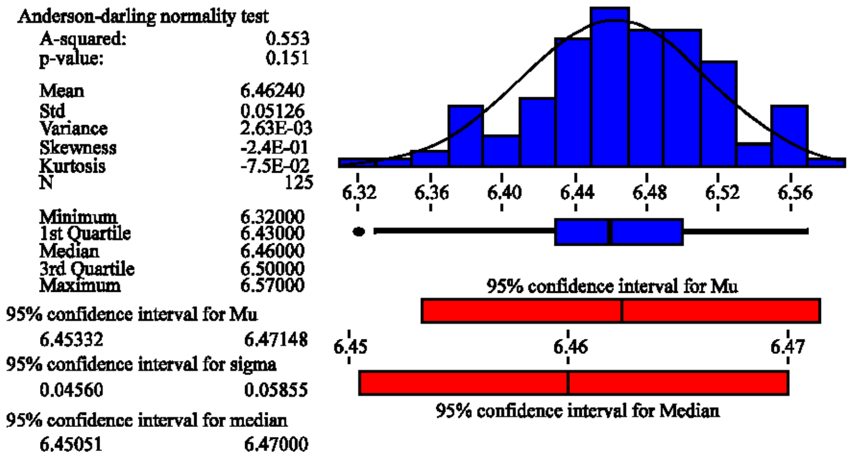



Fig. 1: Descriptive statistics for the raw data of armature resistance process using MINITAB software

**Data collection:** The capability of measurement system must be ensured before the system is used for the process control. This prerequisite must be satisfied because capability of the measurement system contributes significantly to over all results and too much variation in a measurement system may mask important variation in the manufacturing process. Therefore, the capability of the measurement system of ohm meter was ensure before data collection. Jamali (2004a) explained the procedure of capability of measurement system in detailed of armature resistance process but that method can be applied to any measurement system capability.

After ensured the ohm meter capability, data were collected. Data were collected at the inspection point (check armature resistance) covering of 25 sample numbers of size 5 after every 2 h. Also samples were grouped of size 2, 3, 4 and 5 to check the effect of sample size in the performance of armature resistance process.

**Data analysis:** After collected the data, it is necessary to analysis and check the pattern of the data to follow any distribution especially normal distribution. This could be done with any software for example MINITAB, MATLAB, SAS, EXCEL and STATISTICA software. Histograms, Normal probability plots, Histograms with normal curves and descriptive statistics with histograms were drawn with any software to check its normality. MINITAB quality software were used to analysis the data by plot either histograms, normal probability to check its normality. We only show here the descriptive statistics for checking of the normality in Fig. 1.

**Design of control charts:** After the data analysis, any time type of control charts can be plotted to check the performance of the process whether the process is in or

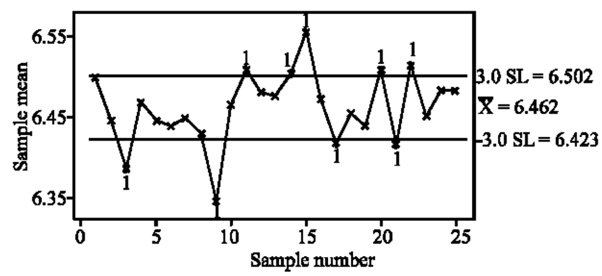
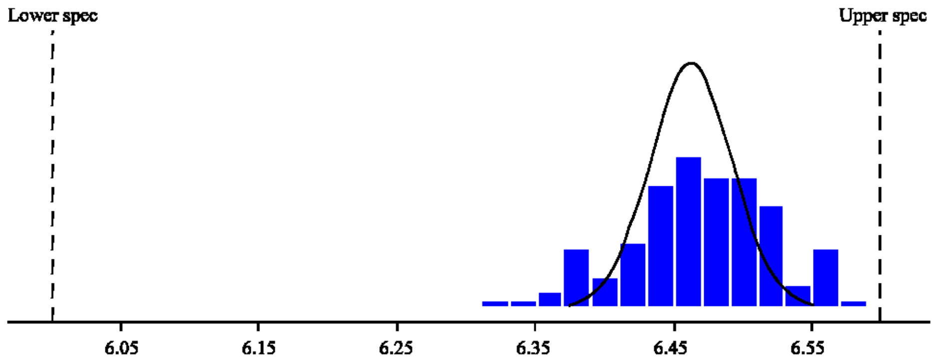



