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Heuristic Strategies Training with the Use of Cooperative Computer-Assisted Instruction in Mathematical Problem Solving

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Abstract: The present study investigates the effects of Cooperative Computer-Assisted Instruction (CCAI) in mathematical problem solving. The main questions are (1) can heuristic strategies training via CCAI make a difference in mathematical problem solving performance and (2) to what extend the students benefit from a heuristic strategy via CCAI. The research contains an experimental study in mathematical problem solving. The participants are 29 college students that learned heuristic strategies with the use of produced computer software via cooperative computer-assisted instruction. The study revealed that when technology is used appropriately in college mathematics, it can have positive effects on students' attitudes toward mathematics learning and create a more constructivist-based learning environment. The results indicate that the explicit mention of the heuristic strategies served to bring those skills to the students' attention and to help them codify and recognize their existing knowledge in such a way that those skills could now be accessed and used more readily.

Key words: Mathematical problem solving, heuristic strategies, cooperative learning, cooperative computer-assisted instruction

INTRODUCTION

All around the world, the discussion about problem solving and enhancement of students ability in performing problem solving has been one of the most important bases in mathematics education. A glance at the history of problem solving shows that the bright, systematic and notable points related to this debate has been found on Polya's works. Polya's books are a rich source of inspiration for teaching problem solving. He suggested a framework for teaching meta-reasoning in mathematical problem solving. In his famous book "How to solve it" (Polya, 1945), he points out the ways and techniques by which we can solve problems. He believed that to solve any mathematics problem, we must go through four processes: understanding the problem, designing the plan, carrying the plan and looking back. In this book these four processes have been described in details.

In the last 1970's and early 1980's Schoenfeld designed courses in problem solving, arranged sessions and prepared video cassettes from these sessions. When analyzed them, he found some behaviors which are useful for problem solving process. The results of his studies have been published in his mathematical problem solving book (Schoenfeld, 1985).

Schoenfeld in his book introduced and discussed four fields related to knowledge and behavior of mathematical problem solving: resources, heuristics, control and beliefs systems. Schoenfeld's work is important because he examined most of polya's thoughts and theories in a fine frame and mixed them with psychological and social aspects and tested them in practice (Farhadian *et al*., 2007).

Cooperative learning is a kind of teaching and learning strategy in which students study together in small groups and follow their common aims. It is one strategy that rewards individuals for participation in the group's effort. In small groups, the attempts of any one are useful for other members in group. A review of the literature on cooperative learning shows that students benefit academically and socially from small-group learning (Gillies, 2002). Cooperative learning is not simply a matter of grouping students heterogeneously but also in understanding that some groups of students are more inclined to function better in group settings than individually (Vaughan, 2002).

Other effective method in learning is computer-assisted instruction. In this method, students learn the lesson by the attractive, intelligent and multimedia software and lesson subjects proposed in pleasant way for student. Some of its advantages are: wide efficacy and

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interesting work with computer, computer simulation, relative reaction, immediate feedback, Instruction, assessment, educational achievement, helping to make mathematics public, to take exam without fear and anxiety.

When the two mentioned instructional methods (cooperative learning and computer-assisted instruction) are combined with each other, a new method is created which is called cooperative computer-assisted instruction. In fact it is one of strategies of cooperative learning. In CCAI method, students work with each other in collective groups and do their duties and assignment that computer designs. The students become less tired because they work by computer and collectively. The computer software guides students intelligently and actively and they work with a multimedia software and communicate two-tailed with lesson. In this method, teaching by computer is used as one of the successful cooperative learning strategies. Usually teaching by computer is designed for individual learning, but what had been done by Dalton *et al*. (1989), Klein and Pridemore (1992), Mevarech (1993) and Xin (1999) indicated that instruction in small group is more useful than individually. There are many reasons to support using cooperative learning methods to improve students' learning and their educational achievement. Some of these reasons are: economic profits, reduction of math-anxiety in low ability students, enhancement of better in problem solving skills in students, reduction of the gap between weak and powerful students. In later, we will analysis a sample of experimental design done in heuristic strategies instruction using a certain method. Some studies have found increased use of technology to be associated with enhanced learning environment and elevated student engagement patterns. Technology can affect a mathematical learning environment by changing a teacher's instructional techniques. Teachers using technology as an integral part of their mathematics instruction have been shown to foster a more constructivist-based learning environment (Guerrero *et al*., 2004).

