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## Application of a Nonlinear Fault Detection and Isolation Scheme to an Aircraft's Nonlinear Closed-Loop System

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**Abstract:** In this study, the nonlinear fault detection and isolation (FDI) scheme is successfully applied to the aircraft's nonlinear closed-loop control system, which is established by using the dynamic inversion theory. The nonlinear FDI scheme consists of a bank of nonlinear adaptive estimators. One of them is the fault detection and approximation estimator, whereas the others are used for fault isolation (each associated with a specific type of fault). A type of fault that has occurred can be isolated if the residual associated with the matched isolation estimator remains below its corresponding adaptive threshold and at least one of the components of the residuals associated with all the other estimators exceeds its threshold at some finite time. The simulation results show the effectiveness of the application.

**Key words:** On-line approximator, nonlinear estimator, closed-loop

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### INTRODUCTION

The failures which occur in the system usually cause enormous loss of productivity, expensive equipment and human lives ultimately, so the design and analysis of fault detection scheme has attached great importance and has received considerable attention. During the last two decades, various approaches to fault detection (FD) using model-based analytical redundancy have been represented. The studies based on these methods mainly focused on the robustness with respect to unknown factors (the disturbance, noise) and the sensitivity to early fault. Several state estimation-based FD methods are proposed, such as Luenberger robust observer (Ibaraki *et al.*, 2005), linear matrix inequalities (LMI) (Wang *et al.*, 2007), H-infinite estimator (Collins and Song, 2000; Collins, 2000 #2) and multiple-model estimation method (Hofbauer and Williams, 2004).

Unlike the fault detection problem, which has been extensively investigated in the literature, the fault isolation (FI) problem has received less attention, especially in the case of nonlinear uncertain systems. In this study, a key design issue of the fault isolation scheme is the adaptive residual threshold associated with each isolation estimator. That is, the residual of each fault isolation estimator is associated with an adaptive threshold, which can be implemented online by using linear filtering methods (Basin *et al.*, 2005). The

occurrence of a particular fault is excluded if at least one of the residual components of the corresponding isolation estimator exceeds its threshold in a finite time. Fault isolation is achieved when all faults but one is excluded.

Meanwhile, the application of FD (without the consideration of FI) scheme aims at LTI and open-loop system. Such as the  $H_\infty$  estimation algorithms to faulty aircraft (Melody *et al.*, 2001). The problem of fault detection was addressed through  $H_\infty$  robust estimator using Popov-Tsyypkin multipliers in (Collins and Song, 2000). Their analysis were based on open-loop linear time-invariant (LTI) system response where the filters were designed for a nominal condition and tested for a second off-nominal LTI plant for robustness analysis. In (Felicio *et al.*, 2002), the  $H_\infty$ -FD Ricatti-based approach is used to design FD filters for an inverted pendulum. But the application was not fully carried out in this case (i.e., no simulations). In fact, most practical conditions of the system are nonlinear in nature and most failures are more accurately modeled as nonlinear functions about the state/output and input variables.

So, in this study, a FDI scheme using some nonlinear estimators to detect and isolate the incipient faults in an aircraft's nonlinear uncertain closed-loop system is significant. In the presence of a failure, the function exported by the on-line approximator which is the important component of the nonlinear estimator can be used as an estimate of the possible nonlinear fault

function. Once the fault is detected, the isolation estimators are activated for the purpose of fault isolation. A type of fault that has occurred can be isolated if the residual associated with the matched isolation estimator remains below its corresponding adaptive threshold and at least one of the components of the residuals associated with all the other estimators exceeds its threshold at some finite time. The simulation results show the effectiveness of the application.

**PROBLEM FORMULATION**

Here, the non-linear system in presence of the fault is formulated as below:

$$\dot{x}(t) = \xi(x, u) + \eta(x, u, t) + \beta(t - T)f(x, u) \tag{1}$$

The differential equation  $\dot{x}(t) = \xi(x, u)$  presents the nominal model,  $\eta = [\eta_1, \dots, \eta_n]^T$  describe the model uncertainty,  $f = [f_1, \dots, f_n]$  presents the fault occurs in the system. Many fault diagnosis schemes simplify the fault function to the time function, actually, most fault is the nonlinear function of the state variables  $x$  and input vector  $u$ .

Each time component  $\beta_i (i = 1, \dots, n)$  can be described as the form below (Trunov and Polycarpou, 2000):

$$\beta_i(t - T_z) = \begin{cases} 0 & t < T_z \\ 1 - e^{-\gamma_i(t - T_z)} & t \geq T_z \end{cases} \tag{2}$$

where,  $\gamma_i$  is an unknown constant and  $\gamma_i > 0$ , it presents the rate at which the fault changes in state variable  $x$  evolves.

