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Transmit Power Control for Optimization of Wireless Sensor Networks

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Abstract: Power conservation is an issue which has to be primarily considered in wireless sensor networks. Since we have the limitation of battery life so an appropriate transmit power has to be chosen while guaranteeing the maximal connectivity at the same time. It is in our best interest to have each node transmit at lowest possible power thus prolonging the network lifetime and preserving the connectivity. So this study investigate the most optimum transmit power while considering the data rates and how densely the nodes are deployed. The effects of the interference powers have to be minimized by reaching towards the minimal transmit power. We have worked out on the interference powers introduced due to different ways of deployments of wireless sensor networks over a certain area and we used the graphical approach to look at the effects of distance from the central receiving nodes and the varying densities on the BER. We have also investigated the phenomena through simulations and the main aim being to optimize the performance of sensor networks by observing their behavior in various regularly placed deployments.

Key words: Sensor networks, transmit power, connectivity, node density, network lifetime

INTRODUCTION

In the wireless sensor networks power conservation is one of the main issues to be dealt with since the nodes operate on limited battery life. The sensor networks do not have a large scale infrastructure and nodes cooperate by relaying packets to ensure that the packets reach their respective destinations. The nodes are usually battery powered, thus it is crucial that relaying and communication strategies be developed to minimize power utilization. Transmitting at low power reduces the amount of excessive interference. Power conservation helps to prolong the lifetime of a node and thus the lifetime of the network as a whole.

One of the things to be considered is that a wireless network should have a good connectivity. The connectivity of a wireless network is dependent upon the transmit power of nodes. If the connectivity is low due to small transmit power, there would be multiple disconnected clusters of node existing instead of a single well connected network. The transmit power also depends upon the density of nodes deployed in a certain area of coverage. Transmitting at excessively high power would lead to higher interference powers and a shorter battery life. For attaining a good connectivity the transmit power must be large enough for a node to communicate with any other node in the network. Thus it is beneficial for each node to use as small power as possible for interference reduction. The reduction in transmit power would also

potentially increase the capacity of sensor networks (Gupta *et al.*, 2000).

In a homogeneous regularly deployed sensor network the node density would be defined as

$$\rho = \pi NR^2/A$$

Where:
N = The number of sensor nodes deployed
R = The range of the area covered by the sensor nodes
A = The deployment area

According to the nature of applications of sensor networks they are placed in locations which are not easily accessible and having the batteries replaced often would lead to the loss of the primary benefit of the sensor networks so longer battery life is essential in sensor networks (Agarwal *et al.*, 2001). One way to achieve ultimate power saving is the adjustment of the transmit power of a node on link-by-link basis (Elbatt *et al.*, 2004). A simple and feasible solution likely for implementation is the use of a common transmit power (Ramanathan and Rosales-Hain, 2000). There is not much difference in terms of performance when considering the power adjustment locally and implementing a common transmit power, especially when the network size is large (Naryanaswamy *et al.*, 2002). Using the approach of common transmit power it is known that as long as the transmission power of each node is of the order

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$$\sqrt{\log(n)/n}$$

the network would be strongly connected without any disconnected clusters of nodes (Rengarajan 2006).

In this study we investigate the optimal transmit power analytically and using simulations. We thoroughly investigate the interrelation between optimal transmit power, data rate and node density. In addition to it we worked on the effects on the node and network lifetime and the maintenance of connectivity by choosing the appropriate transmit power. We also analyze the interference powers and BER effects in a particular network deployment scenario to lead to an ultimate optimum performance of the network as a whole while achieving a well established connectivity at the same time.

NETWORK MODEL AND ASSUMPTIONS

In this study we describe the basic communication network model and the basic assumptions.

In the study we have the scenario of N_A nodes distributed over an area A, the density of the nodes spread over the area would be given by $\rho = N_A/A$. When considering regular topology the network surface has a length L on each edge and the analytical techniques able to be applied for other surfaces. The probability mass function of the number of nodes over a surface of area A in the case of two-dimensional Poisson topology would be given by:

$$P_r(N_A = k) = \frac{(\rho A)^k}{k!} e^{-\rho A} \quad k = 0, 1, 2, 3 \ldots,$$ (1)

where, ρ is the average number of nodes per unit area or the average node spatial density.