Fig. 2: The shewhart $\bar{X}$ control chart for n = 5 of armature resistance process using MINTAB software

out of control. Here $\bar{X}$ control chart were designed and plotted to see the behavior of the process. The MINTAB, EXCEL, SAS, MATLAB and STATISTICA software could be used to draw the $\bar{X}$ control chart. In this study MINITAB were used to plot the $\bar{X}$ control chart with sample sizes 2, 3, 4 and 5. The Shewhart $\bar{X}$ control chart is represented in Fig. 2 of sample size 5 only.

After plotting the 25 values of $\bar{X}$ on the $\bar{X}$ -chart, the points 3, 9, 11, 14, 15, 17, 20, 21, 22 indicates an out of control situation. All the control charts of sample size 2, 3 and 4 show an out-of-control situation. No actions were contemplated because the purposes of this research work to check the performance Shewhart $\bar{X}$ control chart for the armature resistance process.

**Process capability analysis:** After designing the control charts, it is necessary to analysis whether the process is capable or not according to the customer requirements. The capability analysis was conducted through MINITAB quality software. The information obtained only from capability analysis that how many defective products can be produced. The capability analyses of armature resistance are shown in Fig. 3. The detailed of




| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cp | 3.38 | Targ | * | Mean | 6.46240 | %>USL Exp | 0.00 | PPM>USL Exp | 2 |
| CPU | 1.55 | USL | 6.600 | Mean+3s | 6.55105 | Obs | 0.00 | Obs | 0 |
| CPL | 5.22 | LSL | 6.000 | Mean-3s | 6.37375 | %<LSL Exp | 0.00 | PPM<LSL Exp | 0 |
| Cpk | 1.55 | k | 0.541 | s | 0.02955 | Obs | 0.00 | Obs | 0 |
| Cpm | * | n | 125.000 | | | | | | |

Fig. 3: Process capability analysis of armature resistance process using MINITAB software

the capability analysis of armature resistance process could be found out from Jamali (2004b).

**Performance measure of armature resistance process:** The information only obtained from capability analysis about the defective parts produced from manufacturing process. It does not inform operator or engineer after how many samples the process would go out of control. This appropriate, timely and right information is very necessary and could only be obtained from the average run length performance. There is no any built in function available in MINITAB, MATLAB, SAS, EXCEL and STATISTICA software. The performance measure through MINITAB, MATLAB, SAS and STATISTICA required lot of complicated programming. Therefore, an effort was made to evaluate the performance of Shewhart control chart for armature resistance process through EXCEL spread sheet. The beauty of the EXCEL spread sheet model could be applied to any Shewhart control chart for example R, N, P and Np Chart with different sample sizes and different control limits.

As explained in the introduction the adopted criterion of the measure to conform the performance of a $\bar{X}$-chart for the armature resistance process is the Average Run Length (ARL). The OC- curves and the ARL for the $\bar{X}$-chart are explained in Eq. 1 and 2 were applied in this research. ARL curves of $\bar{X}$-chart for monitoring the process mean were calculated for different values of shift level (0.0 (0.25) 3.50, 4.00, 5.00, 6.00) in mean in terms of standard deviation by changing the sample size (n = 2(1)5) to see any influence of sample size on the control chart performance of armature resistance process. The results are shown in Table 1, 2 and graphically represented in Fig. 4 and 5.

