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Analysis of Structure properties of Petri Nets Using Transition Vectors

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Abstract: This study introduces transition vectors based on place transitive matrix derived from graph theory, to study the structure of Petri nets using structure theoretical results that exists in Petri net theory. It has been established that transition vectors provide a simplified and more adequate approach than transitive matrix towards the structural analysis of PN. Some structural classes of Petri nets have been decided and basic concepts about the structure of Petri net have been derived through novel idea of transition vectors. Firstly new representation of place transitive matrix has been introduced for acyclic Petri nets. Secondly Petri net structure has been analyzed and an algorithm to find a directed cycle has been presented with a simplified representation, using transition vectors. Thirdly transition vectors have efficiently been used to identify the particular structures of Petri nets. Finally, useful concepts relevant to the structure of Petri nets have been derived.

Key words: Structure properties, petri net, transition vectors, transitive matrix

INTRODUCTION

Petri Nets (PNs) are promising mathematical and graphical tool for modeling, analysis and design of information processing systems. Petri nets have been frequently used to model and analyze systems that are characterized as being distributed, parallel, concurrent, asynchronous nondeterministic, having discrete events, conflicts, deadlocks or resource sharing (Zurawski and Zhou, 1994; Murata, 1989; Peterson, 1981). Petri nets have a power to analyze the modeled system revealing important information about structure and dynamic behavior of system and same model can be used for performance evaluation. Petri net method for the specification and verification of systems is becoming more and more important as systems increase in size and complexity.

The structural analysis of PN depicts useful information about the behavior of modeled system and the structure of underlying net. Since PN corresponds in a natural way to a directed graph, so the static structure of PN facilitates to study structural i.e., graph theoretic properties depending on the place-transition relationship of underlying net by the flow relation. The study of structure of PNs has the objectives to increase their modeling power and to study the underlying properties of model. The structural analysis of PN has advantage over reachability tree method which exhibit state space explosion problem for larger and concurrent systems. Using structural analysis, important information about the behavior of the modeled system can be achieved. The behavioral properties that are of foremost interest and less easily analyzable in the analysis of systems may be reduced to easier-to-investigate structural properties (Best, 1987). Parallel and cycle structure of PN has been used for classification of solution of matrix equation in order to obtain the firing count vector as a solution of matrix equation (Miyazawa *et al.*, 1996). Up till now, no broad structural classification of PNs has been decided instead of general Pns, ordinary Pns, self-loop free Pns and restricted PNs defined by Peterson (1981). But some important structural sub-classes of PN have been investigated in the literature (Best, 1987; Murata, 1989; Amer *et al.*, 1999). Place and transition invariant method has been used to investigate the structural properties of the underlying unmarked net by Murata (1989). The relationship between PN structure and directed graph has also been discussed and result has been described for a cycle structure of PN (Miyazawa *et al.*, 1998). A method for structural analysis of PN model using transition invariants has been proposed by Cai *et al.* (1993). Authors identify the cycle or/and parallel structure by the solutions of the homogeneous equations of transition invariants. However, study of some basic properties such as boundedness and liveness etc., through structural analysis of PNs is still to be investigated.

Transitive matrix depicts all the place-transition relationships and entries of transitive matrix describe the transferring relation from one place to another place

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through transitions. The characteristic polynomial equation of transitive matrix has been proposed to identify the cyclic structure in PNs (Itoh *et al.*, 1999). Transitive matrix method has also been introduced to study the behavioral properties of PN model (Itoh *et al.*, 1999; Liu *et al.*, 1999). Transitive matrix has been used for decomposition and slicing the basic unit of concurrency in PN model (Lee and Korbaa, 2005; Lee, 2005, 2002).

In this study, new concept of transition vectors to represent and analyze the structure of PN has been proposed. Transition vectors provide a simplified approach to describe the structure of PN and all the information relevant to the structure. New representation of transitive matrix has also been suggested. Some important concepts relevant to structure and classes of PN have been verified using transition vectors.

BASIC DEFINITIONS AND CONCEPTS

Here, some basic definitions, notations, structural classification of ordinary PN (for the sake of simplicity) and its matrix representation are described. The related terminology and notations are mostly taken from (Peterson, 1981; Murata, 1989). Directed and transitive graphs are also explained. The concepts and terminology related to graph theory are taken from (West, 2001).