We have taken different scenarios. In the first scenario we have placed eight nodes in the first tier around the central node and twenty four nodes in the second tier. In the second scenario we have doubled the nodes in the first tier that is sixteen nodes and in the second tier we have placed forty eight nodes. In this way we have varied the densities of the nodes deployed in a certain area to look at the interference power effects and the bit error rates thus analyzing the resultant effects and approaching towards the optimum performance (Fig. 1).

A multihop route would also be existing from source to destination which is a crucial phase in wireless networking scenario. In the regular nodal deployment the
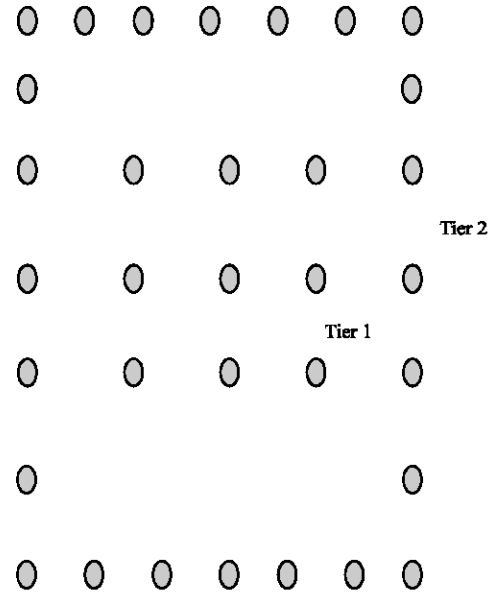

Fig. 1: Tier structure of regularly deployed network with grid topology

distance to the nearest neighbor r_{con} would be fixed and constructing a square lattice of N nodes over a surface area A will be equal to filling N small square tiles of area r^2_{con} into a large square of area A and r_{con} is the distance to the nearest neighbor to be connected. From this we deduce that $Nr_{con}^2 = A$ and therefore the distance to the nearest neighbor can be written as:

$$r_{con} = \sqrt{\frac{A}{N}} = \frac{1}{\sqrt{\rho}}$$ (2)

Here, we would be considering the link BER. It is known generally that the received signal observed at the receiver is the sum of three components (1) the intended signal from a transmitter (2) the interfering signal from other nodes and (3) the thermal noise. Because of the fact that the interfering signals come from other nodes, we assume that the total interfering signal can be treated as an additive noise process independent of the thermal noise process. The received signal r during each bit period can be expressed as:

$$r = s_{trans} + \sum_{j=1}^{N-2} s_j + n_{sig}$$ (3)

where, s_{trans} is the signal from the transmitter, s_j is the signal from an interfering node j and n_{sig} is the thermal noise signal.

NODE DENSITY AND INTERFERENCE POWER

When analyzing a link between the transmitter and the receiver we have to assume that a signal is attenuated with a distance raised to the power γ, where, γ is the pathloss exponent, the power of the intended signal from the transmitter as observed at the receiver can be written as:

$$P_r = \frac{\alpha P_t}{r^\gamma_{con}} \qquad (4)$$

$$\alpha \triangleq \frac{G_t G_r c^2}{(4\pi)^2 f_c^2} \qquad (5)$$

where, P_t is the transmit power, G_t and G_r are the transmitter and receiver antenna gains, f_c is the carrier frequency and c is the speed of light. The antennas at the nodes are assumed to be omni directional ($G_t = G_r = 1$). In case of binary phase shift keying (BPSK) modulation, there can be two phases for the amplitude of the received signal: 1)

$$S_{rec} = \sqrt{P_r/R_b} \triangleq \sqrt{E_b}$$

if a +1 is transmitted and 2)

$$S_{rec} = \sqrt{P_r/R_b} \triangleq -\sqrt{E_b}$$

if a -1 is transmitted ___ E_b is the bit energy of the received signal.
The conditional error probability can be written as:

$$Q\left(\frac{\sqrt{E_b} + \sum_{j=1}^{N-2} S_j}{\sigma} \right) \qquad (6)$$

From this we would be able to calculate the BER.

