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Concatenated Space-Time Block Coding with Asymmetric MPSK TCM for Fast Fading Channels

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Abstract: The field of concatenated space-time block coding (inner) with trellis coded modulation (outer) has recently attracted interest as a means of jointly considering the error correction coding gain and diversity gain possible without bandwidth expansion and power expansion over fading channels. In this research, a concatenated Space-Time Block Coding (STBC) with asymmetric MPSK TCM scheme, based on the design criteria for constructing concatenated space-time block coding with TCM for fast Rayleigh fading channels, is presented by introducing the new optimal signal point assignment. Using parameter comparison and simulation results, the proposed concatenated STBC with asymmetric MPSK TCM is shown to have better coding gain than traditional concatenated STBC with TCM under the same spectral efficiency, decoding complexity.

Key words: Space-time block code, TCM, asymmetric phase shift keying, MPSK

INTRODUCTION

Currently space-time codes have attracted a great amount of research interest in wireless communications. Space-time code is a joint design of channel code, modulations and transmit and receive antenna diversity for Multi-Input Multi-Output (MIMO) systems. Space-time codes can be classified into three major categories: Space-Time Trellis Codes (STTC), Space-Time Block Codes (STBC) and layered space-time architecture. Space-time trellis codes have been introduced in (Tarokh *et al.*, 1998; Baro Hamsmann, 2000) to provide full diversity gain, as well as additional Signal-to-Noise Ratio (SNR) advantage that they call the coding gain. The disadvantage of this approach is that when the number of antennas is fixed, the decoding complexity grows exponentially as function of both bandwidth efficiency and diversity order. In order to combat this problem, STBC have been proposed in the literatures (Alamouti, 1998) that provide full diversity for two transmit antennas. And then, the scheme is generalized to an arbitrary number of transmit antennas (Tarokh *et al.*, 1999). Although a STBC can provide full diversity and a very simple ML decoding scheme, its main goal is not to provide additional coding gain. Hence, powerful outer codes such as Trellis Coded Modulation (TCM) can be concatenated with STBC to have a required coding gain (Alamouti *et al.*, 1998; Yi Gong *et al.*, 2002). Under

different channel environments, different criteria are needed for the design of good concatenated space-time block coding with TCM. For slow flat Rayleigh fading channels, STBC concatenated with the optimal TCM designed for the Additive White Gaussian Noise (AWGN) would have maximum coding gain (Alamouti *et al.*, 1998). In Gong *et al.* (2002), the concatenated STBC with TCM design criteria for fast Rayleigh fading channels (or with perfect interleaving) was summarized as: (1) Maximize the minimum space-time two consecutive symbol-wise Hamming distance between all pairs of distinct codewords; (2) Maximize the minimum product-sum distance over span two consecutive symbol-wise.

Based on the code design rule for fast Rayleigh fading channels, a concatenated STBC with asymmetric MPSK TCM scheme (A-TCM) is presented in this letter. In the following, we will show asymmetry-based optimization in the 8-PSK signal set which results in a performance improvement of up to 1dB for the 4-state over what was reported by Gong *et al.* (2002).

SYSTEM MODEL

For simplicity, we consider Alamouti STBC system (Alamouti, 1998) equipped with 2 antennas and 1 receive antenna and shown that it provides a diversity order of 2. The system may easily be generalized to 2 transmit antennas and M receive antenna to provide a diversity order of 2M. Figure 1 shows the block diagram of the

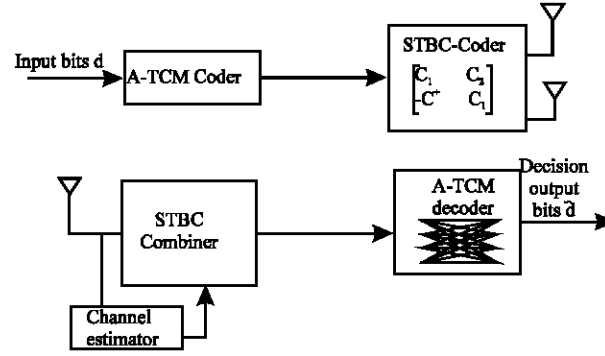


Fig. 1: Block diagram of a concatenated STBC with A-TCM scheme

concatenated STBC with asymmetric MPSK TCM scheme on fading channel considered in this study. At time slot t two signals c_1 and c_2 are transmitted simultaneously from antenna one and antenna two, respectively; at time slot $t+T$, signal c_2^* is transmitted from transmit antenna one and signal c_1^* from transmit antenna two, where c_1^* is the complex conjugate of c_1 , the transmission matrix C is given by Alamouti (1998) (Fig. 1).