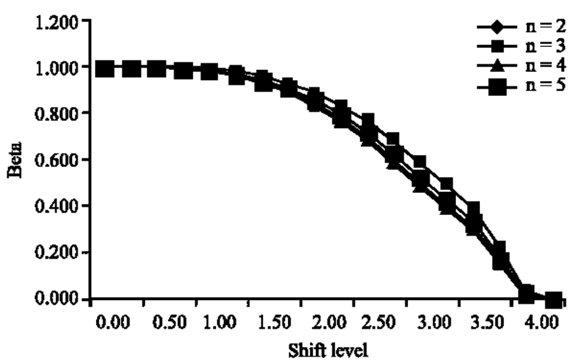



Fig. 4: The operating characteristic curve for the shewhart $\bar{X}$ control chart for armature resistance process with 3- sigma limits using EXCEL software (for n = 2, 3, 4 and 5)

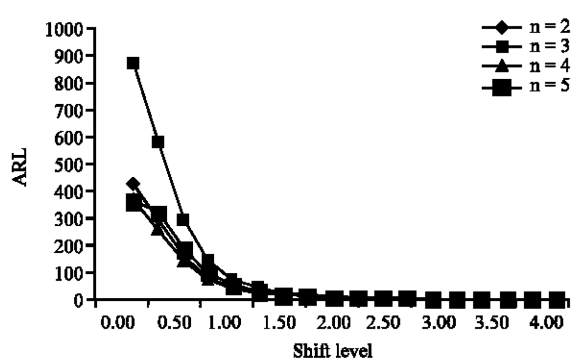



Fig. 5: The ARL for the Shewhart $\bar{X}$ control chart for armature resistance process with 3- sigma limits using EXCEL software (for n = 2, 3, 4 and 5)

Table 1: The numerical data of the OC curves for the shewhart $\bar{X}$ control chart for the armature resistance process with 3- sigma limits using EXCEL software (for n = 2, 3, 4 and 5)

| Shift level (k) | n = 2 | n = 3 | n = 4 | n = 5 |
|---|---|---|---|---|
| 0.00 | 0.9976 | 0.9989 | 0.9973 | 0.9972 |
| 0.25 | 0.9966 | 0.9983 | 0.9960 | 0.9968 |
| 0.50 | 0.9936 | 0.9966 | 0.9927 | 0.9946 |
| 0.75 | 0.9877 | 0.9930 | 0.9860 | 0.9896 |
| 1.00 | 0.9772 | 0.9864 | 0.9744 | 0.9806 |
| 1.25 | 0.9599 | 0.9750 | 0.9554 | 0.9654 |
| 1.50 | 0.9332 | 0.9563 | 0.9265 | 0.9414 |
| 1.75 | 0.8943 | 0.9278 | 0.8849 | 0.9060 |
| 2.00 | 0.8413 | 0.8867 | 0.8289 | 0.8569 |
| 2.25 | 0.7734 | 0.8313 | 0.7580 | 0.7929 |
| 2.50 | 0.6915 | 0.7609 | 0.6736 | 0.7145 |
| 2.75 | 0.5987 | 0.6770 | 0.5793 | 0.6243 |
| 3.00 | 0.5000 | 0.5829 | 0.4801 | 0.5266 |
| 3.25 | 0.4013 | 0.4838 | 0.3821 | 0.4273 |
| 3.50 | 0.3085 | 0.3856 | 0.2912 | 0.3324 |
| 4.00 | 0.1587 | 0.2146 | 0.1469 | 0.1753 |
| 5.00 | 0.0228 | 0.0367 | 0.0202 | 0.0266 |
| 6.00 | 0.0013 | 0.0026 | 0.0011 | 0.0017 |