Definition 1: (Petri net structure) A Petri net structure N, is a four tuple, N = (P, T, I, O) where $P = \{p_1, p_2, ..., p_{|P|}\}$ is a finite set of places, $|P|>0$; $T = \{t_1, t_2, ..., t_{|T|}\}$ is a finite set of transitions, $|T|>0$; I: $T \rightarrow P^{\infty}$ is the input function, a mapping indicating the directed arcs from places to transitions; O: $T \rightarrow P^{\infty}$ is the output function, a mapping indicating the directed arcs from transitions to places, $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

Let I (t_j) represents the set of input place(s) of transition $t_j \in T$, then place $p_i \in P$ is an input place of a transition t_i if $p_i \in I(t_j)$; O (t_j) represents the set of output place(s) and p_i is an output place of t_j if $p_i \in O(t_j)$. Incoming arc to t_j is represented by $(p_i, I(t_j))$ and outgoing arc from t_j be $(p_i, O(t_j))$. The input and output functions can be extended to map set of places P into the set of transitions T. Then arc $(t_j, I(p_i))$ represents the incoming arc to p_i as $t_j \in I(p_i)$ and arc $(t_j, O(p_i))$ represents outgoing arc from p_i as $t_j \in O(p_i)\ \forall\ t_j \in T,\ \forall p_i \in P$.

There exists classification of the topological structure of PN having specific characterizations. The following definitions of structural subclasses of PN are due to (Peterson, 1981; Murata, 1989).

Definition 2: (Self-loop-free PN) A PN structure N = (P, T, I, O) is said to be self-loop-free or pure if and only if $\forall t_j \in T$, $I(t_j) \cap O(t_j) = \emptyset$ i.e., no place may be both an input and an output of the same transition.

Definition 3: (State-machine) A PN structure N is said to be state-machine if and only if $\forall t_j \in T$, $|I(t_j)| = 1 = |O(t_j)|$ i.e., every transition $t_j \in T$ has exactly one input place and exactly one out put place.

Definition 4: (Marked graph) A marked graph is a PN N such that $\forall p_i \in P$, $|I(p_i)| = 1 = |O(p_i)|$ i.e., for every place $p_i \in P$, there exist one and only one input transition and one and only one output transition.

Definition 5: (Free-choice Petri net) PN N is said to be free-choice Petri net if and only if $\forall p_i \in P$, $\forall t_j \in T$, either $I(t_j) = \{p_i\}$ or $O(p_i) = \{t_j\}$.

Definition 6: (Asymmetric choice PN) an asymmetric choice Petri net is a PN N such that $\forall p_i, p_j \in P$: $O(p_i) \cap O(p_j) \neq \emptyset \Rightarrow (O(p_i) \subseteq O(p_j) \vee O(p_j) \subseteq O(p_i))$.

Definition 7: (Matrix definition of PN structure) A PN $N = (P, T, B^-, B^+)$ where B^- and B^+ are $|P| \times |T|$ matrices of input and output functions, respectively, defined by: $B^-[I, j] = 1$, if there is an arc $(p_i.\ I(t_j))$ or 0 otherwise. $B^+[i, j] = 1$, if there is an arc $(p_i, O(t_j))$ or 0 otherwise. The incidence matrix B is defined by $B = B^+ - B^-$. For example, matrix of input function B^-, matrix of out put function B^+ and incidence matrix B of PN in Fig. 1c are, respectively given as $B^- = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B^+ = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$.

Definition 8: (Graph and directed graph); A graph G = (V, E) consists of a set V of vertices and a set E of edges. If E is set of directed arcs, G is called a directed graph or digraph denoted by D.

Since PN corresponds to a directed graph, so the following concepts are taken directly from graph theory and explained in the context of PNs. A directed path is a set of k nodes $x_i \in P \cup T$: $\forall i = 1,....,k$ and set of k-1 arcs $E \subseteq (P \times T) \cup (T \times P)$, the ith arc connects the ith node to (i+1)th node. The directed cycle is a directed path from one node back to itself. PN structure has two important types of connectivity, intensively studied to explain and verify many basic properties of system modeled by PN i.e., connected PNs and strongly connected PNs. A PN is simply connected if, for any pair of nodes $x_i, x_j \in P \cup T$, there is a path either from x_i to x_j or from x_i to x_j i.e., one of