**Assumption 1:** The state variable  $x(t)$  is available for measurement

**Assumption 2:** Each modeling uncertainty function  $\eta_i$  is bounded by a known constant  $\bar{\eta}_i$

$$\bar{\eta}_i = \sup_{x \in X, u \in U, t \in T} |\eta_i(x, u, t)| \quad i = 1, \dots, n \tag{3}$$

**NONLINEAR FAULT DETECTION ISOLATION SCHEME**

A class of  $N+1$  nonlinear adaptive estimators are used in the proposed FDI scheme, where,  $N$  is the number of nonlinear faults. One of the nonlinear adaptive estimators is the fault detection and approximation estimator used to detect faults. The remaining ones are fault isolation estimators that are used for isolation purposes only after a fault has been detected. The structure of the fault detection estimator and the fault isolation estimators are given below, respectively.

**Fault detection and approximation estimator:** Based on (1), a nonlinear FD estimator of the form (4) is considered:

$$\dot{\hat{x}}^d = \xi(x, u) + \hat{f}^d(x, u, \hat{\theta}^d) + G^d(x - \hat{x}^d) \tag{4}$$

where,  $\hat{x}^d \in \mathbb{R}^n$  is the estimated state vector,  $\hat{f}: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^q \rightarrow \mathbb{R}^n$  presents an online approximation model,  $\hat{\theta}^d \in \mathbb{R}^q$  is an adjustable weight vector,  $G^d = (g_1^d, \dots, g_n^d)$  is a positive value. The initial value  $\hat{\theta}^d(0)$  is chosen such that  $\hat{f}^d(x, u, \hat{\theta}^d(0)) = 0$ , which corresponds to the case where the system is normal (no fault).

A key component of the nonlinear adaptive estimator described by (4) is the online approximator, denoted by  $\hat{f}(x, u, \hat{\theta}^d)$ , whose  $i$ th component has the structure as (5):

$$\hat{f}^d(x, u, \hat{\theta}^d) = \sum_{j=1}^q \hat{\theta}_j^d \varphi_j(x, u, \sigma_j) \quad \sigma_j \in \mathbb{R}^k \tag{5}$$

The next step in the construction of the fault detection estimator is the design of the learning algorithm for updating the weights  $\hat{\theta}^d$ . Let  $\epsilon^d = x - \hat{x}^d$  be the state estimation error. Using techniques from adaptive control (Lyapunov synthesis method) (Trunov and Polycarpou, 2000), the learning algorithm of the online approximator is chosen as follows:

$$\dot{\hat{\theta}}^d = P_{\epsilon^d} \{ \Gamma^d Z^T D[\epsilon^d] \} \tag{6}$$

The projection operator  $P$  restricts the weights estimate  $\hat{\theta}^d$  to a predefined region  $M_\delta \subset \mathbb{R}^q$ .  $\Gamma^d$  is a symmetric positive definite learning rate matrix and  $Z$  denotes the gradient matrix of the online approximator with respect to its adjustable weights, i.e.,  $Z = \partial \hat{f}(x, u, \hat{\theta}^d) / \partial \hat{\theta}^d$ . The dead-zone operator  $D[\epsilon^d]$  is defined as follows:

$$D[\epsilon^d] = \begin{cases} 0 & |\epsilon_i^d| - \bar{\epsilon}_i^d < 0 \quad i = 1, \dots, n \\ \epsilon_i^d & \text{otherwise} \end{cases} \tag{7}$$

The dead-zone operator  $D[\epsilon^d]$  which acts on the variable  $\epsilon_i^d$  such that the output of the operator is zero while no fault in system, that is  $|\epsilon_i^d| < \bar{\epsilon}_i^d$ . The detection of the fault occurs if at least one state  $\epsilon_i^d$  of the state estimation error exceeds its dead-zone boundary  $\bar{\epsilon}_i^d$ . More precisely, the fault detection time is defined as the first instant of time such that  $|\epsilon_i^d(t)| > \bar{\epsilon}_i^d(t)$ , for  $t > T_z$ , that is:

$$T_d = \bigcup_{i=1}^n \{ t \geq T_z : |\epsilon_i^d(t)| > \bar{\epsilon}_i^d(t) \} \tag{8}$$

By Eq. 1 and 4 it can be easily verified that each component of the state estimation error satisfies:

$$|\epsilon_i^d(t)| = \left| \int_0^t e^{-\alpha(t-\tau)} \eta_i(x(\tau), u(\tau), \tau) d\tau \right| \quad (9)$$

Because of  $|\epsilon_i^d| < \bar{\epsilon}_i^d$ , then using the Eq. (3), the FD threshold  $\bar{\epsilon}_i^d$  can be get as follow:

$$\bar{\epsilon}_i^d(t) = \int_0^t e^{-\alpha(t-\tau)} \bar{\eta}_i(x(\tau), u(\tau), \tau) d\tau \quad (10)$$

Next, the sensitivity of the fault detection scheme will be considered. The faults occur at some unknown time  $T_z$  and develop with the rate  $\gamma_i$ .

