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Spectral Efficiency Analysis for the Uplink Generalized Distributed Antenna System with Different Signal Combining Algorithms

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Abstract: In this study, the closed-form expressions of the means and cumulative distribution functions of the spectral efficiency for centralized antenna system and distributed antenna system with different signal combining algorithms are derived. These theoretical results demonstrate how overall performance is affected by system parameters such as shadowing intensity and number of antennas. Through numerical experiments it is shown that these results can consistently produce highly accurate results.

Key words: Generalized distributed antenna system, diversity combining, spectral efficiency analysis

INTRODUCTION

Generalized Distributed Antenna System (GDAS) has been proposed as an access technique for future wireless communication networks (Roh and Paulraj, 2002; Zhou *et al*., 2003; Choi and Andrews, 2007; Somekh *et al*., 2007). However, due to the nature of widely spaced antenna array, the channel model for the GDAS should encompass not only small scale fading but also large scale fading. In recent years, many researchers have studied the spectral efficiency of GDAS in a composite fading channel. Yet, due to the complexity of the composite Rayleigh-lognormal distribution, most of the results are based on the Monte-Carlo simulation (Zhuang *et al*., 2003) or numerical integration (Choi and Andrews, 2007). In (Roh and Paulraj, 2002), by using numerical integration, the outage performance is compared between GDAS and CAS. It is shown that GDAS can exploit spatial micro-diversity and macro-diversity simultaneously and then the outage performance is improved. Different from (Roh and Paulraj, 2002), in this study here is provided the closed-form expressions of the spectral efficiency for CAS and GDAS with different signal combining algorithms. The cumulative distribution functions (CDFs) of the spectral efficiency for corresponding receivers are also given. Then, the impact of the different parameters on the spectral efficiency, such as, shadowing variance and number of the RAU, are demonstrated clearly.

The notation adopted in this study conforms to the following convention. Vectors are column vectors and are denoted in lower case bold: **x**. Matrices are upper case bold: **A**. $\mathbf{I}_k$ denotes the identity matrix of size k×k. $(\cdot)^T$ and $(\cdot)^H$ represent transpose and Hermitian transpose, respectively. The operator $E(\cdot)$ denotes expectation and $V(\cdot)$ denotes variance. erfc $(\cdot)$ is the complementary error function defined by

$$\operatorname{erfc}(x) = \frac{2}{\pi}\int_x^{\infty} e^{-t^2}\,dt$$

SYSTEMS DESCRIPTIONS

We consider a noise-limited (N, L) GDAS system, where N is the number of radio access unit (RAU) and L denotes the antenna number per RAU. The antenna number of user equipment is 1. The discrete-time data model can be written as follows:

$$\mathbf{r} = \sqrt{\rho}\mathbf{h}u + \mathbf{n} \tag{1}$$

where, u is the transmit symbol, $\mathbf{r} = [r_1 \; L \; r_{NL}]$ is the receive vector and **n** is the complex additive white Gaussian noise vector with covariance matrix of $E(\mathbf{nn})^H = \mathbf{I}_N$, ρ is the transmitted power and vector **h** denotes the channel gains from the user to all of NL antennas. For analytical convenience, similar to (Roh and Paulraj, 2002), we assume that the path losses between the user and all RAUs are the same. The channel gain between the user and the l-th antenna of n-th RAU is assumed to be:

$$h_{n,l} = \sqrt{cs_n}h_{n,l}^w \tag{2}$$

where, c denotes the path loss, in this study, here assume that c = 0 dB (c = 1), $h_{n,l}$ denotes the small-scale Rayleigh fast fading and s_n is a log-normal shadow fading variable, where $10\log_{10}s_n$ is a zero-mean Gaussian random variable with standard deviation σ_{sh}. Throughout the study, it is assumed $h_{n,l}^w$ is independent of s_n.