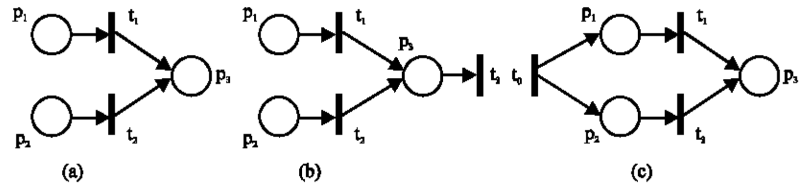

Fig. 1: Different petri net models; (a) without source and sink transition (b) with sink transition and (c) source transition

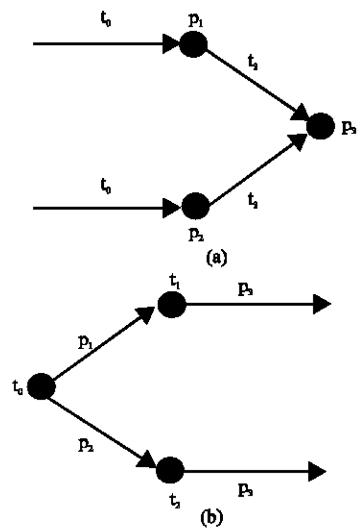

Fig. 2: (a) The place transitive graph of Fig. 1c (b) The transition transitive graph of Fig. 1c

them is reachable from other. A PN is said to be strongly connected if any node $x_i \in P \cup T$ is reachable from any other node $x_j \in P \cup T$. A connected PN is strongly connected if and only if for each directed arc (x_i, x_j) there is a directed path leading from x_i to x_j.

Definition 9: (Transitive graph) A directed graph $D_p = (V_P, E_P)$ (or $D_T = (V_T, E_T)$) for PN N is called a place (or transition) transitive graph, where $V_P = P$ (or $V_T = T$) is the set of vertices and $E_P = T$ (or $E_T = P$) is the set of directed arcs and $t_k \in E_P$ (or $p_k \in E_T$) is input transition (or place) of p_i (ort_i) and the output transition (or place) of place p_j (ort_j), respectively. For example, Fig. 2a shows the place transitive graph and Fig. 2b is a transition transitive graph for PN model shown in Fig. 1c.

TRANSITION VECTORS

Here, definitions of transitive matrix and labeled place transitive matrix are presented which are based on (Itoh *et al.*, 1999; Liu *et al.*, 1999). New representation of place transitive matrix and the idea of transition vectors are also being introduced in order to analyze the structure of PN.

Definition 10: (Transitive matrix) Let $V = P \cup T$ be the set of vertices and $E \subseteq (P \times T) \cup (T \times P)$ the set of directed arcs. D = (V, E) is a directed graph of the PN N. The adjacent matrix A and transitive matrix S are $A = \begin{bmatrix} 0 & (B^+)^T \\ B^- & 0 \end{bmatrix}$ and $S = AA = \begin{bmatrix} (B^+)^T B^- & 0 \\ 0 & B^-(B^+)^T \end{bmatrix}$. Then $B_P = B^-(B^+)^T$ and $B_T = (B^+)^T B^-$ are the place transitive matrix and transition transitive matrix, respectively.

Definition 11: (Labeled place transitive matrix) Let L_{B_P} be the labeled place transitive matrix, defined by $L_{B_P} = B^- \mathrm{diag}(t_1, t_2, \ldots t_{|T|})(B^+)^T$, where t_i $(i = 1, 2, \ldots |T|)$ represent the labels. The entry in the ith row and jth column as $L_{B_P} = [i, j] = t_k$ means the transitive relation from input place p_i (row i) to the output place p_j (column j) through firing of the transition t_k.

But L_{B_P} of PN having source or/and sink transition(s) can not include all the transitions appearing in the structure of PN. If $t_j \in T$ is a source transition of given Petri net such that I $(t_j) = \varnothing$ and $p_i \in$ O (t_j) is the output place of source transition t_j, then all the entries of ith column of L_{B_P} are zero. Similarly if t_j be a sink transition in PN as O $(t_j) = \varnothing$ and $p_i \in$ I (t_j), then all the entries of ith row of L_{B_P} are zero. So place transitive matrix representation of PN having source or/and sink transition(s) does not give the complete portrait of PN model e.g., PNs in Fig. 1a-c all have the same transitive matrix i.e., $L_{B_P} = \begin{bmatrix} 0 & 0 & t_1 \\ 0 & 0 & t_2 \\ 0 & 0 & 0 \end{bmatrix}$.

