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# A Novel Objective Image Quality Metric for Image Fusion Based on Renyi Entropy

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**Abstract:** In this study, a novel objective image quality metric is proposed. The proposed metric can be used for image fusion evaluation without a reference image. The proposed metric is an extension of Mutual Information (MI) metric based on Renyi entropy which is a single parameter generalization of Shannon entropy. Renyi MI measures the total amount of information that fused image contains about source images. Also, the overlapping information problem is considered by using Generalized Normalized MI to avoid its influence. Experimental results show that the presented metric well correlated with human subjective evaluations and it is much better than conventional MI metrics based on Shannon entropy and Tsallis entropy.

**Key words:** Image quality evaluation, mutual information, Renyi entropy, Shannon entropy, Tsallis entropy, overlapping information

## INTRODUCTION

With the rapid development of image and computer technologies, more and more image fusion techniques are becoming available to integrate different source images into a single enhanced quality image (Ardeshir, 2007; Moira and Heather, 2005). In the Image fusion, image quality evaluation plays an important role. On the one hand, as the image fusion algorithms are developing quickly, it is necessary to evaluate the performance of fusion schemes efficiently. On the other hand, image quality evaluation can optimize the parameters of image fusion algorithms to improve the image fusion performance. In recent years, many studies have been conducted to develop image quality metrics (Dixon, 2007).

Generally speaking, the existing image quality evaluation methods can be classified into two categories; subjective evaluation and objective evaluation. In practice, subjective evaluation is usually a time consuming and expensive work. It can not guarantee the same test conditions (Wang *et al*., 2002). This leads to a rising demand for objective evaluation methods to exactly compare the good or bad fusion images. However, objective assessment is a difficult problem due to the variety of different application requirements and the lack of a clearly defined ground-truth.

Basically, two main categories can be distinguished in objective image quality evaluation. First of all, there are mathematically defined measures such as the widely used MSE and PSNR. Secondly, there exists a class of measurement methods, which try to incorporate the characteristics of human visual system, appearing as a valuable alternative (Pappas and Safranek, 2000; Lambrecht, 1998). Several schemes have been proposed for the development of performance metrics. Especially, some of these methods do not use reference image. Recently, based on the assumption that the HVS is highly adapted to extract structural information from the viewing field, a new image quality measurement was proposed by Wang and Bovik (2002) and then improved by Piella and Heijmans (2003) with no reference image. Xydeas and Petrovic (2000) evaluated the fusion performance by calculating the relative amount of edge information that is transferred from the input images to the fused image. Meanwhile, Mutual Information (MI) is employed for evaluating fusion performance by Qu *et al.* (2002) and then Cvejic *et al.* (2006) used Tsallis entropy as the fusion performance metric. The assessment of fused images mainly focuses on finding appropriate computational metrics that correlate well with subjective quality assessment, although such metrics often fail in finding such correlations (Dixon, 2006).

The aim of this study is to provide an objective performance evaluation metric by using MI based on Renyi entropy. Experimental results show that our metric well correlate with human subjective evaluations.

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## THE DEFINITION OF RENYI ENTROPY MEASURES

Researchers in the past several decades have devoted remarkably efforts to develop proper methods in describing the information within signals. Shannon (1948) first developed the modern concept of information and logical entropy which opened the door to information theory in the late 1940s. Information theory dealing with the science of data communications and the definition of entropy initially is a measure of the information content of given data. Shannon entropy H(X) is defined as:

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log p(x_i) \tag{1}$$

where, $p(x_i)$ is the probabilities of $x_i$.

Shannon entropy is a measure of the data spread. Data with a broader and flatter probability distribution will lead to higher entropy. Data with a narrower peaked distribution will have lower entropy.