The literature of mathematics education is full of studies about heuristic strategies. One of the effective factors in mathematical problem solving is attending the heuristic strategies. Schoenfeld defined heuristic strategies as rules of thumb for successful problem solving, general suggestions that help an individual understand a problem better or make progress toward its solution. Such strategies include exploiting analogies, introducing auxiliary elements in a problem or working auxiliary problems, arguing by contradiction, working forward from the data, decomposing and recombining, exploiting related problems, drawing figures and etc. Heuristics are usually general designs which can be used in all of mathematics domains. In spite of this fact that heuristic attracts a lot of writers' attention, there is not an exact definition for heuristic strategies. Obviously they are methods which help anybody to find and learn something by himself or herself. For seeing details and examples of heuristic strategies, the book (Schoenfeld, 1985) is proposed.

Our preliminary studies indicated that students in various college levels have very little awareness of mathematical heuristic strategies or, even those who have some awareness, have no adequate ability to use it. The present study aims to see, the students who received explicit training in the use of five particular heuristic strategies would be able to use those strategies to solve posttest problems comparable to the instruction problems via CCAI method.

MATERIALS AND METHODS

Participants: The participants in the study were 29 first and second-year students of science and mathematics majors in college. All of the students in the workshop participated voluntarily in this study. They were asked to participate in an experimental design with the use of computer-assisted cooperative learning. This project in the form of a workshop was performed in the computer Lab in Mathematics and Computer Department. To analyze the workshop results, we assigned randomly a number from 1 to 29 to each student. The worksheets given to students at any part of the workshop were identified by these numbers only known to the students. The mathematical backgrounds of all the students were comparable.

Materials: The instructional material consisted of 16 problems which included the five pretest problems. The training sessions took place over the four sessions in computers Lab. In each of the training sessions, the four problems which could be solved by the heuristic strategies presented to groups by computers.

In this experiment we used the following scoring scheme. Any problem had 5 scores. If a student proposed a special way or evidence for solving problem such as pointing to induction strategy but he (she) had not followed his (her) proposal, got 1score. When he (she) pursued his (her) proposal but he (she) had a little progress, got 2 scores. The student who had some progress in solving problem received 3 scores. If a student almost solved a problem with a particular approach but it marred by an incorrect calculation received 4 scores and the student who solved a problem

Table 1: For every problem : 5 scores

Evidence	Pursuit	Progress	Scores
+	-	-	1
+	+	-	2
+	+	Some	3
+	+	Almost	4
+	+	Complete	5

The plus sign is used for achieved aim and the minus Sign is used for unachieved aim

completely with one approach received 5 scores. Table 1 shows this scoring (Farhadian *et al*., 2007).

Procedure: At the beginning of the workshop, after a short discussion about the way it works, the students took a pretest containing five problems which could be solved respectively by the five heuristic strategies: induction, fewer variables, drawing diagram, contradiction, determination of subgoals. Each of the problems could be solved by at least two different ways. For any problem, the allowed time was 10 to 15 min. Any problem had been written on a single page of size A4 paper with a provided blank space for its solution and followed by two questions on the bottom of the page: (1) Have you seen this problem before? and (2) Have you seen any problem similar to this?. All of the students answered "NO". With a 5-point scale for each problem, the maximum score for each of the five-problem examinations is 25 points.

Because there was no suitable software for heuristic strategies instruction, the software was produced by instructors with the use of multimedia builder software. The problems which were presented by the software, could be solved by heuristic strategies. The process and the frame of the work will be described in the following.

After describing the cooperative work to students, they were asked to form groups of three or four members. The groups were formed and they began to learn the heuristic strategies by computer. The outline of the first session was as follows.

The first problem was shown on the monitor as follows.

