

<http://ansinet.com/itj>

ITJ

ISSN 1812-5638

# INFORMATION TECHNOLOGY JOURNAL

**ANSI***net*

Asian Network for Scientific Information  
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

## A Robust Doppler Shift Estimator in Wireless Communications

<sup>1</sup>Limin Meng, <sup>1,2</sup>Jingyu Hua, <sup>1</sup>Zhijiang Xu and <sup>1</sup>Gang Li

<sup>1</sup>College of Information Engineering, Zhejiang University of Technology,  
Hangzhou 310032, China

<sup>2</sup>National Communication Research Laboratory,  
Southeast University, Nanjing, China

---

**Abstract:** In this study, we propose a novel estimator under the time-varying multi-path channel and with its performance robust to Signal-to-Noise Ratio (SNR). This estimator is derived with the help of noise influence analysis and realized through the iterative process. We verify our algorithm by computer simulations and observe a great performance improvement in a wide range of velocities and SNRs.

**Key words:** Doppler shift, wireless communications, rate of minimum

---

### INTRODUCTION

Rapid fading is a central problem in mobile communications (Jakes, 1974). It degrades the Bit Error Rate (BER) and frequently introduces an irreducible BER, or error floor. Many authors have studied the use of pilot signals to mitigate the effects of fading. By the appropriate pilot signals we can track the fading (Cavers, 1991).

The fading rate of a channel depends on its maximum Doppler shift ( $f_d$ ) and Doppler shift is related to the velocity of Mobile Terminal (MT). According to the requirements of 3G system, mobile terminals should operate in a velocity range of 0 to 500 km h<sup>-1</sup>. Hence either an efficient estimation of the MT velocity is of great importance in mobile communications and it ultimately provides a good power control performance (Monk and Milstein, 1995), a sound handoff performance (Sampath and Holtzman, 1993) as well as effective dynamic channel estimation (Ma *et al.*, 2003).

In order to estimate the MT velocity, Sampath and Holtzman (1993), Xiao *et al.* (2001), Baddour and Beaulieu (2003) and Tepedelenioglu *et al.* (2001) used the logarithm envelope characteristic, the channel autocorrelation characteristic and the spectral analysis techniques, respectively. Moreover, Stuber (2001) and Kawabata (1994) present us with estimators based on Level Crossing Rate (LCR) and diversity reception.

In this study, we paid our attention to the Rate of Minimum (ROM) based Doppler shift estimator in Rayleigh fading channels (Abdi and Kaveh, 1998;

Abdi *et al.*, 2003; Abdi and Nader, 2003), which was robust to the transmission environments (Abdi and Kaveh, 1998) but whose applicable range was limited due to its large estimation bias. Thus we analyzed this estimation bias theoretically and proposed a modified Doppler shift estimator based on decimator. We verified our algorithm with Monte Carlo simulation and got accurate and robust estimation results in a wide range of velocities and SNRs.

### SYSTEM MODEL

Suppose that a band-limited pilot signal is transmitted over a Rayleigh fading channel and the distinguishable multi-path fading channels are wide-sense stationary and mutually uncorrelated scattering (WSSUS) processes. After synchronously matching the pilot signal, the received information of the strongest path can be written as:

$$r(n) = c(n)d_p(n) + v(n) \quad (1)$$

where,  $c(n)$ ,  $d_p(n)$  and  $v(n)$  represent the channel coefficients, the pilot symbol and noise, respectively and  $n$  is the discrete time index. The channel coefficients ( $c(n)$ ) is modeled as a wide-sense stationary discrete-time complex Gaussian random process with variance  $\sigma^2$  and  $v(n)$  is modeled as a zero-mean additive complex white Gaussian noise (AWGN). An estimate based on Eq. 1 is:

$$\hat{c}(n) = \frac{r(n)d_p^*(n)}{|d_p(n)|^2} \triangleq c(n) + z(n) \quad (2)$$

where, (d)\* denotes the conjugate manipulation and z(n) (with the variance  $\sigma_z^2$ ) is obtained after manipulating the white Gaussian noise v(n) by the estimation process. With these channel estimates, we can calculate the ROM of fading channel envelope and estimate the Doppler shift.

### ORIGINAL DOPPLER SHIFT ESTIMATION

Doppler shift estimation can be done by ROM detection. Rice (1945) gave the analytical expression of ROM, but it was too complicated to be put into practice. Hence, Abdi derived a novel approximate expression for ROM (Abdi and Nader, 2003):

$$N_{\text{approx}} = \frac{1}{2\pi} (b_0 b_4 + 3b_2^2 - 4b_1 b_3)^{1/2} \quad (3)$$

where,  $b_n$  denotes the n-th order moment of the channel Power Spectral Density (PSD). Different from the results of S.O. Rice, the channel PSD can be dissymmetrical for Eq. 3, resulting in more general transmission environments.