**Theory 1:** Consider the nonlinear fault diagnosis scheme described by Eq. 4 and 6. If there exists an interval of time  $[T_z+t_1, T_z+t_2]$ , such that at least one component  $f_i(x(t), u(t))$  of the fault vector  $f(x(t), u(t))$  satisfies:

$$\left| \int_{T_z+t_1}^{T_z+t_2} g_i^d e^{-\alpha(T_z+t_2-\tau)} (1 - e^{-\gamma_i(\tau-T_z)}) f_i(x(\tau), u(\tau)) d\tau \right| > 2\bar{\eta}_i \quad (11)$$

then the fault can be detected, that is  $|\epsilon_i^d(T_z+t_2)| \geq \bar{\epsilon}_i^d$ ,  $T_d = T_z+t_2$  (the proof can be found in Frank and Seliger (1991)).

**Fault isolation estimators:** Once a fault has been detected, the isolation scheme is activated. Then construct the following nonlinear adaptive estimators as isolation estimators:

$$\begin{aligned} \hat{x}^l &= \xi(x, u) + \hat{f}^l(x, u, \hat{\theta}^l) + G^l(x - \hat{x}^l) \\ \hat{f}^l(x, u, \hat{\theta}^l) &= \left[ (\hat{\theta}_1^l)^T \varphi_1^l(x, u), \dots, (\hat{\theta}_n^l)^T \varphi_n^l(x, u) \right]^T \end{aligned} \quad (12)$$

where,  $\hat{\theta}^l \in \mathbb{R}^{q^l}$  for  $i = 1, \dots, n, l = 1, \dots, N$  is the estimate of the fault parameter vector in the  $i$ th state variable. For notational simplicity and without loss of generality, assume that  $g_i^l = g_i$ , for all  $l = 1, \dots, N$ .

The design of fault isolation estimators is similar to the design of the fault detection estimator. Each isolation estimator corresponds to one of the possible types of nonlinear faults. Specifically  $\epsilon_i^l = x_i - \hat{x}_i^l$ , if let  $\epsilon_i^l$  be the  $i$ th component of the state estimation error vector of the  $l$ th estimator, then the learning algorithm is chosen as (Zhang *et al.*, 2002):

$$\dot{\hat{\theta}}_i^l = P_{\epsilon_i^l} \{ \Gamma_i^l \varphi_i^l(x, u) \epsilon_i^l \} \quad (13)$$

where,  $\Gamma_i^l$  is a symmetric positive definite learning rate matrix.

The fault-isolation decision scheme is based on the generalized observer scheme (GOS) principle: if the  $l$ th fault occurs at some time  $T_z$  and is detected at time  $T_d$ , then a set of adaptive thresholds  $\lambda_i^l(t)$ ,  $1, \dots, n$  can be

designed such that the  $i$ th component of the state estimation error associated with the  $l$ th estimator satisfies  $|\epsilon_i^l(t)| \leq \lambda_i^l(t)$ , for all  $t \geq T_d$ . If for each  $re \in \{1, \dots, N\} \setminus \{l\}$ , there exist some finite time  $t' \geq T_d$  and some  $ie \in \{1, \dots, n\}$  such that  $|\epsilon_i^l(t')| > \lambda_i^l(t')$ , then the possibility that the fault  $l$  may have occurred can be deduced. The absolute fault isolation time is defined as  $T_s^l = t'$ .

**Theorem 1:** If the incipient fault  $l$  occurs, then for all  $t \geq T_d$  and for all  $ie \in \{1, \dots, n\}$ , the  $i$ th component of the state estimation error of the  $l$ th isolation estimator satisfies the following inequality:

$$|\epsilon_i^l(t)| \leq \epsilon_i^l(T_d) e^{-\alpha t} + \int_{T_d}^t e^{-\alpha(t-\tau)} \left[ |\Psi_i^l(\tau)| + \bar{\eta}_i(x(\tau), u(\tau), \tau) + e^{-\gamma_i(t-T_z)} |(\hat{\theta}_i^l(\tau))^T \varphi_i^l(x(\tau), u(\tau))| \right] d\tau \quad (14)$$

Where:

$$\Psi_i^l(t) = (\theta_i^l - \hat{\theta}_i^l)^T \varphi_i^l(x(t), u(t)) \quad (15)$$

represents the fault function estimation error in the case of a matched fault.

