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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 7 (7): 1077-1080, 2008
ISSN 1812-5638

Applying Singular Value Decomposition Perturbation to Wideband Signal Direction-of-Arrival Estimation

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Abstract: This study proposes a novel algorithm for wideband signal directions-of-arrival (DOA) estimation based on the Singular Value Decomposition (SVD) perturbation. In our algorithm, the SVD perturbation of focusing matrix is derived and the range of the perturbation parameter is also given. Compared to conventional wideband DOA estimators, our algorithm enjoys the following advantages: (i) consider the nonideal arbitrary sensor arrays, the effects of imperfection and the mutual coupling; (ii) does not require preliminary DOA estimates. As illustrated in simulations, the proposed algorithm exhibits high DOA resolution performance even if the errors of the actual element positions, gains and phases are presented.

Key words: Wideband, DOA estimation, singular value decomposition, two sources

INTRODUCTION

In recent years, estimating signal location parameters like the directions-of-arrival (DOA)'s from observed sensor array data has been of considerable interest. In many classical DOA estimation algorithms, the assumption is that the signals are narrow band. Apparently, those algorithms are not adapted to the wideband or ultra wideband signals. Then, it is necessary to investigate appropriate algorithms (Hong and Russer, 2003a; Valaee and Kabal, 1995) that are adapted to the wideband signals. Currently, there mainly exist two types of wideband DOA estimation algorithms: Maximum Likelihood (ML) (Feder and Weinstein, 1988) methods and signal subspace methods (Su and Morf, 1983).

Recently, several efficient wideband signal subspace methods using beamforming technique have been proposed (Ward *et al*., 1998; Agrawal and Prasads, 2000; Lee, 1994). However, these methods need to carry out many times SVD. It's well known that the computational load of the SVD is excessive. In the DOA estimation, the number of matrices that needs SVD presents exponential increase if the number of the sources increases. Therefore, it is necessary to find high efficient numerical techniques to carry out the SVD. In addition, these methods obtain DOA results under the premise that the sensor array is ideal. However in practice, the measuring errors of the sensor array isn't ignored in the case of high resolution digital DOA. Furthermore, due to the performance change of the array components resulted from temperature and humidity change, ageing of Radio Frequency (RF) elements and mutual coupling among array elements, the sensors array exists error itself. All of errors can make the array manifold matrix imprecise. Therefore, it is obligatory that the sensor array error should be calibrated. From this perspective, we here propose a novel wideband DOA estimation algorithm based on SVD perturbation in the presence of the nonideal arbitrary sensor arrays, the effects of imperfection and the mutual coupling.

THE SVD PERTURBATION

The basic ideas of SVD perturbation can be depicted as follows. On the one hand, assume the SVD decomposition of a m×n real matrix $A_0 \in R^{m\times n}$ is:

$$A_0 = U_0 S_0 V_0^{\prime}$$

where, $U_0 = [u_{01}, u_{02}, \cdots, u_{0m}]$ and $V_0 = [v_{01}, v_{02}, \cdots, v_{0n}]$ satisfy

$$A_0 A_0^T u_{0i} = \sigma_{0i}^2 u_{0i} \quad 1 \le i \le m$$

$$A_0^T A_0 v_{0i} = \sigma_{0i}^2 v_{0i} \quad 1 \le i \le n$$

On the other hand, assume another matrix $A_0 \in R^{m\times n}$ coming from A_0 by a perturbation modification, i.e.,

$$A = A_0 + \varepsilon A_p \quad 0 < |\varepsilon| < 1 \quad A \in R_r^{m\times n} \quad A_p \in R_r^{m\times n} \tag{1}$$

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where, εA_p is a small perturbation matrix and ε is the perturbation parameter. Given such assumes, it has been proved in (Lv, 1997, 1991) that we could asymptotically approximate (Σ_n, U, V) by using (Σ_{0n}, U_0, V_0).

THE SIGNAL MODEL AND THE FOCUSING PRINCIPLE

We consider a wavefield generated by N wideband sources in the presence of noise. It is sampled temporally and spatially by a passive sensor array of M. The N sources impinge on the array from distinct directions (θ_i, ψ_i), I = 1,L, N. The variables θ_i and ψ_i are azimuth and elevation, respectively. For the m-th element, we have:

$$x_m(\omega_k) = e^{j2\pi\omega_k t_m(\theta)} s(\omega_k) + n_m(\omega_k) \quad (2)$$

Considering that the gains $\alpha_i(\omega_k)$, phases $\xi\alpha_i(\omega_k)$ and positions $d_i(\omega_k)$ of the ith elements, the discrete model of the M elements' received wideband signal can be rewritten in matrix form1:

$$X(\omega_k) = A(\omega_k)S(\omega_k) + N(\omega_k)$$

In practice, the true values of the gains, phases and position are unknown. However, their measuring values $\bar{\alpha}_i(\omega_k), \bar{\xi}_i(\omega_k)$ and $\bar{d}_i(\omega_k)$ are known.