Tsallis (2001) entropy is a new entropy measure in order to generalize the traditional entropy to nonextensive physical systems. Due to the presence of nonadditive information content in images, Tsallis-entropy based thresholding approach is presented to attempt to tackle with such nonadditive information. Tsallis entropy is defined as:

$$S_q(X) = \frac{1}{q-1}\left(1-\sum_{i=1}^{n} p(x_i)^q\right) \tag{2}$$

where, the parameter q is a real number associated with the nonextensitivity of the system ($q \neq 1$).

Renyi entropy (Ricardo, 2005) is another generalization form of (Shannon, 1948) entropy with one parameter like Tsallis. The $\alpha$ order Renyi entropy is defined as:

$$R^{\alpha}(X) = \frac{1}{1-\alpha}\log\left(\sum_{i=1}^{n} p(x_i)^{\alpha}\right) \tag{3}$$

where, $\alpha \geq 0$ and $\alpha \neq 1$. When $\alpha \rightarrow 1$, by using L'Hopital's rule, Renyi entropy degenerates to Shannon entropy.

$$H(X) = R^1(X) = \lim_{\alpha\to 1} R^{\alpha}(X) = -\sum_{i=1}^{n} p(x_i)\log p(x_i) \tag{4}$$

For image, entropy describes the total amount of image information. For a pair of random variables X, Y with joint distribution $p_{x,y}$, the joint entropy H(X, Y) of Shannon, Renyi and Tsallis are defined as Eq. 5-7, respectively:

$$H(X,Y) = -\sum_{x,y} p_{x,y}\log(p_{x,y}) \tag{5}$$

$$S_q(X,Y) = \frac{1}{q-1}\left(1-\sum_{x,y} p_{x,y}{}^q\right) \tag{6}$$

$$R_{\alpha}(X,Y) = \frac{1}{1-\alpha}\log\left(\sum_{x,y} p_{x,y}{}^{\alpha}\right) \tag{7}$$

Although Renyi entropy measures of joint distributions formally look like the Tsallis entropy, they do not have the same properties. Gabarda and Cristóbal (2007) used the generalized Renyi entropy for selecting the best image among a set of processed images. A distinct feature of this measure is that it is capable of distinguishing the noise presence in images by decreasing its value when noise is present. Atif *et al.* (2004) also applied Renyi entropy to resolve the multimodal registration problem, improving the speed and the accuracy of intensity-based alignment. Renyi entropy has more advantages for analyzing information of images than other entropies.

## THE PROPOSED METRICS FOR IMAGE QUALITY EVALUATION

In probability theory and information theory, the MI is used to measure the mutual dependence of random variables. It is a useful concept to measure the amount of information shared between two or more variables (Qu *et al.*, 2002). However, MI metric also has severe disadvantages. Therefore, a modified statistic metric based on Tsallis entropy was proposed by Cvejic *et al.* (2006). Although, this metric has better performance than MI due to its statistics properties, extensive tests have shown that the MI based on Tsallis entropy metric does not always agree with human subject perceive. We found that this metric lose part of the overall information.

**Mutual information of source and fusion images based on Renyi entropy:** The Kullback-Leibler divergence is usually used to measure the difference between two probability distributions; a true probability distribution X to an arbitrary probability distribution Y. The divergence measure of Renyi entropy is defined as:

$$D_{Re}(X\|Y) = \frac{1}{\alpha-1}\log\left(\sum_i x_i{}^{\alpha} y_i{}^{1-\alpha}\right) \tag{8}$$

where, $\alpha \geq 0$ and $\alpha \neq 1$, $x_i$ and $y_i$ denote the probability distribution associated with the distribution X and Y.

The Renyi generalized divergence is always non-negative. Renyi MI can also be expressed as a Kullback-Leibler divergence:

$$I^{\alpha}_{XY}(x,y)=\frac{1}{\alpha-1}\log\left(\sum_{x,y}p_{XY}(x,y)^{\alpha}\bullet(p_X(x)\bullet p_Y(y))^{1-\alpha}\right) \quad (9)$$

where, $p_{XY}(x, y)$ is the joint distribution and $p_X(x)\, p_Y(y)$ is the distribution associated with the case of complete independence.