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SPECTRAL EFFICIENCY OF CAS WITH L ANTENNAS

For CAS, the large-scale fading is the same for different antennas. When, micro selection diversity (m-SD) is adopted at the receiver, the instantaneous data rate in bits per second per Hertz (bps/Hz) can be expressed as:

$$Y = \log_2(1+\rho s\gamma_{m\text{-}SD}) \quad (3)$$

Where:

$$\gamma_{m-SD} = \max_{l=1,\dots,L}\left(\left|h_l^w\right|^2\right)$$

Since h_l^w is independently and identically distributed (i.i.d) circular symmetric Gaussian random variable, according to the order statistics of i.i.d random variables (Balakrishnan and Cohen, 1991), the PDF of $\gamma_{m\text{-}SD}$ can be given by:

$$p_{\gamma_{m\text{-}SD}}(x) = Le^{-x}\left(1-e^{-x}\right)^{L-1} \quad (4)$$

Then, the averaged data rate can be given by:

$$\bar{Y} = E\log_2(1+\rho s\gamma_{m\text{-}SD}) \quad (5)$$

Unfortunately, it is hard to obtain the closed-form expression of 5 due to the existent of the shadow fading. Then, we resort to the tight bound.

Theorem 1: The tight lower bound of 5 can be expressed as:

$$\bar{Y} \ge \log_2\left[1+\rho\exp\left(\sum_{n=1}^{L}\frac{(-1)^n(\ln n+\varepsilon)L!}{n!(L-n)!}\right)\right] \quad (6)$$

Proof: Since, $\log_2(1+ae^x)$ is a convex function in x for a>0, using Jensen's inequality, we can lower bound 5 as:

$$\bar{Y} \ge \log_2\left[1+\rho e^{E\ln(\gamma_{m\text{-}SD})}\right] \quad (7)$$

where, it is used the fact that E (1n s) = 0. In the following, Here will compute E ln ($\gamma_{m\text{-}SD}$). First, using binomial expansion, we can write:

$$p_{\gamma_{m\text{-}SD}}(x) = \sum_{n=1}^{N}\frac{L!}{(n-1)!(L-n)!}(-1)^{n-1}e^{-nx}$$

Then, we have:

$$\begin{aligned} E\ln(\gamma_{m\text{-}SD}) &= \sum_{n=1}^{L}\frac{L!}{(n-1)!(L-n)!}(-1)^{n-1}\int_0^\infty \ln(x)e^{-nx}dx \\ &= \sum_{n=1}^{L}\frac{(-1)^n(\ln n+\varepsilon)L!}{n!(L-n)!} \end{aligned} \quad (8)$$

where, ϵ = 0.577215 the Euler-Mascheroni constant. The result follows by substituting 8 into 7.

For m-SD, it is difficult to obtain the CDF of the data throughput 3. Here is used the following approximation. Firstly, we average the data rate 3 over $\gamma_{m\text{-}SD}$ and then we have

$$E_{\gamma_{m\text{-}SD}}(Y) \ge \log_2\left[1+\rho\exp\left(\sum_{n=1}^{L}\frac{(-1)^n(\ln n+\varepsilon)L!}{n!(L-n)!}\right)s\right]$$

At high SNR, $E_{\gamma m\text{-}SD}(Y)$ can be approximated as:

$$E_{\gamma_{m\text{-}SD}}(Y) \approx \log_2(\rho) + \frac{1}{\ln 2}\sum_{n=1}^{L}\frac{(-1)^n(\ln n+\varepsilon)L!}{n!(L-n)!} + \log_2 s$$

Then, we can see that the spectral efficiency can be approximated as a Gaussian random variable:

$$N\left(\log_2\rho + \frac{1}{\ln 2}\sum_{n=1}^{L}\frac{(-1)^n(\ln n+\varepsilon)L!}{n!(L-n)!}, \frac{\lambda^2\sigma_{sh}^2}{(\ln 2)^2}\right)$$

where, $\lambda \triangleq \ln 10/10$, $V(\ln s) = \lambda^2\sigma_{sh}^2/(\ln 2)^2$.