In order to overcome the vague representation of place transitive matrix of Petri net having source or/and sink transition(s), new representation of L_{B_P} is suggested. This advantageous representation admits all the transitions of PN model to appear in L_{B_P}, provides the complete place-transition relationship for PN having source or/and sink transition(s) and accommodating the synthesis to Petri net model.

Property 1: If Petri net N has source (sink) transition t_j such that $I(t_j) = \varnothing \wedge p_i \in O(t_j)\left(O(t_j) = \varnothing \wedge p_i \in I(t_j)\right)$, then the first entry of ith zero column (row) of transitive matrix will be labeled as $L_{B_P} = [1,i] = 0/t_j\,(L_{B_P}[i,1] = 0/t_j)$.

The example of new representation of place transitive matrix of PNs in Fig. 1b and c can be shown as $L_{B_P} = \begin{bmatrix} 0 & 0 & t_1 \\ 0 & 0 & t_2 \\ 0/t_3 & 0 & 0 \end{bmatrix}$ and $L_{B_P} = \begin{bmatrix} 0/t_0 & 0/t_0 & t_1 \\ 0 & 0 & t_2 \\ 0 & 0 & 0 \end{bmatrix}$, respectively.

Property 2: Let N be a PN having $|P| \times |P|$ labeled place transitive matrix L_{B_P}, then $T_R = \left[\sum_{i=1}^{|P|} L_{B_P}[i,1] \;\; \sum_{i=1}^{|P|} L_{B_P}[i,2] \ldots \sum_{i=1}^{|P|} L_{B_P}[i,|P|] \right]$ is a row $|P|$-vector of transitions and $T_C = \left[\sum_{j=1}^{|P|} L_{B_P}[1,j] \;\; \sum_{j=1}^{|P|} L_{B_P}[2,j] \ldots \sum_{j=1}^{|P|} L_{B_P}[|P|,j] \right]^T$ is a column $|P|$-vector of transitions. Transition vector T_R is mapping as $T_R : P \rightarrow I(p_i)\ \forall i = 1,2,\ldots,|P|$ where I:P→T is input function; T_C is a mapping where $T_C : P \rightarrow O(p_i)$ $\forall i = 1,2,\ldots,|P|$ where O:P → T is output function.

So ith component of T_R which is T_R (p_i) gives the set of input transitions of place p_i and ith component T_C (p_i) is a set of output transitions of place p_i.

The components of T_R and T_C are finite linear combinations of transitions with positive integer coefficients. Coefficients of transitions appearing in transition vector T_R (T_C) represents number of incoming arcs i.e., indegree or number of input places (number of outgoing arcs called outdegree or number of output places) of transitions. The ith zero component of transition vector T_R (T_C) can be interpreted as source place p_i (sink place p_i) such as I (p_i) = Ø (O (p_i) = Ø). Labeled zero component e.g., $0/t_j$ appearing in transition vector T_R (T_C) indicates the zero incoming arc to (outgoing arc from) t_j and hence t_j stands for source (sink) transition. As the first labeled zero entry(ies) of zero column(s) (zero row(s)) belong only to zero column(s) (zero row(s)) of transitive matrix L_{B_P}, therefore they can only be added to T_R (T_C) e.g., transitions vectors T_R and T_C of PN in Fig. 1c can be written as $T_R = [0/t_0 \;\; 0/t_0 \;\; t_1{+}t_2]$ and $T_C = [t_1 \;\; t_2 \;\; 0]^T$, where first and second component of T_R which is $0/t_0$ indicate the zero indegree of transition t_0 i.e., source transition while third zero component of T_C communicates p_3 as sink place.

ANALYSIS OF STRUCTURE PROPERTIES OF PN USING TRANSITIONS VECTORS

Here, some structural classes and fundamental properties about the structure of PN are presented in terms of transition vectors in order to analyze the structure of PN. An algorithm to find a directed cycle is also presented.