Let S (S1, S2) and F represent the source and the fused images, respectively. The MI between source and fusion images based on Renyi entropy can be calculated as:

$$\begin{aligned} I^{\alpha}_{FS1}(f,s1) &= \frac{1}{\alpha-1}\log\left(\sum_{f,s1}p_{FS1}(f,s1)^{\alpha}\bullet(p_F(f)\bullet p_{S1}(s1))^{1-\alpha}\right) \\ I^{\alpha}_{FS2}(f,s2) &= \frac{1}{\alpha-1}\log\left(\sum_{f,s2}p_{FS2}(f,s2)^{\alpha}\bullet(p_F(f)\bullet p_{S2}(s2))^{1-\alpha}\right) \end{aligned} \quad (10)$$

Although, all the proposed measures based on information entropy reflect the total amount of information that fused image F contains about source images S1, S2. Unfortunately, the overlapping information problem still exists when calculating mutual information based on Renyi entropy.

**The proposed objective image fusion performance metric:** Partial image overlapping is an important problem in fusion performance assessment. The purpose of any image fusion method is to combine multimodal or multispectral images into a single one, including all of the important features in the source images. The source images often have strong correlations since the same area is covered with complementary imaging features. Thus, the overlapping information will be calculated more than once (Fig. 1). In Fig. 1, it is shown that the overlapping information I(X; Y; Z) will be calculated twice according to the MI metric proposed by Qu *et al.* (2002) and Cvejic *et al.* (2006). In an objective evaluation of the effectiveness of a fusion algorithm, the overlapping information should be considered only once and this problem has not been solved at present (Tsagaris and Anastassopoulos, 2004).

There are some solutions to reduce the influence of the overlapping information, such as the conditional MI (Tsagaris and Anastassopoulos, 2004). Although these methods are variable in theory, the result can not be satisfied in practice since the interaction information can be negative. Therefore, Generalized Normalized MI is utilized in this study, which can be expressed as:

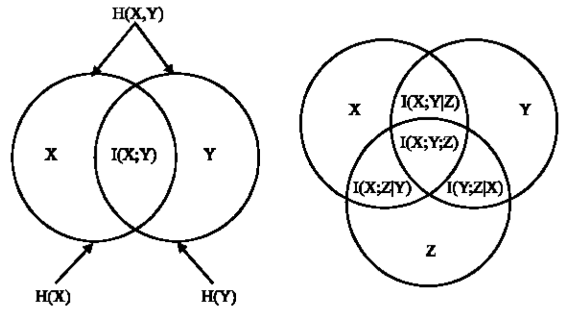



Fig. 1: Venn diagram representation of MI for two and three variables

$$R_f(X,Y)=\frac{I_f(X;X)+I_f(Y;Y)}{I_f(X;X)+I_f(Y;Y)-I_f(X;Y)} \quad (11)$$

where, f is a substitute of Renyi MI.

Then considering the overlapping information as described above, the Generalized Normalized MI is utilized in order to reduce the influence:

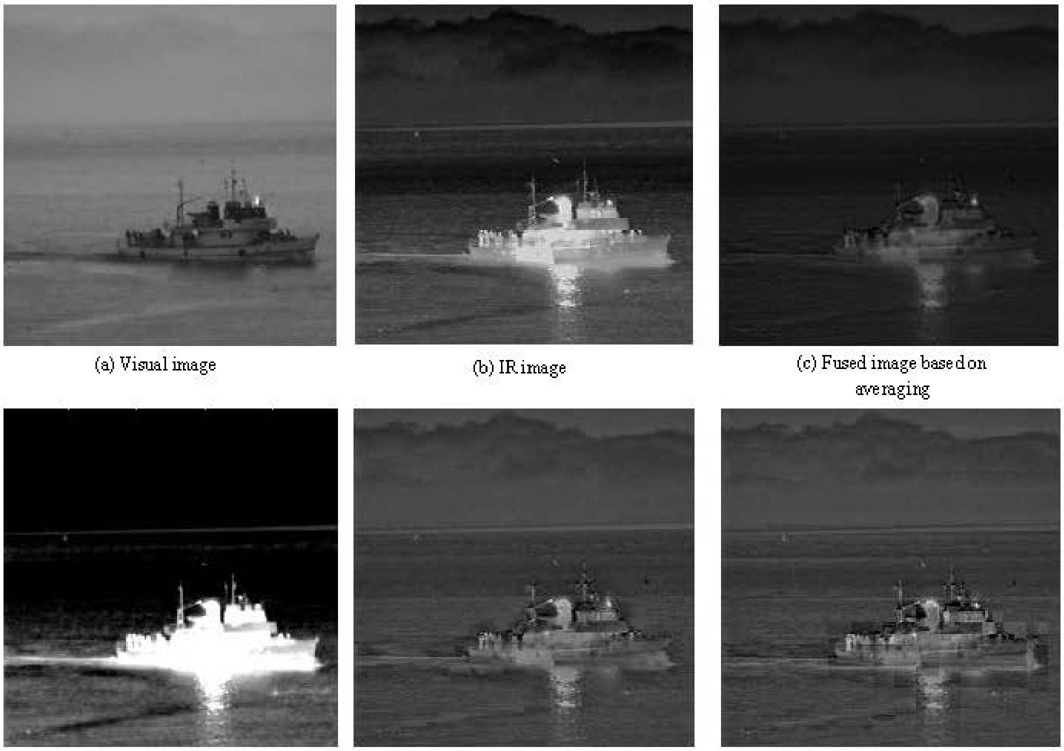
$$R_{Re}(F,S)=\frac{I^{\alpha}_{FF}(F;F)+I^{\alpha}_{SS}(S;S)}{I^{\alpha}_{FF}(F;F)+I^{\alpha}_{SS}(S;S)-I^{\alpha}_{FS}(F;S)} \quad (12)$$

Finally, the proposed objective image fusion performance metric is defined as:

$$MI^{\alpha}_{FS1S2}=R_{Re}(F,S1)+R_{Re}(F,S2) \quad (13)$$

## RESULTS AND DISCUSSION

Here, we verified the effectiveness of the proposed image quality metric by applying it to different fusion schemes and comparing the proposed image quality metric with other standard objective metrics. Four representative algorithms are testified: the image fusion approach based on two source images averaging, the Principal Component Analysis (PCA) algorithm, the Laplace Pyramid (LP) based Multi-Resolution (MR) image fusion approach and the Haar Wavelet Transform (HWT) based multi-resolution image fusion approach. We perform five-level decomposition in all MR cases. The MR fused images are reconstructed by selecting the coefficients with the maximum absolute value of detail images at each position and averaging values of approximation images. We take $\alpha$ = 2 to manipulate quadratic Renyi entropy in order to improve both the computational efficiency and performance. Four standard objective metrics are selected for comparison: the standard MI (Qu *et al.*, 2002), the Tsallis entropy (Cvejic *et al.*, 2006), the Xydeas' metric (Xydeas and Petrovic, 2000) and the Piella's metric (Piella and Heijmans, 2003).



(a) Visual image

(b) IR image

(c) Fused image based on averaging

(d) Fused image based on PCA

(e) Fused image based on Laplacian pyramid

(f) Fused image based on HWT

Fig. 2: Image fusion application case in military scouting

Image fusion is widely recognized as a valuable tool for improving image quality in image-based applications. In experiments, we use two typical image fusion application cases to analyze the performance of image quality metric. The first application is in military scouting, where the source images come from IR and vision sensors (Fig. 2). The second application is to integrate images with different focus to a single image which contains all focused regions of the source images (Fig. 3).