**Proof:** On the basis of (1) and (12), in the presence of the fault  $l$ , the  $i$ th component of the error dynamics of the  $l$ th isolation estimator for is given by:

$$\begin{aligned} \dot{\epsilon}_i^l(t) &= -g_i \epsilon_i^l(t) + (1 - e^{-\gamma_i(t-T_z)}) (\theta_i^l)^T \varphi_i^l(x(t), u(t)) \\ &\quad - (\hat{\theta}_i^l)^T \varphi_i^l(x(t), u(t)) + \eta_i(x(t), u(t), t) \\ &= -g_i \epsilon_i^l(t) - e^{-\gamma_i(t-T_z)} (\hat{\theta}_i^l)^T \varphi_i^l(x(t), u(t)) + (1 - e^{-\gamma_i(t-T_z)}) \\ &\quad (\theta_i^l - \hat{\theta}_i^l)^T \varphi_i^l(x(t), u(t)) + \eta_i(x(t), u(t), t) \end{aligned}$$

Then the solution to the previous differential equation is

$$\begin{aligned} \epsilon_i^l(t) &= \int_{T_d}^t e^{-\alpha(t-\tau)} (1 - e^{-\gamma_i(t-T_z)}) \Psi_i^l(\tau) d\tau \\ &\quad - \int_{T_d}^t e^{-\alpha(t-\tau)} e^{-\gamma_i(t-T_z)} (\hat{\theta}_i^l(\tau))^T \varphi_i^l(x(\tau), u(\tau)) d\tau \\ &\quad + \int_{T_d}^t e^{-\alpha(t-\tau)} \eta_i(x(\tau), u(\tau), \tau) d\tau + \epsilon_i^l(T_d) e^{-\alpha t} \end{aligned}$$

where,  $\Psi_i^l(t)$  is defined as (15). By taking norms, the Eq. 14 can be get

$$|\epsilon_i^l(t)| \leq \epsilon_i^l(T_d) e^{-\alpha t} + \int_{T_d}^t e^{-\alpha(t-\tau)} \left[ |\Psi_i^l(\tau)| + \bar{\eta}_i(x(\tau), u(\tau), \tau) + e^{-\gamma_i(t-T_z)} |(\hat{\theta}_i^l(\tau))^T \varphi_i^l(x(\tau), u(\tau))| \right] d\tau$$

So, the proof is complete.

Because in Eq. 14 the fault approximation error  $\Psi_i^l(t)$ , the fault evolution rate  $\gamma_i$  and the fault occurrence time  $T_z$  are unknown, it cannot be directly used as a threshold function for fault isolation. However, as the estimate  $\hat{\theta}_i^l$

belongs to the known compact parameter set  $\Theta_i^l$ , letting the parameter set  $\Theta_i^l$  be a hypersphere with center  $o_i^l$  and radius  $R_i^l$ ,  $k_i^l(t) = R_i^l + |\hat{\theta}_i^l - o_i^l|$  can be get. Then

$$|\psi_i^l(t)| = |(\theta_i^l - \hat{\theta}_i^l)^T \varphi_i^l(x(\tau), u(\tau))| \leq k_i^l(t) \left| \varphi_i^l(x(\tau), u(\tau)) \right| \quad (16)$$

Meanwhile, for the incipient fault time profile given by Eq. 2, assume that the unknown fault evolution rate satisfies  $\gamma_i > \bar{\gamma}_i$ , where,  $\bar{\gamma}_i$  denotes a known lower bound on the unknown fault evolution rate  $\gamma_i$ . So, the Eq. 7 can be get:

$$e^{-\bar{\gamma}_i(t-T_d)} \leq e^{-\bar{\gamma}_i(t-T_z)} \quad (17)$$

Hence, based on (14), (16) and (17), the following threshold functions for fault isolation are chosen as below

$$\lambda_i^l(t) = \left| e_i^l(T_d) e^{-\bar{\alpha}t} + \int_{T_d}^t e^{-\bar{\alpha}(t-\tau)} \left[ \left( k_i^l(t) + e^{-\bar{\gamma}_i(t-T_d)} |\hat{\theta}_i^l(\tau)| \right) \left| \varphi_i^l(x(\tau), u(\tau)) \right| + \bar{\eta}_i(x(\tau), u(\tau), \tau) \right] d\tau \right| \quad (18)$$

Next, the fault isolability condition of the proposed FDI scheme will be analyzed. Intuitively, faults are easier to isolate if they are sufficiently mutually different in terms of a suitable measure. In the following analysis, a fault mismatch function in the form is introduced:

$$m_i^l(t) = (1 - e^{-\bar{\gamma}_i(t-T_d)}) (\theta_i^l)^T \varphi_i^l(x(t), u(t)) - (\hat{\theta}_i^l)^T \varphi_i^l(x(t), u(t)) \quad (19)$$

$r, l = 1, \dots, N, r \neq l$

which can be interpreted as the difference between the actual  $l$ th fault function in the  $i$ th state equation, represented by  $(1 - e^{-\bar{\gamma}_i(t-T_d)}) (\theta_i^l)^T \varphi_i^l(x(t), u(t))$  and the estimated fault function  $(\hat{\theta}_i^l)^T \varphi_i^l(x(t), u(t))$  associated with any other isolation estimator whose structure does not match the actual fault. The following theorem characterizes the class of incipient nonlinear faults that are isolable by the proposed FDI scheme.