Using Gaussian approximation, the closed-form approximation of the maximum rate at the outage probability δ (i.e., $\Pr(Y<Y^\delta)=\delta$) can be given by:

$$Y^\delta = \bar{Y} + \sqrt{2V(Y)}\,\mathrm{erfc}^{-1}(2(1-\delta)) \quad (9)$$

When, micro-maximum ratio combining (m-MRC) is employed at the receiver, the instantaneous data rate in bps/Hz can be expressed as:

$$Y = \log_2(1+\rho s\gamma_{MRC}) \quad (10)$$

Where:

$$\gamma_{MRC} = \sum_{l=1}^{L}\left|h_l^w\right|^2$$

is a chi-squared random variable with 2L degrees of freedom (In this study, a chi-squared random variable with k degrees of freedom is defined as the sum of the squares of k i.i.d Gaussian random variables with zero mean and variance 1/2) and its PDF can be written as:

$$p_{\gamma_{MRC}}(x) = \frac{x^{L-1}e^{-x}}{(L-1)!}$$

Then, the following theorems used.

Theorem 2: The averaged data rate of 10 can be expressed as:

$$\bar{Y} \geq \log_2\{1 + \rho \exp[\psi(L)]\} \quad (11)$$

where, $\psi(\cdot)$ is the digamma function. For integer x, this function may be expressed as

$$\psi(x) = \sum_{p=1}^{x-1} \frac{1}{p} - \varepsilon.$$

Proof: Following to the similar steps in the proof of Theorem 1 and using E ln $(\gamma_{MRC}) = \psi(L)$ we obtain the desired result.

Now, we focus on the CDF of the spectral efficiency of m-MRC in the high SNR regime. For large ρ, (10) can be approximated as:

$$Y \approx \log_2 \rho + \log_2 s + \log_2 \gamma_{MRC}$$

As shown in Hochwald *et al.* (2004), for large L, ln γ_{MRC} can be approximated as a Gaussian random variable with mean E ln $(\gamma_{MRC}) = \psi(L)$ and variance (Zhang *et al.*, 2005; Oyman *et al.*, 2003) V ln $(\gamma_{MRC}) = \psi(L)$, where $\psi'(\cdot)$ is the first derivative of the digamma function and it can be expressed as (Oyman *et al.*, 2003)

$$\psi'(x) = \sum_{p=1}^{\infty} \frac{1}{(p+x-1)^2}.$$

Then, a numerically accurate approximation of the distribution of Y is given by:

$$N\left(\log_2 \rho + \frac{\psi(L)}{\ln 2}, \frac{1}{(\ln 2)^2}[\lambda^2 \sigma_{sh}^2 + \psi'(L)]\right)$$

SPECTRAL EFFICIENCY OF (N, L) GDAS

In this Section, a hybrid combining (HC) and the macro/micro-MRC (M/m-MRC) are chosen among many possible schemes to be implemented (N, L) GDAS. In the hybrid combining scheme, each port performs L-diversity MRC and their results are compared so that the port with the largest instantaneous combiner output SNR is selected. In the macro/micro-MRC scheme, all antennas at all RAUs are used in the MRC circuit.

At the base station, by using L-diversity MRC, the instantaneous data rate between the user and the n-th RAU can be expressed as:

$$Y_n = \log_2(1 + \rho s_n \gamma_{MRC,n})$$

where, $\gamma_{MRC,n}$ is a chi-squared random variable with 2L degrees of freedom. For HC, the selection of the largest SNR is equivalent to the selection of the largest the instantaneous data rate, that is:

$$Y_n = \log_2\left(1 + \rho \max_{n=1,\ldots,N}(s_n \gamma_{MRC,n})\right) = \max(Y_1, \ldots, Y_N) \quad (12)$$

For GDAS, we assume that s_n (n = 1,2, ... ,N) are i.i.d lognormal random variables. Then, as shown in subsection 3.1, Y_n (n = 1,2, ... ,N) can be approximated as i.i.d Gaussian random variables. For i.i.d Gaussian random sequence, we give the following definition.