**Performance test of image fusion in military scouting:** Fusing the images generated from IR and vision sensors in military scouting is one of the key applications (Fig. 2a, b). The military targets (people on the ship in this test) are usually hidden in a complicated texture background and much smaller than the surroundings, which makes the targets easily neglected. Figure 2c-f show the fused images of different schemes. Obviously, the Laplacian method (Fig. 2e) clearly outperforms the other three methods. The average and PCA methods (Fig. 2d) lose many details and the HWT method (Fig. 2f) has led to image distortion in spite of preserving some details. In the subjective test, observers are required to check the fused images and give the order of them. Table 1 also shows the subjective rank of four fused images. So, it is clear that the proposed MI metric based on Renyi entropy correlates better with the subjective quality of the fused images than standard MI metric and Tsallis metric. Results obtained by the Piella's and Xydeas' metrics in Table 1 also confirm the rank of the fusion methods obtained by our proposed metric.

Table 1: Comparison of different metrics in image fusion application case in military scouting

| Fusion algorithms | Standard MI | Tsallis entropy | Xydeas ($Q_X$) | Piella ($Q_E$) | Proposed (Renyi) | Subjective test (rank) |
|---|---|---|---|---|---|---|
| Average | 1.729 | 6.741 | 0.128 | 0.258 | 5.346 | 4 |
| PCA | 4.067 | 11.734 | 0.380 | 0.387 | 5.648 | 3 |
| Laplace | 1.418 | 7.874 | 0.664 | 0.608 | 6.690 | 1 |
| HWT | 1.233 | 6.763 | 0.572 | 0.509 | 5.977 | 2 |

**Performance test of image fusion from different focus images:** Due to the limited eyeshot of optical lenses, it is impossible to get an image with all objects in focus (Fig. 3a, b). Comparison of different matrics according to Fig. 3 is given in Table 2. So, it is important to fuse different focus images into a single image which contains all of the focused objects. Figure 3c-f show the fused

(a) Image with focus on the right

(b) Image with focus on the left

(c) Fused image based on averaging

(d) Fused image based on PCA

(e) Fused image based on Laplacian pyramid

(f) Fused image based on HWT

Fig. 3: Fusion images from different focus images

Table 2: Comparison of different metrics according to Fig.3

| Fusion algorithms | Standard MI | Tsallis entropy | Xydeas ($Q_X$) | Piella ($Q_P$) | Proposed (Renyi) | Subjective test (rank) |
|---|---|---|---|---|---|---|
| Average | 7.017 | 32.740 | 0.520 | 0.663 | 9.661 | 4 |
| PCA | 6.918 | 31.015 | 0.518 | 0.663 | 9.708 | 3 |
| Laplace | 7.084 | 35.715 | 0.790 | 0.812 | 10.243 | 1 |
| HWT | 6.475 | 28.545 | 0.739 | 0.782 | 9.727 | 2 |

images of different schemes. It is shown that the Laplacian (Fig. 3e) and HWT (Fig. 3f) methods are comparable and outperform the other two schemes subjectively. The most important objects are transferred to the fused images. The subjective test results are confirmed by the presented metric that gives the highest rank to the Laplacian and HWT. The standard MI metric and Tsallis entropy give higher scores for the average algorithm, which is not true contrasting to the subjective quality. So, present proposed metrics still correlate the best with the subjective quality test among all compared metrics.

## CONCLUSION

In this study we propose a new MI metric based on Renyi entropy for image fusion, which does not require a reference image. Compared with other standard objective quality metrics, our proposed metric well correlates with the subjective quality test. In addition, the proposed metric is easy to implement and can be applied into different fusion schemes. In the future, we will do more thorough research in information theory with the expectation that our objective measures are capable of much broader applications. The aim of this future study is to consider the Renyi entropy information based on fixed windows and image segmentation regions. We also plan to study how to associate the MI approaches based on Shannon, Tsallis and Renyi with fusion algorithm as a guide to improve the fusion performance.

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