Here, $\sigma = \sqrt{FkT_0/2}$. F is the noise figure, $k = 1.38 \times 10^{-23}$ J K^{-1} is the Boltzmann's constant, T_0 is the room temperature and B is the transmission bandwidth. The received thermal noise signal would be $n_{sig} = \sqrt{FkT_0B}$. If there is an interfering node j at a distance r_{con} from the receiver, the interference power from node j would be written as:

$$P_{intj} = \frac{\alpha P_t}{(r_{con})^\gamma} \qquad (7)$$

All links in average multihop route would be characterized by path loss exponent $\gamma = 2$ so the Eq. 7 can be rewritten as:

$$P_{intj} = \frac{\alpha P_t}{(r_{con})^2} \qquad (8)$$

Since we have derived the relation between r_{con} and ρ thus the equation (8) would become:

$$P_{intj} = \alpha P_t \rho \qquad (9)$$

The interference power P_{int} experienced by the central receiving node would be due to the nodes in the first tier and nodes in the second tier but the effect due to the nodes in the first tier would be much more due to their proximity to the central node.

For every interfering node j the amplitude of the interfering signal can be classified into one of these three cases: (1)

$$s_j = \sqrt{P_{intj}/R_b}$$
if a "+1" is transmitted (2)
$$s_j = -\sqrt{P_{intj}/R_b}$$
if a "-1" is transmitted (3)
$s_j = 0$ if node j does not transmit

Since each node transmits packets with fixed length L (dimension:[b/pck]), the interference probability can be shown to be equal to the probability that an interfering node transmits during a vulnerable interval of duration L/R_b. This probability can be written as:

$$p_{trans} = 1 - e^{-\lambda_t L/R_b} \qquad (10)$$

In case of linear binary modulation we can assume that the transmitter transmits +1.Thus in this case, $s_{trans} = \sqrt{E_b}$.

When considering linear binary modulation we can assume that the transmitter transmits +1. Thus in this case, $\overrightarrow{S_{int}} = \{s_1, s_2, ..., s_{N-2}\}$. When defining a random $\overrightarrow{S_{int}} = \{s_1, s_2, ..., s_{N-2}\}$ vector where, s_j is the amplitude of the signal of an interfering node j received at the receiver. Note that $\overrightarrow{S_{int}}$ is random because each s_j can take one of these three different values with the probability given below:

$$s_j = \sqrt{\frac{P_{intj}}{R_b}} \qquad \text{with probability } \tfrac{1}{2} p_{tran}$$

$$s_j = -\sqrt{\frac{P_{intj}}{R_b}} \qquad \text{with probability } \tfrac{1}{2} p_{tran}$$

$$s_j = 0 \qquad \text{with probability } 1-p_{tran}$$

Assuming the threshold for bit detection placed at 0, the bit error probability can be written as:

$$P\{BitError\} = BERcon = \sum_{\overrightarrow{S_{int}}} P\left\{\sqrt{E_b} + \sum_{j=1}^{N-2} s_j + n_{sig} < 0 \mid \overrightarrow{S_{int}}\right\} P\left\{\overrightarrow{S_{int}}\right\} \tag{11}$$

Having the random vector $\overrightarrow{S_{int}}$ the conditional error probability can be expressed as:

$$P\left\{\sqrt{E_b} + \sum_{j=1}^{N-2} s_j + n_{sig} < 0 \mid \overrightarrow{S_{int}}\right\} = Q\left(\frac{\sqrt{E_b} + \sum_{j=1}^{N-2} s_j}{\sigma}\right) \tag{12}$$

Where:

$$\sigma = \sqrt{FkT_0 / 2}$$

Assuming that a bit detected erroneously at the end of a link is not corrected in successive links, the BER at the end of a route with $\bar{n}_{grid}$ links, denoted as BER_{route}^{grid} can be written as:

$$BER_{route}^{grid} = 1 - \left(1 - BER_{link}\right)^{\bar{n}_{grid}} \tag{13}$$