As shown in Hong and Russer (2003a), the focusing matrix $T(\omega_k)$ is determined by least squares fitting:

$$\min_{T(\omega_k)} \left\| \bar{A}(\omega_k) - T(\omega_k)A_c(\omega_k) \right\|_F \quad (3)$$

where, $\|.\|_F$ is Frobenius matrix norm. The matrix $\bar{A}(\omega_0) = [\bar{a}(\theta_1,\omega_0),\cdots,\bar{a}(\theta_N,\omega_0)]$ is the target manifold matrix. The solution of Eq. 3 is given by:

$$T(\omega_k) = L_k R_k^{\dagger}$$

where L_k and R_k are the left and right singular vectors of the product $\bar{A}(\omega_0) = [\bar{a}(\theta_1,\omega_0),\cdots,\bar{a}(\theta_N,\omega_0)]$. The symbol † denotes pseudo-inverse operation. In the focusing, we know every angle point needs to carry out SVD for frequency bins $\omega_k, 1 \le k \le h$ in the range of the scanning angle.

THE PROCESSING OF PERTURBATION

According to Eq. 3, the focusing matrix $T(\omega_k)$ is determined by $\bar{A}(\omega_k)A_c^{\dagger}(\omega_k)$. For calculating $\bar{A}(\omega_k)$ we only resort to the measuring value $\bar{\alpha}_i(\omega_k), \bar{\xi}_i(\omega_k)$ and $\bar{d}_i(\omega_k)$ $(1 \le i \le M)$. However, the matrix A (ω_k) is calculated by using true value $\alpha_i(\omega_k), \xi_i(\omega_k)$ and $d_i(\omega_k)$. Unfortunately, $\alpha_i(\omega_k), \xi_i(\omega_k)$ and $d_i(\omega_k)$ could not be obtained in practice. At the same time, we have to use the measuring value to calculate the array manifold matrix $Ac(\omega_k)_{inherent} = C(\omega_k)\cdot A(\omega_k)$ in the presence of mutual coupling. Therefore, we have to resort to the perturbation method.

In the perturbation method, the imprecision resulted from the differences between measuring value and true value is revealed by the perturbation parameter ε. So how to obtain the range of ε is the key problem that remedies the differences. Assume the preliminary matrix is $\bar{A}_0 Ac_{inherent}^{\dagger}|\theta_0$ for the preliminary angle θ_0. Thus for arbitrary angle θ_i, the matrix is $\bar{A}_0 Ac_{inherent}^{\dagger}|\theta_i$. The difference between $\bar{A}_0 Ac_{inherent}^{\dagger}|\theta_i$ and $\bar{A}_0 Ac_{inherent}^{\dagger}|\theta_0$ is given by:

$$B_{inherent} = \bar{A}_0 Ac_{inherent}^{\dagger}|\theta_i - \bar{A}_0 Ac_{inherent}^{\dagger}|\theta_0 \quad (4)$$

For arbitrary angle θ_i, the $\bar{A}_0 Ac_{real}^{\dagger}|\theta_i$ is calculated by true $\alpha_i(\omega_k), \xi_i(\omega_k)$ and $d_i(\omega_k)$. The difference is expressed as:

$$B_{real} = \bar{A}_0 Ac_{real}^{\dagger}|\theta_i - \bar{A}_0 Ac_{inherent}^{\dagger}|\theta_0 \quad (5)$$

From (4) and (5), we know

$$B_{real} - B_{inherent} = \bar{A}_0[(A_{real}^{\dagger}|\theta_i)C_{real}^{\dagger} - (A_{inherent}^{\dagger}|\theta_i)C_{inherent}^{\dagger}] \quad (6)$$

In practice, the position error in DOA estimation can be omitted. We obtain:

$$B_{real} - B_{inherent} = \bar{A}_0 \bar{K}_{\theta_i}^{\dagger}(\gamma^{\dagger}C_{real}^{\dagger} - \bar{\gamma}^{\dagger}C_{inherent}^{\dagger}) \quad (7)$$

According to the definition of γ_i, we have:

$$\gamma_i = \alpha_i e^{j\xi_i} = (\bar{\alpha}_i + \tilde{\alpha}_i)e^{j(\bar{\xi}_i + \tilde{\xi}_i)} \quad (8)$$