**Theorem 2:** Consider the fault isolation scheme described by Eq. 12, (13) and (18). The incipient fault is isolable if for each  $r \in \{1, \dots, N\} \setminus \{l\}$  there exist some time  $t^r > T_d$  and some  $i \in \{1, \dots, n\}$  such that the  $i$ th component  $m_i^l(t)$  of the fault mismatch function satisfies the following inequality:

$$\left| \int_{T_d}^{t^r} e^{-\bar{\alpha}(t^r-\tau)} m_i^l(t) d\tau \right| > \int_{T_d}^{t^r} e^{-\bar{\alpha}(t^r-\tau)} \eta_i(x(\tau), u(\tau), \tau) d\tau + 2 \left| e_i^l(T_d) \right| e^{-\bar{\alpha}t^r} + \int_{T_d}^{t^r} e^{-\bar{\alpha}(t^r-\tau)} \left[ \left( k_i^l(t) + e^{-\bar{\gamma}_i(t-T_d)} |\hat{\theta}_i^l(\tau)| \right) \left| \varphi_i^l(x(\tau), u(\tau)) \right| + \bar{\eta}_i(x(\tau), u(\tau), \tau) \right] d\tau \quad (20)$$

**Proof:** Based on (1) and (14), in the presence of the fault, the  $i$ th component of the error dynamics associated with the estimator  $r$  is given by:

$$\dot{\epsilon}_i^r = -g_r \epsilon_i^r + \eta_i(x, u) + m_i^h(t)$$

the solution of the above differential equation is

$$\epsilon_i^r(t) = \epsilon_i^r(T_d) e^{-\bar{\alpha}t} + \int_{T_d}^t e^{-\bar{\alpha}(t-\tau)} m_i^h(\tau) d\tau + \int_{T_d}^t e^{-\bar{\alpha}(t-\tau)} \eta_i(x(\tau), u(\tau), \tau) d\tau$$

By using the triangle inequality, Eq. 21 is obtained

$$\left| \epsilon_i^r(t) \right| \geq \left| \int_{T_d}^t e^{-\bar{\alpha}(t-\tau)} m_i^h(\tau) d\tau \right| - \left| \epsilon_i^r(T_d) \right| e^{-\bar{\alpha}t} - \int_{T_d}^t e^{-\bar{\alpha}(t-\tau)} \eta_i(x(\tau), u(\tau), \tau) d\tau \quad (21)$$

So, the threshold for the state estimation error of the  $r$ th estimator is:

$$\lambda_i^r(t) = \int_{T_d}^t e^{-\bar{\alpha}(t-\tau)} \left[ \left( k_i^r(t) + e^{-\bar{\gamma}_i(t-T_d)} |\hat{\theta}_i^r(\tau)| \right) \left| \varphi_i^r(x(\tau), u(\tau)) \right| + \bar{\eta}_i(x(\tau), u(\tau), \tau) \right] d\tau + \left| \epsilon_i^r(T_d) \right| e^{-\bar{\alpha}t}$$

Therefore, if (20) is fulfilled, the occurrence of the fault  $r$  is excluded at time  $t^r$ , i.e.,  $|\epsilon_i^r(t^r)| > \lambda_i^r(t^r)$ . If this is satisfied for each  $r \in \{1, \dots, N\} \setminus \{l\}$ , then the  $i$ th fault can be isolated, thus concluding the proof.

### AEROCRAFT'S NONLINEAR CLOSE-LOOP SYSTEM

**Aircraft's nonlinear model:** Consider the tail-controlled pitch-axis missile airframe (Kim *et al.*, 2004) depicted in (Fig. 1).

The aircraft nonlinear model for the longitudinal axis are presented.

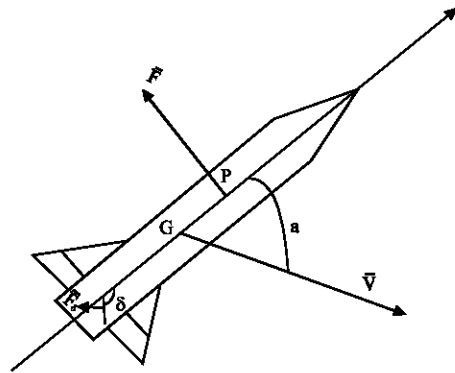


Fig. 1: Aircraft airframe

Table 1: Model parameters

$a_n$	19.373	$a_m$	40.4400
$b_n$	-31.023	$b_m$	-64.0150
$c_n$	-9.717	$c_m$	2.9220
$d_n$	-1.948	$d_m$	-11.8030
$K_\alpha$	0.0207	$K_q$	1.2319
$K_\eta$	0.6659	$\tau_f$	0.0020