REACHING FOR OPTIMUM TRANSMIT POWER

Through out the study assuming that packets would be transmitted with rate λ(pck/s). Also the transmit power on each link would attenuate with

$$\alpha \triangleq \frac{G_t G_r c^2}{(4\pi f_c)^2}$$

where, γ is the pathloss exponent and the signal-to-noise ratio (SNR) at the receiving node of the link can generally be written as (Panichpapiboon *et al.*, 2005).

$$SNR_{con} = \frac{\alpha P_t}{r_{con}^{\gamma} \dfrac{R_b}{B}\left(P_{thermal} + P_{int}\right)} \tag{14}$$

Where:

$$\alpha \triangleq \frac{G_t G_r c^2}{(4\pi f_c)^2}$$

G_t and G_r are transmitter and receiver antenna gains, f_c is the carrier frequency, P_t is the transmit power, $P_{thermal}$ the additive white Gaussian thermal noise power , P_{int} the interference power, R_b is the data rate(kbps) and B is the bandwidth (Hz). The thermal noise power $P_{thermal}$ is FkT_0B, F is the noise figure, $k = 1.38 \times 10^{-23}$ J K^{-1} is the Boltzmann's constant and T_0 is the room temperature. The interference power P_{int} experienced by the receiving node at the centre is due to all the transmitting nodes and also depends on the distance and position of the nodes with respect to the central receiving node. The total interference power P_{int} is the sum of the powers from the interfering nodes. If all the nodes simultaneously transmit it can be shown that the total interference power experienced by the central receiving node is given by

$$P_{int}^{total} = \alpha P_t I_{grid} \tag{15}$$

Where:

$$I_{grid} \triangleq \frac{1}{r_{con}^{\gamma}} \sum_{i=1}^{2}\left[\frac{4}{i^{\gamma}} + \frac{4}{(\sqrt{2}i)\gamma}\right] \tag{16}$$

Here, $i = 2$ is the maximum tier order since we have deployed nodes in 2 tiers around the central receiving node in all the scenarios.

If a node generates packets with fixed length L(b/pck) according to Poisson process with parameter λ_t(pck/s), the probability of interference is equal to the probability that a node transmits during the interval of duration L/R_b.

$$p_{transmit} = 1 - e^{-\frac{\lambda_t L}{R_b}} \tag{17}$$

The average interference power would be given by

$$P_{int}^{avg} = p_{transmit} P_{int}^{total} = (1 - e^{-\frac{\lambda_t L}{R_b}}) P_{int}^{total} \tag{18}$$

Assuming the interfering noise to be Gaussian the BER on each link could be written as:

$$BER = Q\left(\sqrt{\frac{2\alpha P_t}{r_{con}^{\gamma}\dfrac{R_b}{B}(P_{thermal} + P_{int}^{avg})}}\right) \tag{19}$$

Where:

$$Q(x) \triangleq \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

Thus finally the optimum transmit power for the nodes can be expressed as:

$$P_t = FkT_0 R_b\left[\frac{2\alpha}{r_{con}^{\gamma}\psi} - \alpha\left(1 - e^{\frac{-\lambda_t L}{R_b}}\right)I_{grid}\right]^{-1} \tag{20}$$

Where:

$$\psi \triangleq \left\{Q^{-1}\left[1 - (1 - BER)^{1/n_{grid}}\right]\right\}^2 \tag{21}$$

where, n_{grid} is number of nodes in the network which we have varied from about 10 to 500 in different scenarios.

For the different network scenarios used in present analysis we would take the noise Figure (F) = 6 dB, Temperature = 300 K, Carrier frequency f_c = 2.4 GHz, Packet Length L = 1000 bits, Packet arrival rate at each node λ_t = 0.5 pck/s, Pathloss exponent γ = 2.