Problem 1: Let a, b and c be given the real numbers and each of which lies between 0 and 1. Prove the following inequality.

$$(1-a)(1-b)(1-c) > 1-a-b-c$$

The students were asked 5 min to think about the problem. They got engaged in problem solving on their worksheet named "The solution without hints" and the results written on the worksheet.

When the first 5 min passed, a bell rang by computer and the first hint appeared on the monitor. The hint read as "Review each of the following strategies: (1) induction, (2) drawing diagram, (3) similar problem with fewer variables, (4) contradiction and (5) determination of subgoals". Another 5 min was assigned for them to think about the problem with this hint and they were engaged with their work on the worksheets named "The solution after the first hint". After the second 5 min, the computer rang again and the students noticed to the second hint which obviously pointing to useful strategies for problem solving. For example, the second hint for problem 1 was shown on the monitor as follows. Use similar problem with fewer variables strategy. Then they started their efforts during 5 min on "The solution after the second hint" section. When the hints for the first problem were finished, a bell rang to assign the end of time for solving the first problem. The same process was repeated for the second, third and fourth problem.

If some of the problems needed more hints, with repeating the mentioned process, the computer allowed them to spend more time on that related to that hint. In the every stage, the worksheets were collected and graded.

In the end of the first session, the solutions of problems were shown on the monitor. For example, the solution of the first problem was shown on the monitor as follows.

The solution of problem 1: You can use of the similar problem with fewer variables. If the problem has a large number of variables and is too confusing to deal with comfortably, construct and solve a similar problem with fewer variables. You may then be able to adapt the method of solution to the more complex problem or take the result of the simpler problem and build up from there.

The one-variable problem:

$$(1-a) > 1-a$$

It is clear that the inequality is not true. Therefore we can not use the one-variable problem.

The two-variable problem:

$$(1-a)(1-b) > 1-a-b \quad (1)$$

If we multiply out the left, the inequality 1 is true if only if $1-a-b+ab > 1-a-b$. Since a and b are both positive, we have $ab > 0$. This proves the inequality 1.

The original problem:

$$(1-a)(1-b)(1-c) > 1-a-b-c$$

The number c is between 0 and 1, so (1-c) is positive. Multiplying both sides of the two-variable case by (1-c), we get:

$$(1\text{-}a)(1\text{-}b)(1\text{-}c) > (1\text{-}a\text{-}b)(1\text{-}c)$$

or

$$(1\text{-}a)(1\text{-}b)(1\text{-}c) > 1\text{-}a\text{-}b\text{-}c\text{+}ac\text{+}bc.$$

Since ac and bc are both positive, we obtain

$$(1\text{-}a)(1\text{-}b)(1\text{-}c) > 1\text{-}a\text{-}b\text{-}c$$

In the end, the follow text was shown on the monitor "Can you extend the problem to more variables".

The students compared their solutions with the correct ones and discussed with themselves. The instructors answered the questions asked by members of groups.

After the fourth session, the students left the university for a period of two week holidays. On the first class after the holidays, we gave them a test similar to the pretest. On the first class after the holidays, we gave them a test similar to the pretest. We called it the posttest and consisted five problems which were different from what were taught and tested in the workshop and pretest but, of course solvable by the previous strategies were presented to them. Also in posttest the students were asked to answer 2 questions: (1) Have you seen this problem before? and (2) Have you seen any problem similar to this?. All of the students answered "NO" to the first question and only eight students answered "YES" to the second question.

RESULTS AND DISCUSSION

To find the possible significant differences between the tests at pretest and posttest, the scores were computed and the paired samples test and wilcoxon signed ranks test revealed significant differences between the tests. The results are shown in Table 2-11.

The means of scores on the tests at pretest and posttest are shown in Table 11. The data show that heuristic strategies training via CCAI dose make a significant difference in problem solving performance.

The results of our research indicated that the students had progress in the five strategies, but this progress was little in the last two strategies respect to the other strategies.

In the first session of workshop, most groups solved the problems after the second hint. In the second session, some of the groups were able to solve the problems after the first hint and most groups could be solved the problems in the third and fourth sessions before the hints.