Property 3: A PN N is acyclic if its transition vectors T_R or/and T_C has at least one zero component (labeled or/and unlabeled). As explained for property 2, zero component(s) of T_R or/and T_C correspond to source or/and sink place(s), respectively; labeled zero component(s) of T_R or/and T_C correspond to source or/and sink transition(s), respectively. So PN structure is said to acyclic if it has at least one source or/and sink place (or/and transition). An acyclic PN does not imply that it can never contain a cycle.

Property 4: A PN N is cyclic if and only if all the components of both the transition vectors T_R and T_C are non-zero. This condition refers to the situation that there is neither any source and sink place nor any transition in the PN structure. In words, the structure of PN will be cyclic if it doesn't have any source and sink place (and transition).

Property 5: A PN N is self-loop free or pure if and only if the corresponding same components T_R (p_i) and T_C (p_i) don't have identical transition ∀ i = 1, 2,,...,|P| i.e., there does not exist t_k ∈ T such that t_k ∈ T_R (p_i) and t_k ∈ T_C (p_i) ∀ i = 1, 2,....,|P|. In words, if different transition(s) are appearing in each corresponding same components of both T_R and T_C then PN structure is said to be self-loop free.

Property 6: Every acyclic PN has at least one sink place (transition) and at least one source place (transition).

Proof: Since N is acyclic, so by property 3 its transition vectors $T_R$or/and T_C must have at least one zero (labeled or/and unlabeled) component. Choose any node x_1 ∈ P∪T, if x_1 is a source place e.g., p_i ∈ P (transition e.g., t_k ∈ T), then ith component of T_R will be zero (labeled zero), otherwise there will be a transition t_k ∈ T_R (p_i) such that t_R ∈ I (p_i) (p_i ∈ O (t_k)). Similarly, by scanning other entries of T_R, there must be source place (transition). Now, choose any other node x_2 ∈ P∪T, if x_2 is sink place e.g., p_j ∈ P (transition t_S ∈ T), then jth component of T_C will be zero (labeled zero), otherwise there will be a transition t_S ∈ T_C (p_j), such that t_S ∈ O (p_j), (p_j ∈ I (t_S),). Similarly, scanning other entries in T_C, there must be sink place (transition).

Property 7: Every self-loop free cyclic PN is strongly connected.

Proof: By property 4 Cyclic PN, every entry of transition vectors T_R and T_C must have at least one transition. As no transition and place of PN is on a self-loop. So it is possible to leave any place p_i ∈ P through an arc (p_i I (t_j)) such as $t_j \in T_C$ (p_i) and to enter any place $p_k \in P$ through an arc (p_k O (t_j)) such as $t_j \in T_R$ (p_k). So there exist directed path from any node $x_i \in P \cup T$ to any other node $x_j \in P \cup T$. These arguments establish the statement of the property.

Property 8: A cyclic PN having place $p_i \in P$ on self-loop is strongly connected iff ith entry of transition vector T_C has at least two transitions or/and the transition on self-loop with p_i has coefficient at least 2.

Proof: Suppose that place $p_i \in P$ is on self-loop with transition $t_j \in T$ in the strongly connected cyclic PN N. So by property 5, $\exists t_j \in T_R$ (p_i) and $t_j \in T_R$ (p_i). By definition of strongly connectedness, there must be a directed path from any node $x_i \in P \cup T$ to any other node $x_j \in P \cup T$. Firstly, we can not leave place $p_i \in P$ through an arc (p_i, I (t_j)) as p_i is on a self-loop with t_j. The condition of strongly connectedness is violated if we don't have an arc (p_i, I (t_k) such that $t_k \in T_C$ (p) other than $t_j \in T_C$ (p). So, ith component of transitions vector T_C must have at least two transitions. Secondly, we can not leave t_j which is on a self-loop, as there is only one out going arc from t_j to self-loop place p_i, which violates the strongly connectedness condition. So, t_j must have coefficient at least 2 which indicates the outdegree of t_j at least 2, in order to find directed path from self-loop to any node of N. So both or one of these two are necessary conditions for strongly connectedness.

Conversely, suppose $p_i \in P$ on a self-loop with $t_j \in T$ in cyclic PN N, ith component of T_C has at least one transition other than $t_j \in T_C$ (p_i) or/and t_j has coefficient at least 2. Then we can leave place p_i through transition $t_k \in T_C$ (p_i) other than $t_j \in T_C$ (p_i) or/and t_j can be left to enter a place other than p_i. As given PN is cyclic having no zero entry in transition vectors T_R and T_C, so from property 7, there exist directed path from any node $x_i \in P \cup T$ to any other node $x_i \in P \cup T$. Hence the given cyclic PN is strongly connected.

