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Stability and Maximum Delay Bound of Networked Control Systems with Multi-Step Delay

[1]Limin Cheng, [1]Li Kong, [1]Changlin Ma and [2]Xiaoling Zhu
[1]Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China
[2]Department of Mechanical Engineering, Wuhan Institute of Technology, Wuhan, Hubei 430074, China

Abstract: A new method to obtain a maximum delay bound of networked control systems whose network-induced delay is longer than a sampling period is proposed in terms of linear matrix inequalities, which is shown to render corresponding system asymptotically stable. Simulation results verify the correctness of the proposed theory.

Key words: Networked control systems, maximum delay bound, asymptotically stable

INTRODUCTION

Feedback control systems in which the control loops are closed through a real-time network are called networked control systems (NCSs) (Nilsson and Wittermark, 1998; Zhang *et al.*, 2001). The network will inevitably induce some nondeterministic phenomena, especially the network-induced delays, which will affect the performance of NCSs. Conventional control theories must be reevaluated before they are applied to NCSs because of the network-induced delay.

Nilsson and Wittenmark (1998) studied stochastic time delay and the optimal controllers of NCS whose network-induced delay is shorter than a sampling period. However, the network-induced delay is longer than one sampling period in many cases. As for the transfer delay longer than one sampling period, Yu *et al.* (2000) proposed a model of NCS that has time-driven sensor and actuator and event-driven controller.

The network-induced delay can degrade the performance of control system and even destabilize the system by reducing the system stability region. There have been some results on the stability of NCSs (Yang *et al.*, 2004; Hu and Zhu, 2003; Lian *et al.*, 2003) but these were concerned with obtaining stability conditions of the NCS with a given maximum delay. It is important to find the network-induced delay bound within which stability of the system is guaranteed. A method was proposed to obtain the maximum delay bound of NCS with network-induced delay shorter than a sampling period (Kim *et al.*, 2003). However, the network-induced delay is longer than one sampling period in many cases. It is of great significance to seek the maximum delay bound for the NCS with multi-step network-induced delay.

In this study, a new method to obtain the maximum delay bound of Yu *et al.* (2000) model is proposed in terms of linear matrix inequalities (LMI), which guarantees the stability of NCS with multi-step delay.

Description of stochastic NCS model: The stochastic delay mainly comes from two resources: sensor-to-controller delay τ_{sa} and controller-to-actuator delay τ_{ca}. Assume that the network-induced delay τ has a known probability distribution function and $\tau = \tau_{sc}+\tau_{ca} \leq pT$ (p is a positive integer and T is the sampling period of the sensor). Yu *et al.* (2000) proposed a mathematical model of NCS under the following control mode: sensor and actuator is time-driven and controller is event-driven. Now we describe the NCS model. We assume the state equation of plant is linear time-invariant which is expressed as (1):

$$\begin{aligned} x(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where x (t) R^n, u (t)R^m. A, B and C are matrices of appropriate sizes. Discretilizing Eq. 1 and considering the affect of control mode above over a sampling interval [kT, (k + 1) T], we yield a stochastic NCS model (Yu *et al.*, 2000):

$$\begin{aligned} x_{k+1} &= A_c x_k + \sum_{j=0}^{p} B_j^k u_{k-j}, \\ y_k &= Cx_k, \end{aligned} \tag{2}$$

Corresponding Author: Li Kong, Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China

where

$x_k = x(kT), y_k = y(kT), A_c = e^{AT}, B_j^k = \beta_j\Gamma = \beta_j\int_0^T e^{At}dt \cdot B, \beta_0, \cdots, \beta_k \in \{0,1\}$

$B_j^k (j = 0,\cdots,p)$ are stochastic variables. A controller is given as follows:

$$u_k = Gx_k \tag{3}$$

Now we introduce a new state variable

$$z_k = [x_k^T, x_{k-1}^T \cdots, x_{k-p+1}^T]^T \in R^{pn}, \tag{4}$$

then system (2) can be expressed as follows:

$$z_{k+1} = A_k z_k + B_k z_{k-p+1} \tag{5}$$

where

$$A_k = \begin{bmatrix} A_c + B_0^k G & B_1^k G & \cdots & B_{p-2}^k G & 0 \\ I_n & 0 & \cdots & 0 & 0 \\ 0 & I_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 \end{bmatrix},$$

$$B_k = \begin{bmatrix} B_{p-1}^k G & B_p^k G & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

STABILITY ANALYSIS AND MAXIMUM DELAY BOUND OF NCSs

In order to obtain the maximum delay bound guaranteeing stability of NCSs with multi-step delay, the following Lemma plays an important role.