Definition: Let Q_1, Q_2, K, Q_K be a sequence of i.i.d. Gaussian random variables with mean μ and variance σ^2. Define $M_K = \max(Q_1, Q_2, K, Q_K)$ and

$$\varsigma_K \triangleq \frac{M_K - \mu}{\sqrt{2\sigma^2 \ln K}}$$

The CDF of M_K is given by:

$$\varsigma_K \triangleq \frac{M_K - \mu}{\sqrt{2\sigma^2 \ln K}}$$

According to the order statistics of Gaussian random variables (Balakrishnan and Cohen, 1991), the mean of M_K (or ς_K) does not have explicit expression when K>5. However, we know that the mean of ς_K, denoted as $\bar{\varsigma}_K$ is only a function of K. Using numerical integration, we find the relation of them and it is shown in Fig. 1. As shown in (Hochwald *et al.*, 2004), $\bar{\varsigma}_K \to 1$ in probability when K goes to infinity.

Then, for GDAS with HC, the averaged data rate can be approximated by:

$$\bar{Y} = E(Y_n) + \bar{\varsigma}_K \sqrt{2V(Y_n)\ln N}$$

Using the results mentioned above, we can obtain the closed form expression of $\bar{Y}$ as:

$$\bar{Y} = \log_2 \rho + \frac{\psi(L)}{\ln 2} + \frac{\bar{\varsigma}_N \sqrt{2 \ln N[\lambda^2 \sigma_{sh}^2 + \psi'(L)]}}{\ln 2} \quad (13)$$

For large N, $\varsigma_N \approx 1$ and we can see that the spectral efficiency increases with the logarithm of N. Note that as L goes to infinity, $\psi'(L) \to 0$. Then, it can be seen that for large L the spectral efficiency is linearly increasing with σ_{sh}. These conclusions coincide with the results in Roh and Paulraj (2002) by using numerical integration.

With Gaussian approximation, the CDF of the data rate can be obtained immediately:

$$F(y) = \left\{1 - 0.5\text{erfc}\left(\frac{y\ln 2 - \ln\rho + \psi(L)}{\sqrt{2\left[\lambda^2\sigma_{sh}^2 + \psi'(L)\right]}}\right)\right\}^N \tag{14}$$

By using macro/micro-MRC, the instantaneous data rate in bps/Hz can be expressed as:

$$Y = \log_2\left[1 + \rho\sum_{n=1}^{N} s_n \gamma_{MRC,n}\right] \tag{15}$$

At high SNR,

$$Y = \log_2\rho + \log_2\left(\sum_{n=1}^{N} s_n \gamma_{MRC,n}\right) \tag{16}$$

Note that ln (s_n $\gamma_{MRC,n}$) can be approximated by a Gaussian random variable. Then, s_n $\gamma_{MRC,n}$ can be approximated by a lognormal random variable. It is well accepted that the distribution of the sum of log-normal variables can be approximated by another log-normal distribution (Schwartz and Yeh, 1982). Then, 16 can also be approximated by a Gaussian random variable. In this paper, we use Schwartz and Yeh's method (Schwartz and Yeh, 1982).

NUMERICAL RESULTS

Numerical examples along with simulation results are given this section to investigate the spectral efficiency under various system configurations. Unless specified otherwise, log-normal shadow fading standard deviation σ_{sh} is 8dB.

Figure 1 and 2 give the comparisons of the mean and the CDF of the spectral efficiency for CAS with m-SD or m-MRC receiver. It is seen that the analytical approximations of the mean and the CDF of the spectral efficiency are very accurate in the high SNR regime. As expected, the performance of m-MRC is better than that of m-SD.

Figure 3 and 4 give the comparison of the mean and the CDF of the spectral efficiency for GDAS with HC or M/m-MRC receiver. For HC receiver, It is seen that the analytical expressions of the mean 13 and the CDF 14 of