Based on Eq. 3, A. Abdi proposed a new Doppler shift estimator for general wireless environments (Abdi and Kaveh, 1998). But this estimator required the knowledge of SNR in isotropic scattering Rayleigh channels and of SNR and Angle of Arrival (AOA) distribution in anisotropic scattering Rayleigh channels at least. Obviously, these knowledge were not always known to the receiver, hence, we first arrive at the estimator of infinite SNR in isotropic scattering Rayleigh channels. Then we will analyze estimation error from both theoretical and simulation perspective.

Given Jakes PSD and Rayleigh fading channel, Eq. 3 can be simplified as Abdi *et al.* (2003):

$$N_{\text{approx}} = \frac{1}{2\pi} \sqrt{\frac{b_4 + 3b_2}{b_2}} = 1.5f_d \quad (4)$$

we have:

$$f_d(n) \approx \hat{N}_{\text{approx}}(n)/1.5 \quad (5)$$

where,  $\hat{N}_{\text{approx}}(n)$  represents the number of envelope minimum in time index n.

By Eq. 5, we can obtain the Doppler shift estimation  $\hat{f}_d(n)$ . At first, we must store K channel estimations, where K should be large enough to ensure that the time length T between the first and the K-th channel estimations is much larger than the fading periods. Secondly, at every time instant we can report the instantaneous Doppler shift estimation according to the following ROM iteration method:

$$\hat{N}_{\text{approx}}(n+1) = \hat{N}_{\text{approx}}(n) + a \quad (6)$$

where,  $a \in \{-1, 1, 0\}$  is determined by whether the first and the last channel estimates are minimum or not.

### NOISE ANALYSIS

Generally speaking, the noise will inevitably affect ROM. After a carefully analysis, we find that this influence is aroused by the PSD moments (Stuber, 2001):

$$b_m = (2\pi)^m \int_{-f_d}^{f_d} (S_s(f) + S_n(f)) f^m df \quad (7)$$

where,  $S_s(f)$  is the fading channel PSD and  $S_n(f)$  is the AWGN PSD including background noise and interference. In noisy cases,  $S_n(f)$  does not equal to 0, which makes ROM biased from noise-free scenarios. Analogous to Stuber (2001), the ratio of the estimated to the actual Doppler shift can be used to describe this bias:

$$\eta = \frac{\hat{f}_d}{f_d} = \frac{\hat{N}_{\text{approx}N}}{N_{\text{approx}S}} = \sqrt{\frac{b_4 + \frac{3b_2}{b_0}}{b_{s4} + \frac{3b_{s2}}{b_{s0}}}} \quad (8)$$

where,  $\hat{N}_{\text{approx}}(n)$  and  $N_{\text{approx}S}$  represent ROMs in noisy and noise-free cases respectively and  $\{b_{s4}, b_{s2}, b_{s0}\}$ ;  $\{b_4, b_2, b_0\}$  represent moments in noise free case and noisy case, respectively. Considering Jakes model, we have (Stuber, 2001):

$$b_m = \frac{P_{\text{av}}}{\pi} \int_{-f_d}^{f_d} \frac{(2\pi f)^m}{\sqrt{f_d^2 - f^2}} df + \int_{-B/2}^{B/2} \frac{N_0 (2\pi f)^m}{2} df \quad (9)$$

where, B and  $P_{\text{av}}$  denote noise bandwidth and channel mean power, respectively. We can calculate  $b_4$ ;  $b_2$ ;  $b_0$ ;  $b_{s4}$ ;  $b_{s2}$ ;  $b_{s0}$  by Eq. 9 and substitute them into Eq. 8, thus resulting in:

$$\eta = \sqrt{\frac{3\gamma + (B/f_d)^4/10}{9(\gamma + (B/f_d)^2/6)} + \frac{6\gamma + (B/f_d)^2}{9(\gamma + 1)}} \quad (10)$$