Table 2: Ranks in the drawing diagram strategy

		N	Mean rank	Sum of ranks
Y-X	Negative ranks	0[a]	0.00	0.00
	Positive ranks	29[b]	15.00	435.00
	Ties	0[c]		
	Total	29		

[a]: Y < x, [b]: Y > X, [c]: Y = X
x = Pretest score Y = Posttest score

Table 3: Test statistics[b] in the drawing diagram

	Y-X
Z	-4.750[a]
Asymp. Sig. (2-tailed)	0.000
Exact. Sig. (2-tailed)	0.000
Exact. Sig. (1-tailed)	0.000
Point probability	0.000

[a]: Based on negative ranks
[b]: Wilcoxon signed ranks test

Table 4: Ranks in the fewer variables strategy

		N	Mean rank	Sum of ranks
Y-X	Negative ranks	0[a]	0.00	0.00
	Positive ranks	29[b]	15.00	435.00
	Ties	0[c]		
	Total	29		

[a]: Y < x, [b]: Y > X, [c]: Y = X
x = Pretest score Y = Posttest score

Table 5: Test statistics[b] in the fewer variables

	Y-X
Z	-4.846[a]
Asymp. Sig. (2-tailed)	0.000
Exact. Sig. (2-tailed)	0.000
Exact. Sig. (1-tailed)	0.000
Point probability	0.000

[a]: Based on negative ranks
[b]: Wilcoxon signed ranks test

Table 6: Ranks in the induction strategy

		N	Mean rank	Sum of ranks
Y-X	Negative ranks	0[a]	0.00	0.00
	Positive ranks	28[b]	14.50	406.00
	Ties	1[c]		
	Total	29		

[a]: Y < x, [b]: Y > X, [c]: Y = X
x = Pretest score Y = Posttest score

Table 7: Test statistics[b] in the induction strategy

	Y-X
Z	-4.658[a]
Asymp. Sig. (2-tailed)	0.000
Exact. Sig. (2-tailed)	0.000
Exact. Sig. (1-tailed)	0.000
Point probability	0.000

[a]: Based on negative ranks
[b]: Wilcoxon signed ranks test

The students who were instructed by cooperative computer-assisted instruction were more comfortable than in the traditional classes. The students in each group talked to each other, made tricks and class was less tedious for them. The time period of classes, in most of schools and universities, is generally about 100 min and the role of the teacher is as a lecturer and instructor in these classes is usually limited. Such classes are very

Table 8: Paired samples test in the contradiction strategy

	Paired differences							
				99% confidence interval of the difference				
	Mean	SD	SEM	Lower	Upper	t	df	Sig. (2-tailed)
Y-X	2.68966	1.64975	0.30635	1.84313	3.53618	8.780	28	0.000

x = Pretest score Y = Posttest score

Table 9: Ranks in the determination of subgoals strategy

		N	Mean rank	Sum of ranks
Y-X	Negative ranks	0[a]	3.50	3.50
	Positive ranks	25[b]	13.90	347.50
	Ties	3[c]		
	Total	29		

[a]: Y < x, [b]: Y > X, [c]: Y = X
x = Pretest score Y = Posttest score

Table 10: Test statistics[b] in the determination of subgoals

	Y-X
Z	-4.411[a]
Asymp. Sig. (2-tailed)	0.000
Exact. Sig. (2-tailed)	0.000
Exact. Sig. (1-tailed)	0.000
Point probability	0.000

[a]: Based on negative ranks
[b]: Wilcoxon signed ranks test

Table 11: Individual pretest and posttest scores

Problem	Heuristic strategy	Average score for problem (N = 29)
1	Induction	1.2 (4.5)
5	Determination of subgoals	0.2 (2.4)
4	Contradiction	0.1 (2.8)
2	Fewer variables	0.0 (4.2)
3	Drawing diagram	0.0 (3.5)
Total		1.5 (17.4)

Individual posttest scores appear in parentheses

tedious for both teachers and students. Weariness in cooperative computer-assisted instruction is very less. In addition, it is easily seen that the students enjoy using the cooperative learning advantages.