$$C = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \quad (1)$$

In this study, we assume that signals transmitted from different antennas undergo independent fast Rayleigh fading (channels fading coefficients are constant over two consecutive symbol transmission periods $2T$ and vary from $2T$ symbol periods to another independently). Alternatively, this is also applicable to slow fading channels where the perfect 2-symbols-wise interleaver is exploited. The received signals over two consecutive symbol periods, denoted by for time slot t and $t+T$, respectively, can be expressed as:

$$\begin{aligned} r_1 &= r(t) = h_1(t)c_1 + h_2(t)c_2 + n_1 \\ r_2 &= r(t+T) = -h_1(t)c_2^* + h_2(t)c_1^* + n_2 \end{aligned} \quad (2)$$

Where, h_i ($i = 1, 2$) is the fading coefficient for the path from transmit antenna i to receive antenna and is complex Gaussian distributed with mean zero variance 0.5 per dimension; n_i ($i = 1, 2$) is the additive complex white Gaussian noise by receive antenna with mean zero variance $N_0/2$ per dimension. It is assumed that the channel fading coefficients are perfectly known to the receiver but unknown to the transmitter, then the received signals can be combined in the following two equations to generate estimates:

$$\begin{aligned} \hat{c}_2 &= h_2^*r_1 - h_1r_2^* = (|h_1|^2 + |h_2|^2)c_2 + h_2^*n_1 - h_1n_2^* \\ \hat{c}_1 &= h_2^*r_2 - h_1r_1^* = (|h_1|^2 + |h_2|^2)c_1 + h_2^*n_2 - h_1n_1^* \end{aligned} \quad (3)$$

These combined signals in Eq. 3 are then sent to the A-TCM Viterbi decoder for each of the signals C_0 and C_1 .

PERFORMANCE ANALYSIS AND DESIGN CRITERIA

In this study, we assume that the transmit symbols sequence is

$$C = \{c_1, c_2, -c_2^*, c_1^*, \dots, c_{2L-1}, c_{2L}, -c_{2L}^*, c_{2L-1}^*\}$$

The maximum-likelihood decoder decides in favor of the coded sequence at the receiver is

$$\tilde{C} = \{\tilde{c}_1, \tilde{c}_2, -\tilde{c}_2^*, \tilde{c}_1^*, \dots, \tilde{c}_{2L-1}, \tilde{c}_{2L}, -\tilde{c}_{2L}^*, \tilde{c}_{2L-1}^*\}$$

By using the chernoff bound (Ungerboeck *et al.*, 1982), the pairwise error probability for fast Rayleigh fading channels (or with perfect interleaving) can be got as (Gong *et al.*, 2002):

$$p(C \rightarrow \tilde{C}) \leq \prod_{t=1,3,5,\dots}^{2L-1} \left[1 + \frac{E_s}{4N_0} (|c_t - \tilde{c}_t|^2 + |c_{t+1} - \tilde{c}_{t+1}|^2) \right]^{-2} \quad (4)$$

Then at high signal-to-noise ratios ($\frac{E_s}{4N_0} \gg 1$), (4) can be written as:

$$\begin{aligned} p(C \rightarrow \tilde{C}) &\leq \prod_{t=1,2+}^{2L-1} \left[\frac{E_s}{4N_0} (|c_t - \tilde{c}_t|^2 + |c_{t+1} - \tilde{c}_{t+1}|^2) \right] \\ &= \frac{1}{\left[\left(\frac{E_s}{4N_0} \right)^h d_p(l_n) \right]^2} \end{aligned} \quad (5)$$

Where, l_η is the number of all t for which $c_t \neq \tilde{c}_t$ or $c_{t+1} \neq \tilde{c}_{t+1}$,

$$d_p(l_\eta) = \prod_{t=1,2+}^{2L-1} (|c_t - \tilde{c}_t|^2 + |c_{t+1} - \tilde{c}_{t+1}|^2)$$

is referred to as the product-sum Euclidean distance associated with two consecutive symbols along the error event path ($C \rightarrow \tilde{C}$).