NETWORK AND NODE LIFETIME

An important performance indicator for the wireless sensor network is the node/network lifetime The network life time has been referred to as the fraction of surviving nodes in the network (Wattenhofer *et al.*, 2001). Every node in the network can be taken to have an initial battery energy $E_{battery}$ which we would take as 1J. For a given data rate R_b the time taken to transmit one packet is L/R_b so to transmit one packet the total amount of energy required would be written as:

$$E_{packet} = P_t \times L \Big/ R_b \qquad (22)$$

Since nodes transmit packets with average rate λ_t, the average energy depleted per second is $\lambda_t E_{packet}$. So finally the total time taken for the battery energy to be exhausted can be written as:

$$\tau = \frac{E_{battery}}{\lambda_t E_{packet}} = \frac{E_{battery} R_b}{\lambda_t L P_t} \qquad (23)$$

Most of the energy consumed by a node is during the transmitting mode. For more accurate approach to estimate power consumption the idle and receiving modes could be also taken into account properly.

SIMULATION RESULTS AND DISCUSSIONS

The network parameters to be used in present scenarios would be F = 6 dB, T_0 = 300 K, Pathloss exponent γ = 2, Packet arrival rate λ_t = 0.5 pck/s, Carrier frequency f_c = 2.4 GHz, Packet Length L = 1000 bits. Also considering Binary Phase Shift Keying we will have the bandwidth B = R_b.

Optimal transmit power and data rate: In Fig.2. The transmit power is shown as a function of the data rate. The transmit power-data rate curves are shown for different values of the threshold Bit Error Rate. In terms of the power consumption we have to choose the point with the lowest transmit power corresponding to a given value of data rate R_b . It can be observed from Fig. 2 that the optimal transmit power increases as the data rate increases. The receiving nodes experiences interference from the other nodes and thus the transmit power of a node changes accordingly for sustaining a particular BER
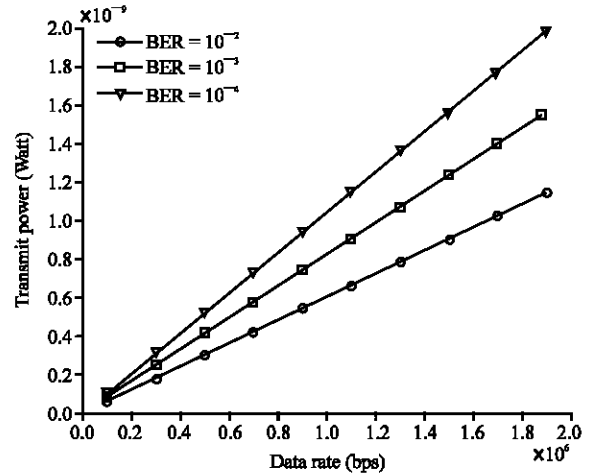

Fig. 2: Transmit power in a network as a function of data Rates (Mbps)
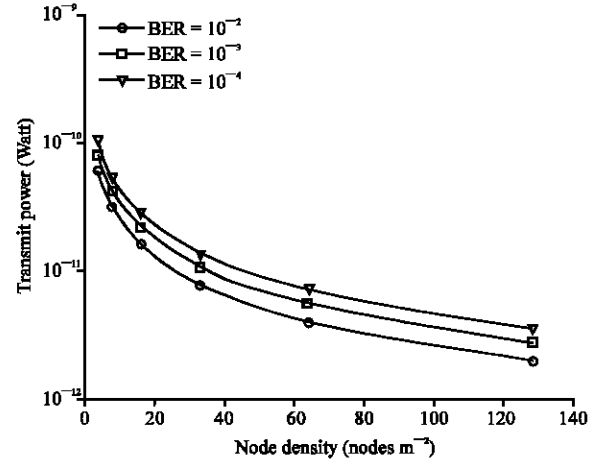

Fig. 3: Transmit power in the network as function of node density

level. Data rate thus plays an important role in the design of wireless sensor networks since for a given node density if we choose the right data rate, the transmit power can be minimized and thus prolonging the network's life time. Since to sustain the BER level the power is varying accordingly so for sustaining a lower BER the optimal transmit power required is larger as the data rate increases and the packet being transmitted at a higher data rate with the interference getting lower. However the thermal noise power also increases and the minimum transmit power required to sustain the network connectivity will increase accordingly.