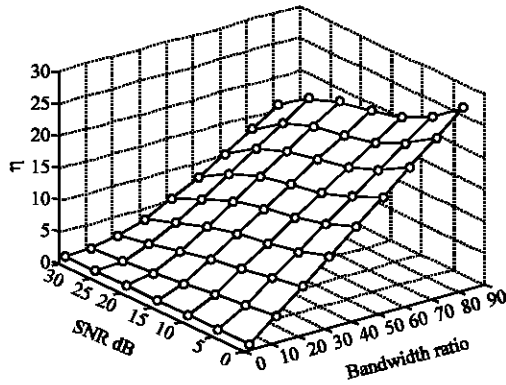


Fig. 1: Estimation bias ( $\eta$ ) with different SNR and bandwidth ratio: SNR  $\in[0,30]$ dB, bandwidth ratio  $\in[2,82]$

where,  $\gamma$  denotes the symbol signal to noise ratio. It can be found from Eq. 10 that this bias is a function of SNR, noise Bandwidth and the actual Doppler shift. In order to decrease the bias, the noise Bandwidth must be matched to the Doppler shift. If we define the bandwidth ratio ( $B/f_d$ ) and it together with  $\gamma$  satisfy a certain condition found by solving the Eq. 10,  $\eta$  will be 1; otherwise  $\eta \neq 1$  will decrease with  $\gamma$  increasing.

Figure 1 shows the Doppler shift estimation bias in the noisy case. It is clear that higher SNRs and more suitable bandwidth ratios lead to smaller biases. Obviously, this bias is very sensitive to the bandwidth ratio and is unacceptable even for moderate bandwidth ratios in practical communication systems. These results indicate the requirement of and the method to revise the original Doppler shift estimator.

**MODIFICATION CONSIDERATIONS FOR ROM ESTIMATOR**

In order to decrease the estimation bias, first, we can improve the symbol SNR, which needs increased pilot length. Then the number of data symbols in a slot must be decreased. Besides, we have found in Fig. 3 that the bias is not sensitive to SNR. The other method, which we prefer, is to decrease the noise bandwidth, which can be realized by Low Pass Filter (LPF) or increased sample interval. Taking into account the wide range of velocities in our CP-SCBT system, we need several LPFs to cover it, as will increase our cost along with the computation load.

Taking into consideration the fact that mobile channel is a narrow-band multiplicative noise, whose

band-width is much less than that of wide-band CP-SCBT signal or AWGN at the receiver. Therefore, a better method for our system is to decrease the sampling rate by decimator. The reason is that the spectrum of the decimated band-pass signal is a periodic extension of the original spectrum, which can be seen from (Oppenheim, 1998):

$$X_s(j\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c(j\Omega - kj\Omega_s) \quad (11)$$

$$\Omega_s = 2\pi/T_s$$

where,  $X_s$  and  $X_c$  denote the spectrums of the decimated signal and the original signal.

From Eq. 11, we can find that different sample intervals ( $T_s$ ) lead to different spectrum periods of the decimated signal. Presuming that the Nyquist sampling theory is satisfied, then this period can be reduced to  $1/M$  as long as the channel coefficient sequence is decimated by  $M$  (the decimating factor), then the noise bandwidth of the decimated sequence is reduced to  $1/M$  of the original bandwidth, resulting in a decrease in bandwidth ratio.

**MODIFICATION BASED ON DECIMATION**

For the sake of simplicity of realization, we divide the velocity range into several bins and an appropriate  $M$  is chosen for each bin. The choice of which is a tradeoff between the noise filtering and the channel tracking. Finally we can find a group of appropriate  $M$ s for any slot structures analogous to the above analysis.

Figure 2 describes the proposed Modification. First of all, we get a coarse Doppler shift by the original ROM method in short observation duration. Then we employ the decimating factor calculator to determine the velocity bin index according to the coarse estimation and select an appropriate  $M$  from a Look-up-table (LUT) pre-calculated. At last if this  $M$  equals to 1, this coarse estimation is the final result, otherwise we must use the decimated channel coefficient sequence to detect ROM again in order to obtain a final result.

Generally speaking, the iteration can be applied several times to further improve the performance, but we find in simulation that performances with only one iteration are good enough, thus we just give the one iteration structure in Fig. 2 as an example without loss of generality.

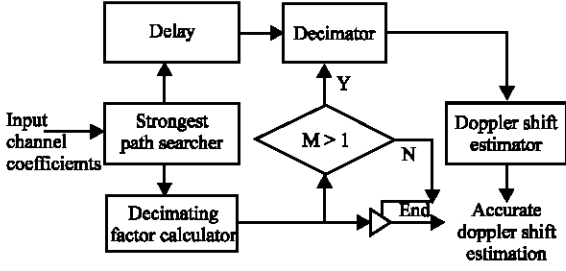


Fig. 2: Modification for Doppler shift estimator

## SIMULATIONS AND ANALYSIS

Some numerical results are presented in this section. First we inspect the performance of the original ROM method and the method in (Sampath and Holtzman, 1993) is also simulated for comparison. The simulation parameters are shown in Table 1.