$$\begin{aligned} \dot{\alpha}(t) &= K_\alpha M C_n(\alpha, \delta, M) \cos \alpha + q \\ \dot{q}(t) &= K_q M^2 C_m(\alpha, \delta, M) \end{aligned} \quad (22)$$

where,  $\alpha$  denotes the angle of attack,  $q$  is the pitch rotational rate,  $\delta$  is the tail fin deflection,  $C_n$  and  $C_m$  are the aerodynamic coefficients, the expressions of which depend on  $\alpha$ ,  $\delta$  and the Mach number  $M$  (1.5~3). The parameters can be find in Table 1.

$$\begin{aligned} C_n &= a_n \alpha^3 + b_n \alpha^2 + (2 - M/3) c_n \alpha + d_n \delta \\ C_m &= a_m \alpha^2 + b_m \alpha + (8M/3 - 7) c_m \alpha + d_m \delta \end{aligned} \quad (23)$$

The output of the system is the normal acceleration.

$$\eta = K_\eta M^2 C_n(\alpha, \delta, M) \quad (24)$$

The aerodynamic surface actuator and reaction jet actuator dynamics are modeled as:

$$\dot{\delta} = \tau_f (\delta_c - \delta) \quad (25)$$

**Nonlinear control law design:** Since the present missile configuration is tail controlled, it has a strong non-minimum phase behavior with respect to the normal acceleration. Consequently, a straightforward application of the feedback linearization technique will not be successful, because the unstable zero dynamics would not assure the internal stability of the system (Devaud *et al.*, 2000).

Two approaches have been proposed to overcome this difficulty. First of these is the time-scale separation of the system dynamics into slow and fast modes. The second approach is to redefine the system outputs to suppress the zero dynamics. The first method will be adopt in the following sections. The first step in the two time-scale design process is to split the system dynamics into time scales based upon the notion of slow and fast dynamic modes. Note that even if a clear separation between the modes are not present in the open-loop dynamics, mode separation can be enforced during control system design. For the present case, the actuator dynamics, together with the pitch rate dynamics are included in the fast time-scale. The normal acceleration dynamics is considered to be the slow time scale mode.

**Control of slow subsystem:** The state is attack angle  $\alpha$  and the output is the pitch angle rate  $q$ . The subsystem is described as follow:

$$\begin{aligned} \dot{\alpha} &= K_\alpha M C_n(\alpha, 0, M) \cos \alpha + q_{e0} \\ \eta &= K_\eta M^2 C_n(\alpha, 0, M) \end{aligned} \quad (26)$$

The desired closed-loop aerodynamic of the slow subsystem can be described by first order module as follow:

$$\frac{\eta}{\eta_c} = \frac{1}{\tau_s s + 1} \quad (27)$$

Then the desired input of the slow subsystem can be computed by Eq. 26 and Eq. 27

$$q_{e0} = -K_\alpha M C_n(\alpha, 0, M) \cos \alpha - \frac{1}{\tau_s} \left( \frac{\partial \eta}{\partial \alpha} \right)^{-1} (\eta - \eta_c) \quad (28)$$

**Control of fast subsystem:** The state and output variants are identical  $q$  and the input is the rudder deflect angle  $\delta$ . The system is described as follow:

$$\dot{q} = K_q M^2 C_m(\alpha, \delta, M) \quad (29)$$

The desired aerodynamic of the fast subsystem can be denoted by a second order modul as follow:

$$\frac{q}{q_{e0}} = \frac{\omega_f^2}{s^2 + 2\zeta \omega_f s + \omega_f^2} \quad (30)$$

Proceeding as before, the nonlinear control law for the aerodynamic surface actuator in the fast time-scale is given by:

$$\delta_c = \frac{\omega_f^2 (q_{e0} - q) - 2\zeta \omega_f \dot{q} - K_q M^2 (3a_m \alpha^2 + 2b_m \alpha + (8M/3 - 7)C_m) \dot{\alpha}}{K_q M^2 d_m \tau_f} + \delta \quad (31)$$

## SIMULATION RESULTS

Here, the proposed FDI scheme is applied to detect and isolate incipient faults in a nonlinear aircraft's control system described as Eq. 22, 24, 25 and 31.