Property 9: If N be a PN without any source and sink place (and transition), then there is a directed cycle in N.

Proof: According to the property 4, N given in the statement of property is cyclic which implies that transition vectors T_R and T_C both have all non-zero components. As all entries of T_C have at least one transition, so each place is the input of at least one transition i.e., $p_i \in I(t_j)$ such that $t_j \in T_C$ (p_i) $\forall$ i = 1, 2,...,|P| and $\forall$ j = 1, 2,...,|T|. All entries of T_R also have at least one transition such that $p_i \in O(t_j)$, $t_j \in T_R$ (p_i) $\forall$ i = 1, 2,...,|P| and $\forall$ j = 1, 2,...,|T|. Then we can leave every node $x_i \in P \cup T$

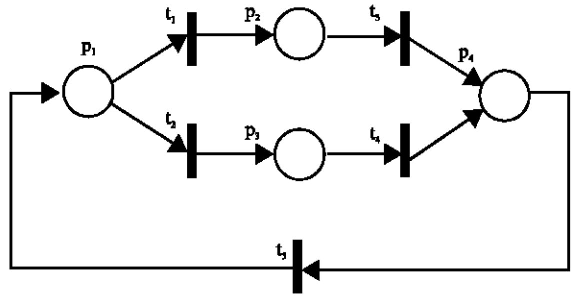

Fig. 3: An example of PN model

through its outgoing arc and can enter to every node $x_j \in P \cup T$ through its incoming arc. So, we stop as soon as we obtain a repeated node in directed path getting directed cycle and give the proof of the statement.

Directed cycles play a key role in the structure theory and analysis of modeled systems by PNs. The existence of directed cycle in the PN structure assists in verifying important structural properties. An algorithm is presented in order to find directed cycles in the cyclic PN structure using transition vectors.

Algorithm to find a directed cycle: INPUT: Transition vectors T_R and T_C.

(1) Select ith entry of T_C i.e., T_C (p_i) for i = 1
(2) If selected place is same as previously selected place then, Go to step 6, else go to next step.
(3) Do
    (i) Select first transitions in the ith component of T_C.
    (ii) If selected transition is same as previously selected transition then go to step 6, else go to next step.
(4) Do
    (i) Scan all components of T_R to locate selected transition in step 3 and select transition in first component having selected transition.
    (ii) Select an entry (place) in T_R corresponding to selected transition in step 4(i)
(5) Go to step 2
(6) OUT PUT: Directed cycle.
(7) END

The algorithm given can efficiently be used to find directed cycle in cyclic PN. For example PN model given in Fig. 3, has transition vectors T_R and T_C obtained from transitive matrix L_{B_P}, given by $T_R = [t_5 \quad t_1 \quad t_2 \quad t_3{+}t_4]$ and $T_C = [t_1{+}t_2 \quad t_3 \quad t_4 \quad t_5]$.

First entry of T_C corresponds to place p_1, having transitions t_1 and t_2 in the first component of T_C. After selecting t_1, second component of T_R contains t_1 which corresponds to second entry i.e., p_2, getting directed path $p_1 \rightarrow t_1 \rightarrow p_2$. Augmenting the iterations of algorithm, directed cycle is given by $p_1 \rightarrow t_1 \rightarrow p_2 \rightarrow t_3 \rightarrow p_4 \rightarrow t_5 \rightarrow p_1$. Similarly, the directed cycle $p_1 \rightarrow t_2 \rightarrow p_3 \rightarrow t_4 \rightarrow p_4 \rightarrow t_5 \rightarrow p_1$ can be obtained using above mentioned algorithm.

IDENTIFICATION OF PARTICULAR STRUCTURES USING TRANSITION VECTORS

Certain PNs have structural characteristics and properties not possessed by the other PN structures. In this section, transition vectors T_R and T_C are efficiently used to identify these structural sub-classes of PNs.