The change of γ_i can be omitted if $\tilde{\xi}_i$ is in the range of [-5°, 5°]. Thus Eq. 8 can be rewritten as:

$$\gamma_i = \alpha_i e^{j\xi_i} = (\bar{\alpha}_i + \tilde{\alpha}_i)e$$

According to the structure of the mutual matrix C, omit the effect of error of the positions, that is, only consider the effect of gains and phases. It is inferred that only the matrix G has much effect on C. The element g_{ij}, $(i \ne j)$ of G is:

$$g_{ij} = (\bar{\alpha}_i + \tilde{\alpha}_i)e^{j(\bar{\xi}_i + \tilde{\xi}_i)} \cdot (\bar{\alpha}_j + \tilde{\alpha}_j)e^{j(\bar{\xi}_j + \tilde{\xi}_j)} \quad (9)$$

Similarly, omit the effect of $\tilde{\xi}_i$, Eq. 9 can be rewritten as:

$$g_{ij} = (\bar{\alpha}_i + \tilde{\alpha}_i)\cdot(\bar{\alpha}_j + \tilde{\alpha}_j)e^{j(\bar{\xi}_i+\bar{\xi}_j)} \approx (\bar{\alpha}_i\bar{\alpha}_j + \bar{\alpha}_j\tilde{\alpha}_i + \bar{\alpha}_i\tilde{\alpha}_j)e^{j(\bar{\xi}_i+\bar{\xi}_j)}$$

As shown in Hong and Russer (2003b), assume each element has the same measuring gain and the random error of gain $\tilde{\alpha}_i$, $1 \le i \le M$, are in the range of [-0.1, 0], thus:

$$\bar{\alpha}_i\bar{\alpha}_j + 2\min\{\bar{\alpha}_j\tilde{\alpha}_i, \bar{\alpha}_i\tilde{\alpha}_j\} \le \bar{\alpha}_i\bar{\alpha}_j + \bar{\alpha}_j\tilde{\alpha}_i + \bar{\alpha}_i\tilde{\alpha}_j \le \bar{\alpha}_i\bar{\alpha}_j \quad (10)$$

Reformulating every element of matrix G in terms of Eq. 10, we have:

$$\|C_{inherent} + H\|_F \le \|C_{real}\|_F \le \|C_{inherent}\|_F \quad (11)$$

Substituting Eq. 11 into (7), we have:

$$B_{real} - B_{inherent} = \bar{A}_0\bar{K}^{\dagger}_{\theta_i}[(\bar{\gamma}+\beta)^{\dagger}(C_{inherent}+H)^{\dagger} - \bar{\gamma}^{\dagger}C^{\dagger}_{inherent}] \quad (12)$$

Equation 12 is simplified as:

$$B_{real} - B_{inherent} = \bar{A}_0\bar{K}^{\dagger}_{\theta_i}[\beta^{\dagger}C^{\dagger}_{inherent} + \gamma^{\dagger}H^{\dagger}) \quad (13)$$

Reformulating the Eq. 4 as:

$$B_{inherent} = \bar{A}_0(\bar{K}^{\dagger}_{\theta_i} - \bar{K}^{\dagger}_{\theta_o})\bar{\gamma}^{\dagger}C^{\dagger}_{inherent} \quad (14)$$

When the steps of the elements of θ_i are in the range of [0.5°, 2°], there exists:

$$0.1 \le \left[\left\|\bar{K}^{\dagger}_{\theta_i} - \bar{K}^{\dagger}_{\theta_0}\right\|_1\right]/(M*M) \le 0.3 \quad (15)$$

If $\tilde{\alpha}_i$, $1 \le i \le M$, are in the range of [0.7, 0.8], we estimate:

$$-2.4\cdot B_{inherent} \le B_{real} - B_{inherent} \le -1.4\cdot B_{inherent} \quad (16)$$

In terms of Eq. 4 and 5 , we know:

$$\bar{A}_0Ac^{\dagger}_{real}|\theta_i = \bar{A}_0Ac^{\dagger}_{inherent}|\theta_0 + B_{inherent} + B_{real} - B_{inherent} \quad (17)$$

Substituting Eq. 16 into 17, it is derived:

$$\bar{\mathbf{A}}_0\mathbf{Ac}^{\dagger}_{real}|\theta_i = \bar{\mathbf{A}}_0\mathbf{Ac}^{\dagger}_{inherent}|\theta_0 + \mathbf{B}_p$$

where, -1.4. $B_{inherent} \le B_p \le$ -0.4.$B_{inherent}$. Let A_p = 2.B_p, we have:

$$\bar{A}_0Ac^{\dagger}_{real}|\chi\theta_i = \bar{A}_0Ac^{\dagger}_{inherent}|\theta_0 + \varepsilon A_p \quad (18)$$

where, $-0.7 \le \varepsilon \le -0.2$. Considering that Eq. 18 has the same form with Eq. 1, it is natural to apply the perturbation method to Eq. 18.