The faults which occur in the aircraft's system are described as (two faults):

$$F = \left\{ \begin{bmatrix} 0 \\ f^1 \end{bmatrix}, \begin{bmatrix} 0 \\ f^2 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0 \\ \theta^1 \sin \alpha \end{bmatrix}, \begin{bmatrix} 0 \\ \theta^2 \cos \alpha \end{bmatrix} \right\}$$

Where,  $\theta^1 \in \Theta^1 = [-1.5, 1.5]$  and  $\theta^2 \in \Theta^2 = [-1.5, 1.5]$ . Assume that the unknown incipient-fault evolution rate  $\gamma$  defined in (2) satisfies:

$$\gamma \geq \bar{\gamma} = 0.1$$

**Fault detection:** Based on Eq. 22, the nonlinear FD estimator is constructed as follow:

$$\begin{aligned} \hat{\alpha}^d(t) &= K_\alpha M C_n(\alpha, \delta, M) \cos \alpha + q + g_1(\alpha - \hat{\alpha}^d) \\ \hat{q}^d(t) &= K_q M^2 C_m(\alpha, \delta, M) + \hat{f}^d + g_2(q - \hat{q}^d) \end{aligned}$$

The online approximator  $\hat{f}^d$  is implemented as a continuous radial basis function (RBF) neural network (Trunov and Polycarpou, 2000), which are described by:

$$\hat{f}^d = \sum_{j=1}^m \hat{\theta}_j^d \exp(-|\alpha - c_j|^2 / \sigma^2)$$

where, the weight  $\hat{\theta}_j^d$  can be tuned by Eq. 6 and choose a uniform width  $\sigma = 0.6$  for the basis functions and 11

fixed centers  $c_j$  (i.e.,  $m = 11$ ), which are evenly distributed in the interval  $[-2, 2]$ . The stability and fault-detectability properties of the fault detection estimator have been investigated in (Trunov and Polycarpou, 2000).

**Fault isolation:** By using the methodology described earlier, a bank of two isolation estimators is designed

$$\begin{aligned} \hat{\alpha}^i(t) &= K_\alpha M C_n(\alpha, \delta, M) \cos \alpha + q + g_1(\alpha - \hat{\alpha}^i) \quad \hat{f}^1 = \hat{\theta}^1 \sin \alpha \\ \hat{q}^i(t) &= K_q M^2 C_m(\alpha, \delta, M) + \hat{f}^1 + g_2(q - \hat{q}^1) \\ \hat{\alpha}^2(t) &= K_\alpha M C_n(\alpha, \delta, M) \cos \alpha + q + g_1(\alpha - \hat{\alpha}^2) \quad \hat{f}^2 = \hat{\theta}^2 \cos \alpha \\ \hat{q}^2(t) &= K_q M^2 C_m(\alpha, \delta, M) + \hat{f}^2 + g_2(q - \hat{q}^2) \end{aligned}$$

The control input is set as Eq. 31. The modeling uncertainty is assumed to arise out of a 5% inaccuracy in the value of  $a_i, b_i, c_i, (i = m, n)$ . The bounding function  $\bar{\eta} = |0.5K_q M^2 C_m|$ . Moreover, set  $g_i = 5(i = 1, 2)$ .

**The experiment results:** Figure 2 shows the simulation results when an incipient fault of type 1, with  $\theta^1 = 0.75$  and the fault evolution rate  $\gamma = 0.2$ , occurs at  $t = 10$  sec.