Property 10: PN structure is said to be state machine if and only if there does not exists any transition $t \in T$ such that

- $t \in T_R(p_i)$ and $t \in T_R(p_j)$ $\forall i, j = 1, 2, \ldots, |P| i \neq j$ and
- $t \in T_C(p_i)$ and $t \in T_C(p_j)$ $\forall i, j = 1, 2, \ldots, |P|, i \neq j$

In words, If every component of T_R and T_C is a distinct transition(s), then state machine structure of PN can be identified.

Property 11: A PN N is said to be a marked graph if and only if $\#(T_R(p_i)) = \#(T_C(p_i)) = 1 \;\; \forall i = 1, 2, ..., |P|$; i.e., single transition (not necessarily distinct) is appearing in all components of transition vectors T_R and T_C, then PN structure is called as marked graph.

Note: $\#(T_R(p_i))$ (respectively, $\#(T_C(p_i))$ is the number of transitions in ith component of T_R, (respectively, T_C).

Property 12: A PN N is said to be a conflict free or persistent if and only if

- $\#(T_C(p_i)) \leq 1 \;\; \forall i = 1, 2, \ldots, |P|$, or
- If $\exists T_C(p_i)$ such that $\#(T_C(p_i)) > 1$ then $\forall t \in T_C(p_i), t \in T_R(p_i)$

In words, a structure is said to be a conflict free Petri net if and only if every component of T_C has at most one transition, or if any component of T_C has more than one transition then these transitions must appear in the corresponding same component of T_R. A PN structure has conflict or choice if at least one component of vector T_C has more than one transition. For T_C having multiple transitions at ith entry, provided self-loop exception of place p_i with these transitions, the case corresponds to the situation of conflict.

Property 13: A PN structure is said to be simple Petri net if and only if

$$\forall t \in T_C(p_i) \Rightarrow t \in T_C(p_j) \;\vee\; \forall t \in T_C(p_j) \Rightarrow t \in T_C(p_i), \;\forall i, j = 1, 2, \ldots, |P|, i \neq j$$

In words, if all the transitions of ith component are included in jth component of T_C or all the transitions of jth component are included in the ith component of T_C, then simple PN structure can be identified.

PN structure represents to have concurrent or parallel activity if and only if at least one entry of vector T_c has coefficient 2 or greater than two. If every component of T_C of PN structure has distinct transition(s) i.e., every transition $t \in T$ has exactly one input place, then PN structure has Basic Parallel Process (BPP). In formal way, there does not exist $t \in T$ such that $t \in T_C(p_i)$ and $t \in T_C(p_j) \; \forall i, j = 1, 2, \ldots, |P| i \neq j$. A PN structure having BPP is called BPP-net (Esparza and Nielsen, 1994).

For example, all the components of both the transition vectors T_R and T_C of PN model shown in Fig. 3 are non-zero and no corresponding same entries of T_R and T_C have identical transition, so given PN structure is cyclic and self-loop free. Property 7 implies that given PN is strongly connected. As all components of transition vectors T_R and T_C are distinct transition(s), so state-machine structure of PN having basic parallel process (BPP) must be identified. First component of transition vector T_C has two transitions i.e., $\#(T_C(p_1)) = 2$ such as $t_1, t_2 \in T_C(p_i)$. Property 12 implies that transitions t_1 and t_2 are on conflict or have choice to fire when p_1 is marked. So, either firing sequence (t_1t_3) or (t_2t_4) can occur.

CONCLUSION

In this study, novel concept of transition vectors has been introduced to analyze the structure of PN. It has been shown that transition vectors provide complete information about the structure of PNs is symmetrical manner as they are bijection of the set of places P to their sets of input and output transitions which can not be observed in transitive matrix. Some structural classes of PNs have been decided and basic concepts about the structure of PN have been established by using the simplified approach of transition vectors. Simple algorithm to find a directed cycle in PN structure has been presented using transition vectors. Transition vectors have efficiently been used to identify the structural subclasses of PNs. New representation of place transitive matrix for acyclic PN has also been introduced which provides the complete list of transitions and elaborate the place-transition relationship in transitive matrix, whenever acyclic PN have source or/and sink transition(s). It has been established that transition vectors provide a simplified and more adequate approach than transitive matrix towards the analysis and depiction of important properties of PN structure. Some fundamental marking-independent properties of PN model of system e.g. boundedness, liveness and conservativeness etc, will be studied using transition vectors.

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