Figure 3 shows the simulation and the theoretical results of the original ROM method, which are consistent with previous bias analysis, i.e., the higher the speed or SNR is, the smaller the bias will be, just as Eq. 10 shows. On the other hand, one can explicitly see an obvious gap between the simulation and the theoretical results regardless of variation of SNRs. It should be noted that both the simulation and the theoretical results possess the same trend, which proves the consistency between them to some extent. For example, when SNR is 10 dB, the ROM values of the theoretical curve at high speeds exceed that at low speeds for the first time, so does the simulation curve. In fact, this turning point phenomenon is universal for all SNRs and a lower SNR leads to a higher speed at the corresponding turning point.

Figure 4 shows the results of the modified ROM method. Apparently, the estimation accuracy is improved to a large extent and it is safe to say that the accuracy is robust to SNR and velocity approximately. Compared with the original ROM method, the modified one can be applied to practical communication systems. Furthermore, decimation would not shorten the total observing duration, thus the channel ergodicity is reserved after it. Taking the shorter sequence length after decimating into consideration, we find the computation load of the proposed algorithm is lower than that of multi-LPFs method.

Mean Square Error (MSE) of Doppler shift estimation is defined as:

$$\text{MSE} = \frac{1}{m} \sum_{i=1}^m E \left[ \frac{|f_{d,i} - \hat{f}_{d,i}|^2}{f_{d,i}^2} \right] \quad (12)$$

Table 1: Simulation parameters

Parameters	Values
Slot length	1056 bit
Chip rate	1.2288 Mbit/s
Pilot length	32 bit
Carrier	2.11 Gkz
Coding	None
Channel model	ITU M.1225 model
Simulation length	1000 Slots
Path No.	6
Pilot interval	0.208 ms
Modulation	QPSK

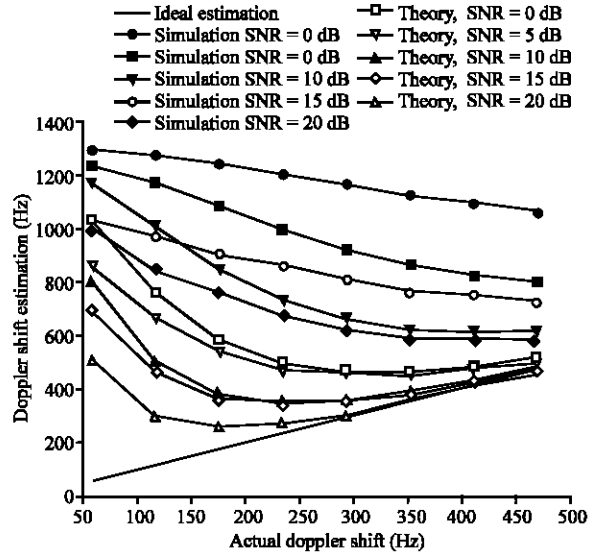


Fig. 3: Accuracy of original ROM method

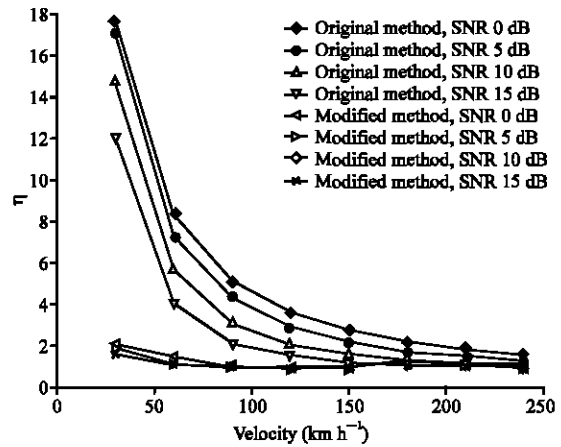


Fig. 4: Accuracy of modified ROM method

where,  $f_{d,i}$  and  $\hat{f}_{d,i}$  are the actual and the estimated Doppler shift and  $m$  is the number of velocities used in simulation. We should bear in mind that the MSE is an averaged version w.r.t velocities in Eq. 12, thus we can look deep into the relations between the MSE and SNR.