SIMULATIONS

In the simulations, we test the proposed DOA estimation algorithm. For convenience, we denote the proposed algorithm P-MUSIC. We consider the azimuth estimation using a uniform circular array of 10 (M = 10) elements, with a spacing of d = $c/2f_0$, where f_0 is the midband frequency and c is the velocity of propagation. Assume the wideband sources are linear frequency modulated signals whose midband is f_0 = 1.2M. The bandwidth is 400 K that spanned by 201 frequency bins. Spatially white Gaussian noise n(t), independent of the signals and with its M elements $n_i(t)$, $1 \le i \le M$, statistically independent, is presented at each array element.

In the simulations, the element gains and phases are uniformly distributed and the range of them are [0.75, 0.85] and [-5, 5], respectively. The element position error (POE), Gain Error (GE), Phase Error (PE) and mutual coupling (MC) are considered. Assume POE, GE and PE follow Gaussian distributions with mean 0 and different standard deviations $\sigma_{position}$ = 0.0035, σ_{gain} = 0.018 and σ_{phase} = 1. In addition, it's assumed that the SNR is 8dB.

We investigate a configuration with two coherent wideband sources. Under the premise that sources impinges on the array from directions of 75° and 80°, we investigate the capability of the P-MUSIC algorithm to resolve coherent sources. In Fig. 1, the P-MUSIC spatial spectrum for two coherent sources scenario is depicted. As it is seen, the estimated errors of the two sources are 0.4° and 1°, respectively. This means that the P-MUSIC can resolve two coherent sources.

Next, we study the effect of scanning angle steps on the resolution capability of the P-MUSIC. Assume the span of scanning angle is 20° and two sources impinge on the array from directions of 78.5° and 81.5°. The

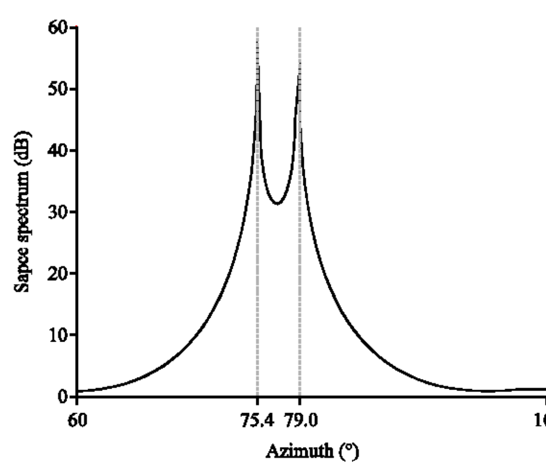

Fig. 1: Spatial spectrum for two coherent sources

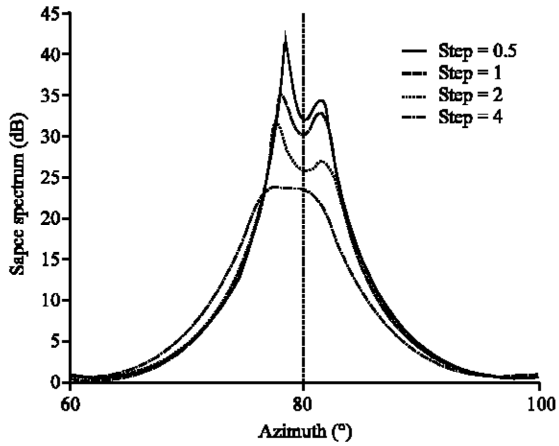

Fig. 2: Spatial spectrum for different scanning angle steps

corresponding spatial spectrums are given in Fig. 2 when the scanning angle steps are varied in the range of [0.5°, 4°]. The figure illustrates that P-MUSIC owns high DOA resolution performance when the step is below 4°.

CONCLUSION

This study proposes a new algorithm for wideband signal DOA estimation based on SVD perturbation method. In this algorithm, the SVD perturbation of focusing matrix is firstly derived under the premise that the errors of the actual element positions, gains and phases are considered. Following that, the range of the perturbation parameter is given. Finally, the DOA of the wideband signals is estimated by summarizing the focusing matrices and by a narrowband DOA estimator such as MUSIC. Theoretical analysis illustrate that the computational load of the proposed algorithm is small. The simulation results verify the feasibility of the proposed algorithm.

ACKNOWLEDGMENTS

The authors would like to express their cordial thanks to Dr. Liu Feng for his valuable advice. The research is supported by the National Natural Science Foundation of China under Grant (No. 60202034) and Gansu Natural Science Foundation of China under Grant (No. 3ZS051-A25-047).

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