Lemma 1: Lee *et al.* (2001) assume that, $a(\cdot) \in R^{n_a}$, $b(\cdot) \in R^{n_b}$ and $\aleph(\cdot) \in R^{n_a \times n_b}$ are defined on the interval Ω. Then, for any matrices $X(\cdot) \in R^{n_a \times n_a}$, $Y(\cdot) \in R^{n_a \times n_b}$ and $Z(\cdot) \in R^{n_b \times n_b}$, the following holds:

$$-2a^T \aleph b \le \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - \aleph \\ Y^T - \aleph^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \tag{6}$$

and

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0.$$

Theorem 1: If there exist P>0, M>0, Q>0, N>0, X,Y,Z, such that

$$\begin{bmatrix} \lambda_{11} & -Y & A_k^T & \lambda_{41}^T \\ -Y^T & -Q & B_k^T & \lambda_{42}^T \\ A_k & B_k & -M & 0 \\ \lambda_{41} & \lambda_{42} & 0 & -(p-1)N \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0, \tag{8}$$

$$\begin{bmatrix} P & I_{pn} \\ I_{pn} & M \end{bmatrix} \ge 0, \tag{9}$$

$$\begin{bmatrix} Z & I_{pn} \\ I_{pn} & N \end{bmatrix} \ge 0, \tag{10}$$

where

$\lambda_{11} = -P + (p-1)X + Y + Y^T Q$,

$\ell_{41} = (p-1)(A_k - I_{pn})$,

$\ell_{42} = (p-1) B_K$.

Then system (5) is asymptotically stable for any network-induced delay τ satisfying $0 \le \tau \le pT$ ($p \ge 2$ is a positive integer).

Proof: Since the following equations hold

$$x_{k-p} = x_{k-1} - \sum_{j=k-p+1}^{k-1} (x_j - x_{j-1}),$$

$$x_{k-p+1} = x_k - \sum_{j=k-p+2}^{k} (x_j - x_{j-1}),$$

system (2) can be written as

$$\begin{aligned} x_{k+1} &= (A_c + B_0^k G + B_{p-1}^k G)x_k + (B_1^k G + B_p^k G)x_{k-1} \\ &+ \sum_{j=2}^{p-2} B_j^k G x_{k-j} - B_{p-1}^k G \sum_{j=k-p+2}^{k} (x_j - x_{j-1}) - B_p^k G \sum_{j=k-p+1}^{k-1} (x_j - x_{j-1}). \end{aligned} \tag{11}$$

Using (11), system (5) can be written as

$$z_{k+1} = (A_k + B_k) z_k - B_k \sum_{j=k-p+2}^{k} (z_j - z_{j-1}) \tag{12}$$

Define a Lyapunov candidate as follows:

$$V_k = V_{1k} + V_{2k} + V_{3k}, \tag{13}$$

where

$$V_{1k} = z_k^T P z_k \tag{14}$$

$$V_{2k} = \sum_{i=2}^{p} \sum_{j=k-i+2}^{k} (z_j - z_{j-1})^T Z (z_j - z_{j-1}), \tag{15}$$

$$V_{3k} = \sum_{j=k-p+1}^{k-1} z_j^T Q z_j \cdot \tag{16}$$

Taking the increment of V_{1k}, we have

$$\Delta V_{1k} = z_{k+1}^T P z_{k+1} - z_k^T P z_k \cdot \tag{17}$$

Substituting (12) into (17), we get

$$\begin{aligned}\Delta V_{1k} &= z_k^T (A_k + B_k)^T P (A_k + B_k) z_k - \\ &2 z_k^T (A_k + B_k)^T P B_k \sum_{j=k-p+2}^{k} (z_j - z_{j-1}) + \\ &(z_k - z_{k-p+1})^T B_k^T P B_k (z_k - z_{k-p+1}) - z_k^T P z_k.\end{aligned} \tag{18}$$

Defining in Eq. 6 as

$$a = z_k, b = z_j - z_{j-1}, \aleph = (A_k + B_k)^T P B_k,$$

and applying Lemma 1 in (18), we conclude

$$\begin{aligned}\Delta V_{1k} &\le z_k^T (A_k + B_k)^T P (A_k + B_k) z_k + \\ &\sum_{j=k-p+2}^{k} \{ z_k^T X z_k + 2 z_k^T [Y - (A_k + B_k)^T P B_k](z_j - z_{j-1}) \\ &+ (z_j - z_{j-1})^T Z (z_j - z_{j-1}) \} + (z_k - z_{k-p+1})^T B_k^T P B_k \\ &(z_k - z_{k-p+1}) - z_k^T P z_k \\ &= z_k^T [A_k^T P A_k + (p-1)X + Y + Y^T - P] z_k + \\ &2 z_k^T (A_k^T P B_k - Y) z_{k-p+1} + z_{k-p+1}^T B_k^T P B_k z_{k-p+1} \\ &+ \sum_{j=k-p+2}^{k} (z_j - z_{j-1})^T Z (z_j - z_{j-1}).\end{aligned} \tag{19}$$