Table 2: The numerical data of the ARL for the shewhart $\bar{X}$ control chart for the armature resistance process with 3- sigma limits using EXCEL software (for n = 2, 3, 4 and 5)

| Shift level (k) | n = 2 | n = 3 | n = 4 | n = 5 |
|---|---|---|---|---|
| 0.00 | 423.552 | 873.575 | 365.888 | 362.448 |
| 0.25 | 293.835 | 577.166 | 253.139 | 317.038 |
| 0.50 | 156.815 | 290.452 | 136.325 | 184.089 |
| 0.75 | 81.388 | 143.149 | 71.552 | 96.376 |
| 1.00 | 43.914 | 73.609 | 39.042 | 51.481 |
| 1.25 | 24.959 | 39.934 | 22.435 | 28.861 |
| 1.50 | 14.968 | 22.884 | 13.600 | 17.065 |
| 1.75 | 9.465 | 13.843 | 8.690 | 10.641 |
| 2.00 | 6.303 | 8.828 | 5.846 | 6.990 |
| 2.25 | 4.413 | 5.928 | 4.133 | 4.830 |
| 2.50 | 3.241 | 4.183 | 3.064 | 3.503 |
| 2.75 | 2.492 | 3.096 | 2.377 | 2.661 |
| 3.00 | 2.000 | 2.397 | 1.923 | 2.112 |
| 3.25 | 1.670 | 1.937 | 1.618 | 1.746 |
| 3.50 | 1.446 | 1.628 | 1.411 | 1.498 |
| 4.00 | 1.189 | 1.273 | 1.172 | 1.213 |
| 5.00 | 1.023 | 1.038 | 1.021 | 1.027 |
| 6.00 | 1.001 | 1.003 | 1.001 | 1.002 |

## RESULTS AND DISCUSSION

The Average Run Length (ARL) is defined as the expected number of samples taken before the process exceeds the control limits, as a performance measure for a control chart. As explained in above sections that the ARL performance for the Armature Resistance Process were calculated through by a new developed EXCEL spread sheet tool for the normal distributions. The beauty of the methodology is that it can be extended to any Shewhart Control Charts for any manufacturing process industry with different sample sizes and different width of the control limits. The results were obtained in above Table 1 to 2 using the probability and combination theory. First an EXCEL spread sheet model were developed to find out the Type I and II error performance for the Shewhart $\bar{X}$ control chart. After verifications and validations of the results obtained, the EXCEL model were then extended to find out the ARL performance of armature resistance process as mathematically modeled in Eq. 1 and 2. From Table 1 and 2 of Shewhart $\bar{X}$ control chart (A single point falls out side the control limits) considering normal distribution, implies that if the process is in control (Shift level zero), there will be a false alarm of 0.9972 probability and above every 362.448 samples of armature resistance process at sample size of 5.

The out-of-control ARL performance (Shift level ranging from 0.25 to 6.00 at the rate of increment of 0.25), considering a shift level of 0.25 the ARL is 317.038 for Shewhart $\bar{X}$ control chart from table 2 of normal distribution armature resistance process implies that it will take 317.038, on average, to detect this shift at sample size of 5.

The results of ARL performance of armature resistance process for sample sizes 2, 3 and 4 can be interpreted and analyzed the same way as explained above.

## CONCLUSIONS

The average run length performance (ARL) measure of armature resistance process for monitoring the process mean was found out by different values of shift levels in the process average in terms of standard deviation and different values of sample sizes. It was found in this study that the performance of Shewhart $\bar{X}$ control chart for armature resistance process when process is in and out-of-control has significant differences of different sample sizes. The in-control ARL performance of armature resistance process of sample size 3 has a very large value of about 873.575. In contrast, the in-control ARL performance of armature resistance process of sample size 4 and 5 has the value 365.888 and 362.448, which is almost equal to the theoretical result of three- sigma limits of 370.40. The same difference in the results of the out-of-control performance of armature resistance process was found in this study.

It was also found by observing the Fig. 5 that the ARL for armature resistance process vary inversely with sample size (n). Using the small sample sizes, they often result in relatively large β-risk. However, there is good chance that the large shifts will be detected reasonably quickly.

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