Based on the union bound, an upper bound of the error event probability P_e can be written as

$$P_e \leq \sum_{L=1}^{\infty} \sum_C \sum_{\tilde{C} \neq C} P(C)P(C \rightarrow \tilde{C}) \quad (6)$$

Where, $P(C)$ is a priori probability of transmitting the symbol sequence C with length $2L$. Substituting Eq. 5 into Eq. 7, we get (Gong *et al.*, 2002)

$$P_e \leq \sum_{l_\eta} \sum_{\alpha(l_\eta)} \frac{\alpha(l_\eta, d_p(l_\eta))}{[(\frac{E_s}{4N_0})^{l_\eta} d_p(l_\eta)]^2} \quad (7)$$

Where, $\alpha(l_\eta, d_p(l_\eta))$ is the average number of error events having the effective length l_η and $d_p(l_\eta)$ is the product-sum distance over span 2. Equation 7 indicates that the concatenated STBC with TCM design criteria for high SNRs and fast Rayleigh fading channels (or with perfect interleaving) was summarized as (Gong *et al.*, 2002): (1) Maximize the minimum space-time span two symbol-wise Hamming distance l_η between all pairs of distinct codeword; (2) Maximize the minimum product-sum distance over span two consecutive symbol-wise.

At low SNRs, Eq. 5 can be rewritten as (Gong *et al.*, 2002):

$$P(C \rightarrow \tilde{C}) \leq [1 + \frac{E_s}{4N_0} \sum_{t=1}^{2L} |c_t - \tilde{c}_t|^2 + \alpha(\frac{E_s}{4N_0})]^{-2} \quad (8)$$

Where, $\alpha(\frac{E_s}{4N_0}) =$ The summation of all the terms which include higher order quantities of $(\frac{E_s}{4N_0})$. Equation 8 indicates that the dominant factor affecting the performance of the concatenated STBC with TCM at low SNRs for fast Rayleigh fading channels is the free Euclidean distance rather than the minimum product-sum distance over span 2.

CODE DESIGN

We know that the optimum 4-state symmetric 8-PSK TCM schemes for fast fading channels have been constructed (Jamali *et al.*, 1991) (referred to as J.L.-TCM

hereafter), which results in the effective length of the code error event paths is 2 and the minimum-squared product distance between signal points is $\delta_1^2 \cdot \delta_4^2 = 2.344E_s$ (Fig. 2a, 3a and 3b). Wilson and Leung (1987) (referred to as W.L.-TCM hereafter) and have suggested such a suboptimum scheme that results in the effective length of the code error event paths is 2 and actual length is 3 and the minimum-squared product distance is $\delta_1^2 \cdot \delta_2^2 = 1.172E_s$ (Fig. 3 c). But based on the design criteria presented previous section, the concatenated STBC with WL-TCM has larger performance gain than JL-TCM for fast Rayleigh fading channels (or with perfect interleaving) (Gong *et al.*, 2002). This is due to the fact that the minimum product-sum distance over span 2 for the concatenated STBC with WL-TCM is:

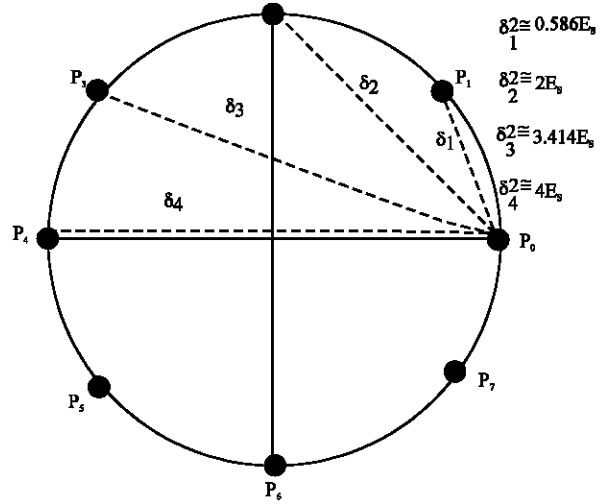


Fig. 2a: Symmetric 8-PSK with distance marked

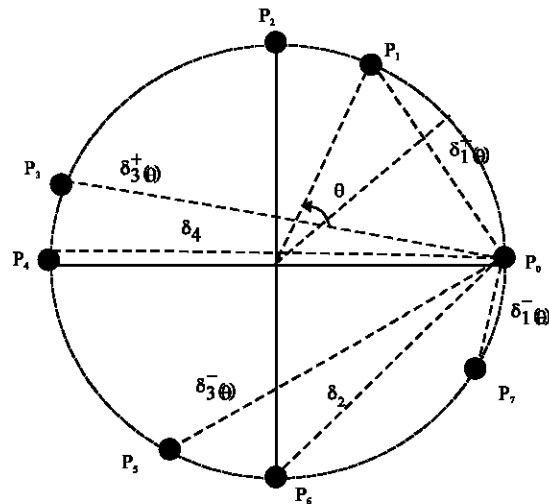


Fig. 2b: Asymmetric 8-PSK (distance marked as well as the pairings of signal points)

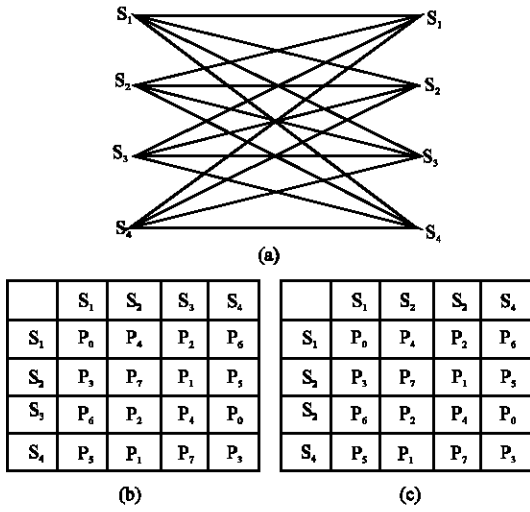


Fig. 3a: 4-state transition diagram of a full connected, TCM scheme (b) JL TCM and (c) WL TCM

$$d_p(l_\eta) = \delta_1^2 + \delta_4^2 = 4.586E_s \quad (9)$$

However, that the minimum product-sum distance over span 2 for the concatenated STBC with J.L-TCM is:

$$d_p(l_\eta) = \delta_2^2 + \delta_2^2 = 4E_s \quad (10)$$

We shown here that following the design rules of the concatenated STBC with W.L TCM for 8-PSK signal set with asymmetry (Fig. 2b) can further increases the $d_p(l_\eta)$.

Observing columns of the matrix (Fig. 3c), it follows that the term δ_1^2 in the above expression appears only in one of the following ways:

$$P_0P_1, P_2P_3, P_4P_5, P_6P_7$$

and in no other way, i.e., no other combination of the form $p_i p_{i+1}$ can appear as branch labels in the same level. If the angle of asymmetry introduced is θ as (Fig. 2b), the signal points p_1, p_3, p_5, p_7 are rotated counterclockwise with respect to the other 4 signal point and the signal points P_0, P_2, P_4, P_6 do not change their position, increases the distance δ_1^2 while the δ_4^2 is undisturbed. Let the new distances marked after introducing asymmetry be $\delta_1^-(\theta), \delta_1^+(\theta), \delta_2^-, \delta_3^-(\theta), \delta_3^+(\theta), \delta_4$. The product-sum distances over span 2 with the effective length $l_\eta = 1$ of the concatenated STBC with WL-TCM and the same product-sum distances either decrease or increase after introducing the angle θ of asymmetry as shown in the

second column (Table 1). The relationship between the minimum product-sum distances over span 2 with the effective length $l_\eta = 1$ and the angle θ of asymmetry is given by:

$$\delta_1^{+2}(\theta) + \delta_4^2 = 4 \sin^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right) + 4 \quad (11)$$