Optimal transmit power and node density: Figure 3 illustrates the optimal transmit power as a function of node spatial density. We have kept the data rate R_b fixed at 0.5 Mb/s and plotting the transmit power P_t against different values of node density ρ at three different

values of BER i.e., 10^{-2}, 10^{-3} and 10^{-4}. It is observed that the optimal node transmit power is decreasing with the increased number of deployed nodes per unit area i.e. the node densityρ. Since at lower node density the nodes are farther from each other and so the transmit power has to be increased in order to preserve the connectivity. With increased number of the nodes deployed over a certain area i.e. in a denser network the nodes are closer to each other and therefore the power required to sustain the connectivity decreases.

Node density and bit error rate: Figure 4 illustrates the relationship of Bit Error Rate and Node density at different date rates. It can be observed that the Bit Error Rate increases with the increase in node density. This can be explained as follows: When the network is sparse i.e. for low node density there would be smaller numbers of nodes communicating and thus the probability of data loss would be lower and consequently a lower BER. We have plotted the graphs for BER at three different data rates. With increasing data transmission rates there is significant increase in bit error rates since higher data rates(increased bandwidth) would lead to decrease in average SNR (since $E_b = P_t \times T_b$ where $T_b = 1/R_b$, assuming thermal noise remains constant) and an increased traffic would lead to the loss of some data also. It would be difficult to maintain connectivity at higher data transmission rates.

Variation of Interference Power with Node Density: Figure 5 depicts the relationship between the node density and the interference power due to the nodes in the first tier and the second tier. The values for the interference due to the nodes in the first tier and nodes in the second tier are plotted against the increasing number of nodes. It is observed that with increasing node density there would be corresponding linear increase in the interference powers. The dominant interference signals come from the nodes in the first tier around the receiver node since the nodes in the first tier are nearer to the central receiving node.

The interference due to nodes in the second tier is lesser than the nodes in the first tier since the distance r_{con} is getting larger. Also our result in Eq. 9 is being highlighted here since the interference power is increasing linearly with the density and if the density is doubled the interference powers due to each of the tiers would also be doubled.

Bit energy and transmission rates: Figure 6 illustrates the fact that the bit energy would be decreasing with an increasing transmission rate The optimal transmit power
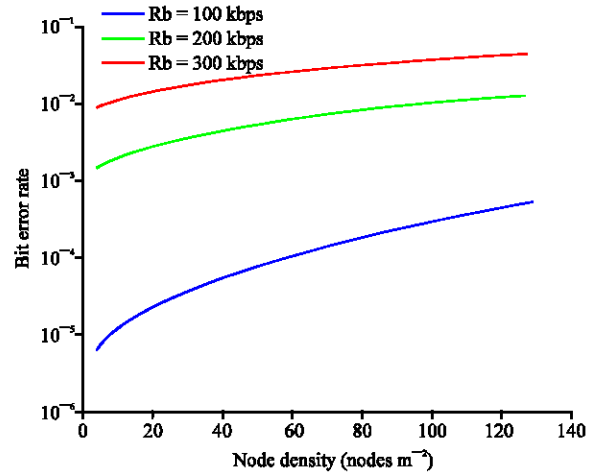

Fig. 4: The variation of Bit Error Rates for node densities at different data rates
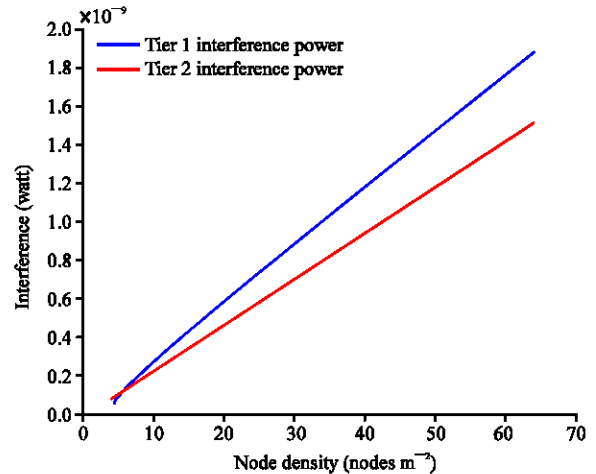