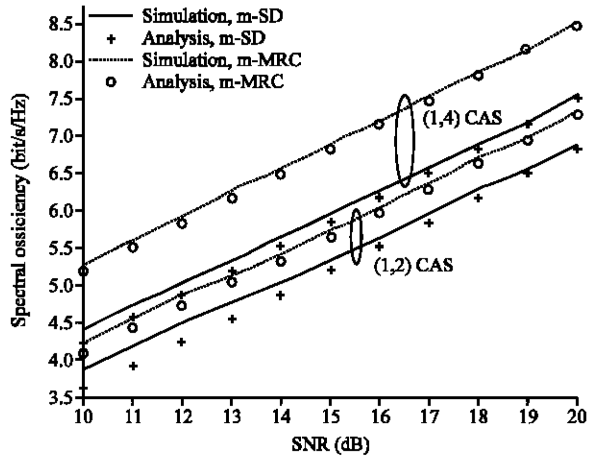

Fig. 1: Mean of the spectral efficiency of CAS

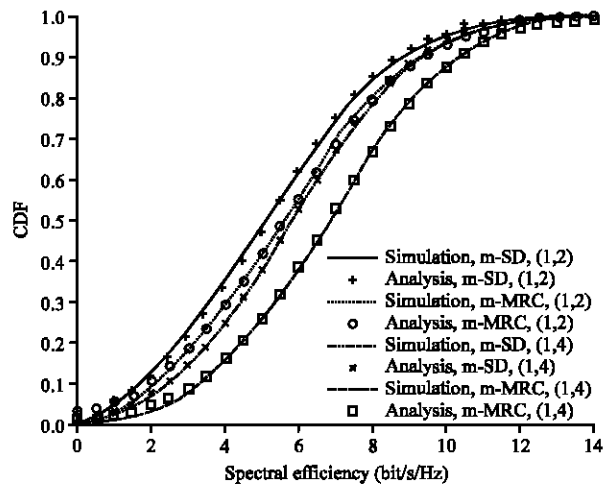

Fig. 2: CDF of the spectral efficiency of CAS, ρ = 15 dB

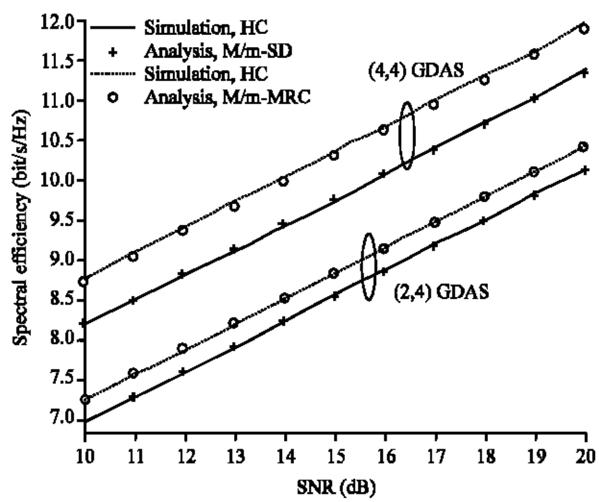

Fig. 3: Mean of the spectral efficiency of GDAS

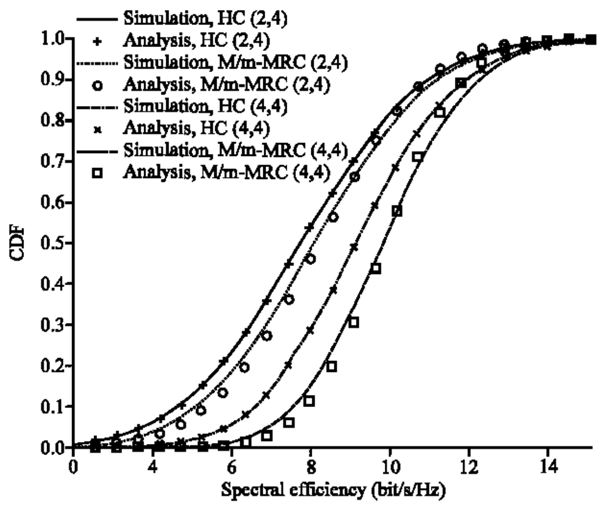

Fig. 4: CDF of the spectral efficiency of GDAS, ρ = 15 dB

the spectral efficiency are extremely accurate. Using Schwartz and Yeh's method, the approximations for M/m-MRC receiver is also very close to the Monte-Carlo simulation.

CONCLUSIONS

In this study, Here is presented some analytical results on the spectral efficiency of CAS and GDAS in a composite channel model. With Gaussian assumption for the data throughput, here is driven the means and the CDFs of the spectral efficiency for different receivers at high SNR. Numerical examples were also presented and discussed to verify the correctness and accuracy of analytical results.

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