It is essential to provide opportunities for students to apply skills previously learned and to extend their learning to solve new problems. In the extensible problems, the groups were able to discuss and to extend them.

The research method is almost similar to our previous study (Farhadian *et al*., 2007). There are two different in the treatment the two researches. The type and number of participants was different and also the performed sessions number in this study was more than our previous study.

In comparison with our previous study, we experimented two other heuristic strategies: contradiction and determination of subgoals. In a research project, Schoenfeld (1985) instructed two groups of students (control and experimental) for a period of two weeks (individually and without using computer). They were instructed 20 problems which were solvable by 5 the mentioned heuristic strategies. In this project, we used some of the problems that schoenfeld used in his study. The primary difference in the treatments the two groups received was that the solutions shown to the experimental group explicitly included mention of the heuristic strategies used to solve the problems. The control group saw the same solutions but without an elaboration of the heuristic method. Two comparisons of pretest-to-posttest gains, indicated that the experimental group significantly out performed the control group. The results indicated that, the students in the experimental group did learn to use three of the five strategies (drawing figure, similar problem with fewer variables and induction) but they did not learn to use the two other strategies (determination of subgoals and contradiction). He pointed that the last two strategies were complex and the instruction time (two weeks) and the instruction amount (20 problems) were very limited.

The results of our research indicated that the students had progress in the last two strategies too, but this progress was little respect to the other strategies. It seems that the results could be better with more repetition and more work in workshop. When the students were in group, they could use their ability and solve some of the problems by the computer's hints which were a strict point to heuristic strategies and by a group discussion.

In this study the conditions and the limitations were different from Schoenfeld's study but the results were more than our expectation. Some of these limitations and differences were as follows.

- The time for solving every problem was 10 to 15 min. We believe that if the assigned time for solving problems was more, the results were better.
- The students participated voluntarily in the experiment. The students were said that we were experimenting the cooperative learning by computer and we wanted them to cooperate with us. In other words there was not obligation or pursuiting for student to cooperate in this project. There was a probability that they did not cooperate in this study. During instruction, one of the groups had come out of instruction program curiosity and had gone to other computer programs and it is one of our reasons for this probability.

- There were two weeks from the ending of instruction to posttest. During this period the students were on their holidays. They did not have any classes and it was impossible for them to see problems similar to the posttest problems and they were not together to discuss heuristic strategies. Even they had not been said that the instructors wanted to test them again. After holidays, in the first session, they took the test.

In this study, we produced the courseware based on cognitive multimedia design approaches. We believe that mathematics teachers with the help of programmers able to produce suitable multimedia softwares to teach mathematical strategies to students. Also the traditional teaching and learning focuses on the primacy of instruction. Most important aspects are the systematic planning of lectures, strict differentiation of domains, teacher-centric classes with the teacher as presenter and controller of learning and embedded assessment. Learners stay mostly inactive and perceptive, because self-direction is largely limited by the instructional design of the teacher and the authors of learning material. In contrast, constructivist learning theories put the learner into control. Knowledge building is an active construction performed by learners based on the interaction with their environment. Thus, instruction plays a less important role. The teacher rather coaches learners in their learning activities and helps them to solve complex and authentic problems (Schroeder and Spannagel, 2006). In this study, we encouraged the students to learn in groups and the educators had only the role of a guide. The individual ability of each member in the group affected the group ability. It is clear that cooperative environment is the best environment for constructivist activities. The students became more actively engaged in mathematical problem solving through cooperative learning. They became more motivated, less competitive, more aware of the problem solving process.

We wanted to use computers as assistant and for management of classroom, time and mathematical problem solving process. It was successful. The aim of the ringings in the software was to have the students' attention to control discussion which has an important role in problem solving process. The problem solvers with a good control can benefit from their knowledge and they can solve difficult problems with least effort. In mathematical problem solving Schoenfeld (1985) says that the instruction of heuristic strategies is effective in the enhancement of solving problems of mathematics. The results of our research and similar studies confirm this idea.

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