Notice that the term $\delta_1^+(\theta)$ always goes on increasing as θ increases (Table 1). The next higher product-sum distances $\delta_2^2 + \delta_3^-(\theta) = 5.4142E_s$ goes on decreasing as θ increases. Thus, while $\delta_1^{+2}(\theta) > 1E_s^2$, the minimum product-sum distances will be given by $\delta_2^2 + \delta_3^-(\theta)$ rather than $\delta_1^{+2}(\theta) + \delta_4^2$. Hence the angle θ of rotation is increased up to that value for which is given by:

$$\delta_1^{+2}(\theta) + \delta_4^2 = \delta_2^2 + \delta_3^-(\theta) \quad (12)$$

which gives the maximum counterclockwise rotation angle $\theta = \pi/12$. Substituting $\theta = \pi/12$ into Eq. 11, we get the maximum $\delta_1^{+2}(\theta) = 1E_s$ and the minimum product-sum distance over span 2 the effective length $l_\eta = 1$ is given by:

$$\delta_1^{+2}\left(\theta = \frac{\pi}{12}\right) + \delta_4^2 = 5E_s \quad (13)$$

which is higher that for the concatenated STBC with W.L-TCM (Eq. 4).

The minimum Euclidean distance d_{free}^2 of the concatenated STBC with WL-TCM after introducing asymmetry of angle $\theta = \pi/12$ is increased from:

$$d_{free}^2 = \delta_1^2(\theta) + \delta_2^2 = 2.586E_s \quad (14)$$

to the value of:

$$d_{free}^2 = \delta_1^{+2}(\theta) + \delta_2^2 = 3E_s \quad (15)$$

However, minimum Euclidean distance d_{free}^2 of the concatenated STBC with J.L-TCM is:

$$d_{free}^2 = \delta_2^2 + \delta_1^2 + \delta_1^2 = 3.172E_s \quad (16)$$

Equation 8 and 14 ~16 indicated using asymmetric 8-PSK signal set in the concatenated STBC with WL-TCM scheme at low SNRs for fast fading channels to gain performance gain over the same schemes with symmetric,

Table 1: Product-sum distance over span 2 for the concatenated STBC with WL-TCM with 4-state 8PSK

The product-sum distances $d_p(l_n)$ with the effective length $l_n = 1$	The product-sum distances $l_n = 1$ after introducing asymmetry of angle θ	
	θ	$\theta = \pi/12$
$\delta_1^2 + \delta_2^2 = 4.586E_s$,	$\delta_1^2(\theta) + \delta_4^2 = 4 + 4 \sin^2(\frac{\pi}{8} + \frac{\theta}{2})$	$5E_s$
$\delta_3^2 + \delta_5^2 = 5.414E_s$,	$\delta_3^2 + \delta_3^2(\theta) = 4 + 4 \cos^2(\frac{\pi}{8} + \frac{\theta}{2})$	$5E_s$
$\delta_2^2 + \delta_4^2 = 6E_s$,	$\delta_2^2 + \delta_4^2 = 6E_s$,	$6E_s$
$\delta_3^2 + \delta_5^2 = 8E_s$,	$\delta_3^2 + \delta_5^2 = 8E_s$,	$8E_s$

Table 2: d_{free}^2 and $d_p(l_n)$ of the concatenated STBC with different TCM schemes

Different TCM schemes	U-TCM	ML-TCM	J.L-TCNM	Asymmetric 8-PSK TCM
The shortest length of error event L	1	2	2	2
l	1	1	1	
$d_p(l_n)$	$4E_s$	$4E_s$	$4.586E_s$	$5E_s$
d_{free}^2	$4E_s$	$3.172E_s$	$2.586E_s$	$3E_s$

but still lost performance gain to the concatenated STBC with JL-TCM scheme. The minimum Euclidean distance d_{free}^2 and the minimum product-sum distance over $l_n = 1$ span 2 of the concatenated STBC with different TCM schemes (Table 2).

SIMULATION RESULTS

Here, we show simulation results for the performance of the concatenated STBC with different TCM scheme on fast Rayleigh fading channels (channel fading is constant in a time interval of duration 2-symbol and vary from 2-symbol to another independently). In the simulations, the performance is measured by the Frame Error Rate (FER) for a frame of 130 symbols and ideal channel state information is known at receiver.