Fig. 5: The Interference powers due to nodes in Tier 1 and Tier 2 with increasing node density
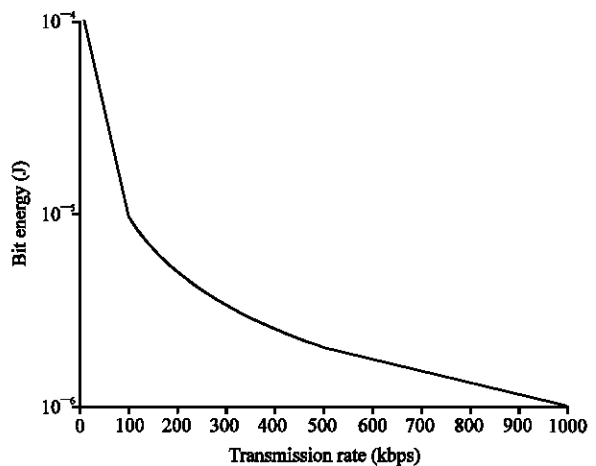

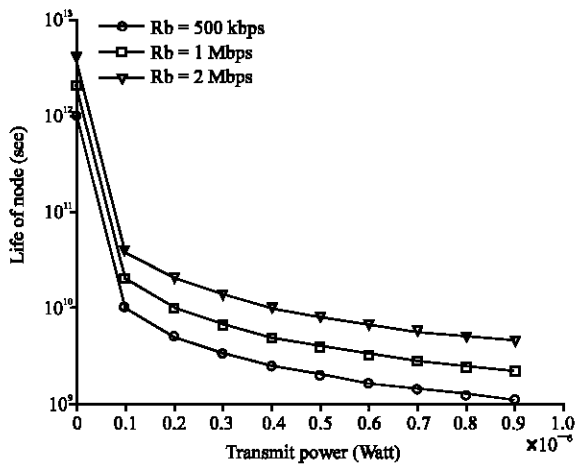Fig. 6: The Bit energy for nodes with increasing transmission rates

Fig. 7: Life time of the node, τ against optimum transmit power

for the deployed nodes has to be increased for increased bit energy but with the increasing transmission rate there would be an adverse effect with interference and noise increasing while the bit energy would be decreasing.

Transmit power and node life time: Figure 7 shows the lifetime of a node, τ as a function of transmit power P_t. The total time taken to exhaust the initial battery energy is $\tau = E_{battery} / \lambda_t E_{packet} = E_{battery} R_b / \lambda_t L P_t$. We have plotted the life time of nodes against transmit powers for three different values of R_b i.e., 500 Kbps, 1 Mbps and 2 Mbps. The node life time decreases for increased value of transmit powers. The effect of the data rate R_b is also important because as evident from the plots the higher data rates contribute to an increase in the node and network life time.

CONCLUSIONS

We have investigated the optimum transmit power for the most suitable performance of wireless sensor networks. While looking at the approach used in present study it would help anyone to look at the design aspects of sensor networks choosing the correct data rates and the required node density thus aiming to prolong the network lifetime due to the most optimum transmit power of the nodes. The data rate if chosen carefully, can guarantee significant savings in terms of transmit power, prolonging the battery life of devices and network lifetime. With the transmit power and the data rate of sensor nodes preconfigured we can also determine the minimum node density required to preserve connectivity. we have also investigated the variation of bit error rate with increasing node densities. The phenomena for increased interference due to nodes in first tier have been observed which is

affected by the respective distance of the interfering node to that of the receiving node. The interference power due to each node individually has been derived in terms of the node density. This is important for network planning when vital issues like deployment and optimal power control have to be considered.

We also observed that for lower data rates the interference powers are also lowered along with decreased bit error rates and this is also a contributing factor to the prolonging of battery life in WSNs. Similarly we can conclude that with the increase in data transmission rates among the network there would be a decrease in bit energy and higher transmission rates would mean an earlier end for the life time of the sensor node.

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