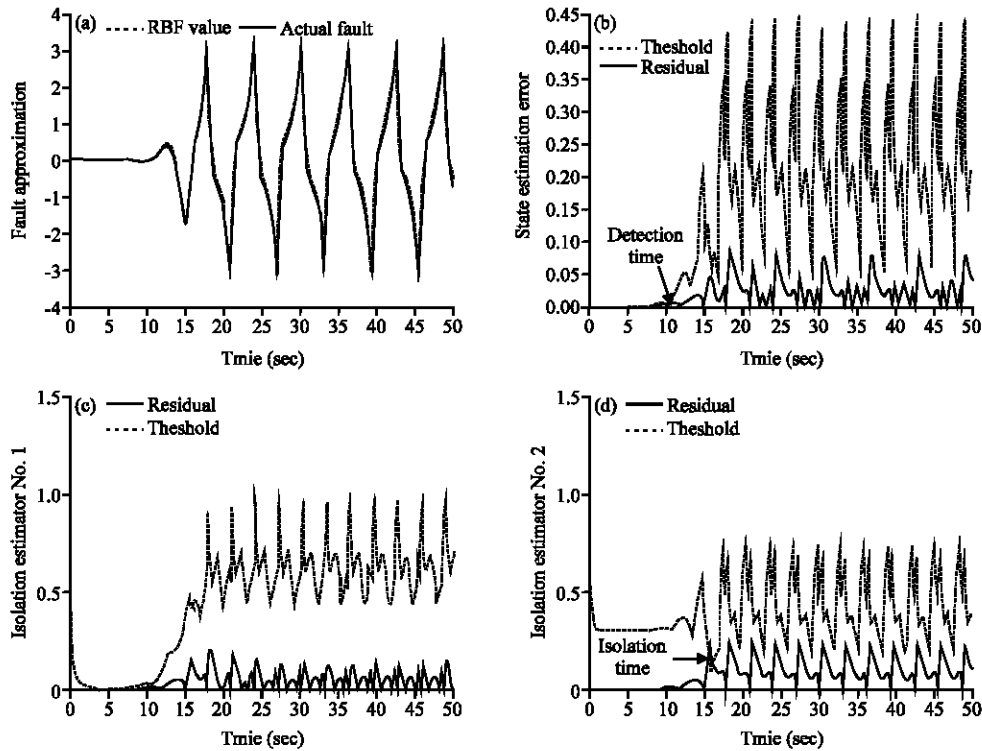


Fig. 2: (a) Time-behaviors of the fault function (solid line) and the RBF neural-network output (dash-dotted line) associated with the fault detection estimator (b) Time behaviors of the state estimation error (solid line) associated with the FDAE and the dead-zone threshold (dash-dotted line) (the fault detection time instant is shown by an arrow) (c) and (d) Time-behaviors of the state estimation errors (solid lines) and the thresholds (dash-dotted lines) associated with the two isolation estimators (the fault isolation time instant is shown by an arrow)

The evolution of the actual fault function  $f$  (solid line) and the output of the neural network approximator  $\hat{f}^d$  (dash-dotted line) associated with the fault detection estimator are shown in Fig. 2a. The state estimation error (solid line) of the fault detection estimator and its corresponding dead-zone threshold (dash-dotted line) are shown in Fig. 2b. As shown, the fault is detected at approximately  $T_d = 11.2$  sec. Moreover, in Fig. 2c and d, the residuals  $\epsilon^l(t)$  (solid lines) and their corresponding thresholds  $\lambda^l(t)$  (dash-dotted lines), associated with each isolation estimator, are shown. It can be seen that the residual of estimator 1 always remains below its threshold, whereas the residual of estimator 2 exceeds its threshold at approximately  $T_s^1 = 15.5$  sec, thus allowing the isolation of fault 1.

### CONCLUSION

In this study, a fault detection and isolation scheme using some nonlinear estimators to detect and isolate the incipient faults is presented and an aircraft's nonlinear closed-loop system using dynamic inversion method is constructed. Then the FDI scheme is applied in the aircraft's faulty system constructed before. The simulation results show the effectiveness of the application.

### REFERENCES

- Basin, M., J. Rodriguez-Gonzalez and R. Martinez-Zuniga, 2005. Optimal filtering for linear state delay systems. *IEEE Trans. Autom. Control*, 50: 684-690.
- Collins, E.G. and T. Song, 2000. Robust H8 estimation and fault detection of uncertain dynamic systems. *J. Guidance Control Dyn.*, 23: 857-864.
- Devaud, E., H. Siguerdidjane and S. Font, 2000. Some control strategies for a high-angle-of-attack missile autopilot. *Control Eng. Practice*, 8: 885-892.
- Felicio, P., J. Stoustrup, H. Niemann and P. Lourie, 2002. Applying parametric fault detection to a mechanical system. *Proceedings of the 2002 American Control Conference (IEEE Cat. No. CH37301)*, May 8-10, American Automatic Control Council, pp: 4537-4542.
- Frank, P.M. and R. Seliger, 1991. Fault Detection and Isolation in Automatic Processes. In: *Control Dynamic Systems*, Leondes, C. (Ed.). Academic, New York, pp: 241-287.
- Hofbauer, M.W. and B.C. Williams, 2004. Hybrid estimation of complex systems. *IEEE Trans. Syst. Man Cybernet. B, Cybernet.*, 34: 2178-2191.
- Ibaraki, S., S. Suryanarayanan and M. Tomizuka, 2005. Design of Luenberger state observers using fixed structured optimization and its application to fault detection in lane-keeping control of automated vehicles. *IEEE Tran. Mech.*, 10: 34-42.
- Kim, S.H., Y.S. Kim and C.H. Song, 2004. A robust adaptive nonlinear control approach to missile autopilot design. *Control Eng. Pract.*, 12: 149-154.
- Melody, J.W., T. Hillbrand, T. Basar and W.R. Perkins, 2001. H8 parameter identification for inflight detection of aircraft icing: The time-varying case. *Control Eng. Pract.*, 9: 1327-1335.
- Trunov, A.B. and M.M. Polycarpou, 2000. Automated fault diagnosis in nonlinear multivariable systems using a learning methodology. *IEEE Trans. Neural Network*, 11: 91-101.
- Wang, J.L., G.H. Yang and J. Liu, 2007. An LMI approach to H-index and mixed H-/H8 fault detection observer design. *Automatica*, 43: 1656-1665.
- Zhang, X.D., M.M. Polycarpou and T. Parisini, 2002. A robust detection and isolation scheme for abrupt and incipient faults in nonlinear systems. *IEEE Trans. Autom. Control*, 47: 576-593.