4-state rate 2/3 8-PSK J.L, W.L and the asymmetric TCM of this paper (counterclockwise rotation angle $\theta = \pi/12$) concatenated with STBC are simulated for $N = 2$ transmit antennas and $M = 1$ receive antenna Fig. 4. The code labeled A-TCM denotes the proposed the concatenated STBC with asymmetric TCM. The performance result of the 4-PSK STBC proposed in (Alamouti, 1998) only is listed for comparison at a spectral efficiency of 2b/s/Hz. It is not surprising that the proposed asymmetric TCM concatenated with STBC has better performance than the other three schemes at high SNRs.

It is a remarkable result that the counterclockwise rotation angle $\theta = \pi/12$ shows best performance than the other rotation angle (at $FER > 1 \times 10^{-1}$) (Fig. 5). This is consistent with the analytical result of Section IV. However, A-TCM with counterclockwise rotation angle $\theta = \pi/12$ has worse performance than clockwise rotation angle $\theta = \pi/12$ (at $FER > 5 \times 10^{-1}$). This is due to the fact that the derivation of the design criteria for the

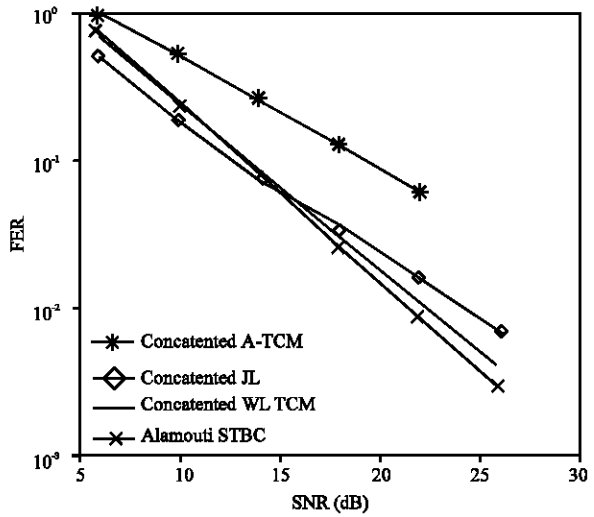


Fig. 4: The concatenated STBC with different TCM

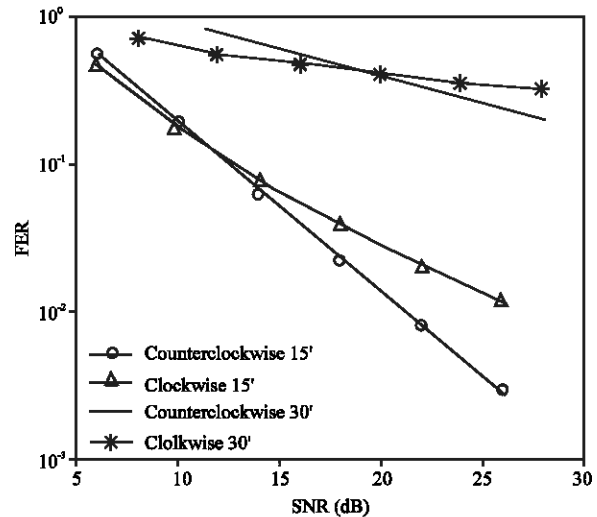


Fig. 5: The concatenated STBC with A-TCM of different rotation angle θ

concatenated STBC with TCM scheme is based on the assumption that the SNR is sufficiently high. At low SNR, the assumption may not be valid and thus the design criteria may not be optimal.

CONCLUSION

In this study, the concatenated STBC with 4-state asymmetric 8-PSK TCM was designed based on the design criteria for fast fading channels introduced in (Yi Gong *et al.*, 2002). Simulation results showed that this code has better performance than the other 4-state symmetric 8-PSK TCM schemes concatenated with STBC at high SNR. This implies power saving over symmetric 8-PSK TCM. Higher SNR may result in more gains but at the optimum asymmetric angle. However, bringing the points too close together makes a system more sensitive to phase jitter due to imperfect carrier synchronization and must be avoided.

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