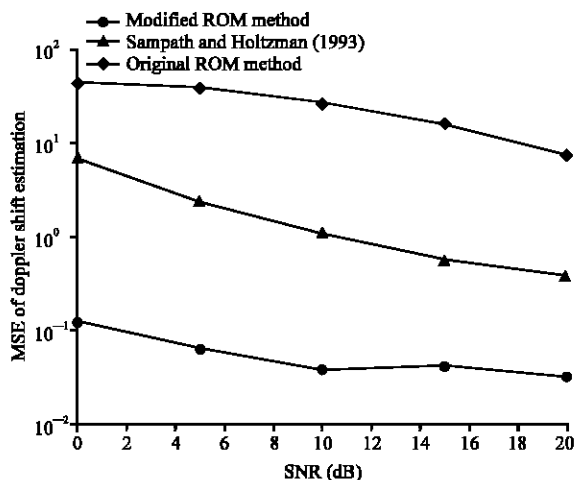


Fig. 5: MSE performance of the original and the modified

Figure 5 shows the MSEs of both the original and the modified method with velocity ranging from 30 to 240 km h<sup>-1</sup>. It is clear from the figure that the MSE of the modified method is robust when SNR is larger than 0 dB and at least gains of two orders of magnitude are obtained by this method when SNR belongs to 0 ~ 20 dB. Furthermore, it is also the best among the three methods in Fig. 5, while the original ROM method is the worst. In addition, because the decimating factor LUT is optimized for SNRs from 5 to 10 dB, which may cause over-decimating at high SNR. Hence the MSE of the modified method increased slightly when SNR is 15 dB. To sum up, when SNR belongs to 5 ~ 10 dB, which is an effective SNR range for most mobile communication systems, the modified method can produce accurate Doppler shift estimation robust to SNR.

### CONCLUSION

In this study, we propose a novel Doppler shift estimator based on ROM, where an adaptive decimator is provided to degrade the effects of noise. Simulations show that the proposed algorithm is robust to SNR and produces a good performance in a wide range of velocities and SNRs.

### ACKNOWLEDGMENTS

The study is supported by the open research fund of National Mobile Communications Research Laboratory, Southeast University, China. Moreover, the authors want to thank the anonymous reviewers for their helpful comments.

### REFERENCES

- Abdi, A. and M. Kaveh, 1998. A new velocity estimator for cellular systems based on higher order crossing. In: Proceedings of IEEE VTC'98, pp: 1423-1427.
- Abdi, A., H. Zhang and C. Tepedelenioglu, 2003. Speed estimation techniques in cellular systems: Unified performance analysis. In: Proceedings of IEEE VTC'03, pp: 1522-1526.
- Abdi, A. and S. Nader, 2003. Expected number of maxima in the envelope of a spherically invariant random process. IEEE. Trans. Inform. Theor., 49: 1369-1375.
- Baddour, K.E. and N.C. Beaulieu, 2003. Nonparametric Doppler spread estimation for flat fading channels. In: Proceedings of IEEE. WCNC'03, pp: 953-958.
- Cavers, J.K., 1991. An analysis of pilot symbol assisted modulation for Rayleigh fading channel. IEEE. Trans. Veh. Technol., 40: 683-693.
- Jakes, W.C., 1974. Microwave Mobile Communications. Johns Wiley Publishing.
- Kawabata, K., 1994. Estimating velocity using diversity reception. In: Proceedings of IEEE VTC'94, pp: 371-374.
- Ma, Z., Y. Yan, C. Zhao and X. You, 2003. An improved channel estimation algorithm based on estimating level crossing rate for CDMA receiver. Chin. J. Elect., 12: 235-238.
- Monk, A.M. and L.B. Milstein, 1995. Open-loop power control error in a land mobile satellite system. IEEE. J. Sel. Areas Commun., 13: 205-212.
- Oppenheim, A.V., 1998. Discrete Time Signal Processing. Chinese Science Publishing.
- Rice, S.O., 1945. Mathematical analysis of random noise. Bell Syst. Technol. J., 24: 46-156.
- Sampath, A. and J. Holtzman, 1993. Estimation of maximum Doppler frequency for handoff decisions. In: Proceedings of IEEE VTC'93, pp: 859-862.
- Stuber, G.L., 2001. Principles of Mobile Communication. Kluwer Academic Publishing.
- Tepedelenioglu, C., A. Abdi, G.B. Giannakis and M. Kaveh, 2001. Estimation of Doppler spread and signal strength in mobile communications with application to handoff and adaptive transmission. Wireless Commun. Mobile Comput., 1: 221-242.
- Xiao, C., K. Mann and J. Olivier, 2001. Mobile speed estimation for TDMA based hierarchical cellular systems. IEEE. Trans. Veh. Technol., 50: 981-991.