Since ΔV_{2k} and ΔV_{3k} can be expressed as follows:

$$\begin{aligned}\Delta V_{2k} &= -\sum_{j=k-p+2}^{k} (z_j - z_{j-1})^T Z (z_j - z_{j-1}) + \\ &(p-1)(z_{k+1} - z_k)^T Z (z_{k+1} - z_k),\end{aligned} \tag{20}$$

$$\Delta V_{3k} = z_k^T Q z_k - z_{k-p+1}^T Q z_{k-p+1}, \tag{21}$$

we have the increment of V as follows:

$$\begin{aligned}\Delta V_k &= \Delta V_{1k} + \Delta V_{2k} + \Delta V_{3k} \\ &\le z_k^T [A_k^T P A_k + (p-1)X + Y + Y^T - P] z_k + \\ &2 z_k^T (A_k^T P B_k - Y) z_{k-p+1} + z_{k-p+1}^T B_k^T P B_k z_{k-p+1} \\ &+ (p-1)(z_{k+1} - z_k)^T Z (z_{k+1} - z_k) + z_k^T Q z_k \\ &- z_{k-p+1}^T Q z_{k-p+1}.\end{aligned} \tag{22}$$

Substituting (5) into (22), we derive

$$\begin{aligned}\Delta V_k &\le z_k^T [A_k^T P A_k + (p-1)X + Y + Y^T - P + \\ &(p-1)(A_k - I_{pn})^T Z (A_k - I_{pn}) + Q] z_k + \\ &2 z_k^T [A_k^T P B_k - Y + (p-1)(A_k - I_{pn})^T Z B_k] z_{k-p+1} \\ &+ z_{k-p+1}^T [B_k^T P B_k + (p-1) B_k^T Z B_k - Q] z_{k-p+1} \\ &= \begin{bmatrix} z_k \\ z_{k-p+1} \end{bmatrix}^T U \begin{bmatrix} z_k \\ z_{k-p+1} \end{bmatrix},\end{aligned}$$

where

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{12}^T & U_{22} \end{bmatrix},$$

$$\begin{aligned}U_{11} &= A_k^T P A_k + (p-1)X + Y + Y^T - P + \\ &(p-1)(A_k - I_{pn})^T Z (A_k - I_{pn}) + Q,\end{aligned}$$

$$U_{12} = A_k^T P B_k - Y + (p-1)(A_k - I_{pn})^T Z B_k$$

$$U_{22} = B_k^T P B_k + (p-1) B_k^T Z B_k - Q$$

Therefore, if

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{12}^T & U_{22} \end{bmatrix} < 0, \tag{23}$$

system (5) is asymptotically stable according to the Lyapunov-Krasovskii stability theorem (Hale and Lunel, 1993). By the schur complement (Boyad *et al*., 1994), Eq. 23 is equivalent to Eq. 7. Now we prove Theorem 1.

The upper bound of network-induced delay can be obtained efficiently using the MATLAB LMI Toolbox.

SIMULATIONS

The simplified model of the inverted pendulum process is as follows:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t).$$

The sampling period is given as T=0.05 s. The discrete-time system is as follows:

$$x_{k+1} = \begin{bmatrix} 1.0013 & 0.05002 \\ 0.05002 & 1.0013 \end{bmatrix} x_k + \begin{bmatrix} 0.00125 \\ 0.05002 \end{bmatrix} u_k.$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k$$

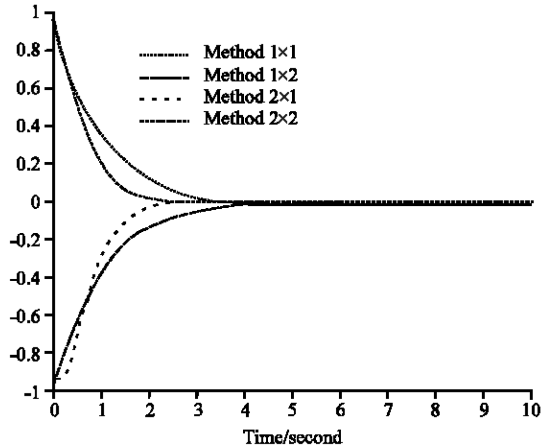

Fig. 1: Curves of state response of the system

The LQG controller for the system without considering the network-induced delay is as follows:

$$G = [-3.9814 \quad -3.9794], u_k = Gx_k$$

Then using Theorem 1, we derive p = 2 satisfying Eq. 7. With the initial state value, x (0) = $[1 -1]^T$ the simulation results of Method 1 (methods without considering the transfer delay) and Method 2 (methods in this paper) are given in Fig. 1. Simulation results show the correctness of the proposed theory.

CONCLUSIONS

A new method to obtain a maximum delay bound of networked control systems whose network-induced delay is longer than a sampling period is proposed in terms of linear matrix inequalities, which is shown to render corresponding system asymptotically stable. Simulation results verify the correctness of the proposed theory. Applying the theoretical results in this paper to the industrial networked control systems will be studied in the next research.

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