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Connectivity Preserving Distributed Coordination Control with Few Long Range Interactions

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Abstract: Distributed coordination control of multi-agent systems raises fundamental and novel problems in recent years. A great new challenge is the development of robust distributed motion algorithms. In this study, a distributed control strategy for connectivity preserving coordinated motion of multi-agent system is presented by introduction small-world connections among mainly local interactions. For arbitrary initial network topology, the group consists of several connected subgroups. Some agents are modeled as virtual leader and steer the disconnected subgroup to flock together. In this way, flocking problem can be solved under more relaxed conditions, which need no the connectedness of the dynamic topology all the time, even the connectedness of the initial graph. Further, we show that the strategy is robust against connection failures between followers and leader in the leader following coordination control. Simulation results are given to validate the method.

Key words: Multi-agent system, connectivity control, flocking motion, local interaction, connected graphs

INTRODUCTION

Recently, there has been a surge of interest among coordination control of multi-agent systems. By using graph theory and other methods, many interesting coordination problems are under investigation. Such as, consensus problem of the multi-agent system with single or double integrator model has been extensively studied by Jadbabaie *et al*. (2003), Ren and Beard (2005) and Ren (2007) and flocking control were considered by Saber (2006) and Tanner *et al*. (2007) based on the strategy of combining artificial potential with velocity consensus. However, all the results critically rely on the assumption that the underlying network is either connected for all time or is jointly connected over infinite sequences of bounded time intervals, or at least there has a spanning tree. Motivated by this, several recent studies (Dimarogonas and Kyriakopoulos, 2007; Zavlanos *et al*., 2007; Ji and Egerstedt, 2005, 2007; Zavlanos and Pappas, 2007) considered the connectivity maintenance problem. Dimarogonas and Kyriakopoulos (2007) and Zavlanos *et al*. (2007) designed novel inter-agent potentials that force agents to remain within this distance for all time, the swarm aggregation and flocking of multi-agent systems were studied. Ji and Egerstedt (2005, 2007) applied nonlinear weights on edges that guarantee the connectivity property of the network, the rendezvous problem and formation control problem were solved. Zavlanos and Pappas (2007) used the dynamics of the Laplacian matrix and its spectral properties to construct artificial potential fields that preserve the connectivity property of the network, a centralized feedback control framework was proposed. In above connectivity control problems, there is still a common assumption that the initial graph of system is connected.

The main contribution of this study is to propose a distributed control strategy that can simultaneously solve the distributed connectivity maintenance problem and the flocking motion control with arbitrary initial positions and velocities. The idea is to introduce the small-world connections among mainly local interactions. That is, to add long-range interactions between the disconnected agents.

PRELIMINARIES

Consider a system consisting of N agents. Let $p_i \in R_m$ and $q_i \in R_m$ denote the position and velocity vectors of agent i, respectively. A continuous time model of the N agents is described as follows:

$$\begin{aligned} \dot{p}_i &= q_i \\ \dot{q}_i &= u_i, i = 1, 2, \cdots, N \end{aligned} \tag{1}$$

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where, $u_i \in R_m$ is the control inputs of agent i and is determined by the mainly local interactions and few long-range interactions.

- **Definition 1:** (Flocking motion) A group of mobile agents is said to flock, when all agents velocity vectors become asymptotically the same, collisions between interconnected agents are avoided and the system approaches a configuration that minimizes all agents' potentials

Since, the design of control laws and stability analysis of the group of agents critically rely on algebraic graph theory, the following is a brief introduction to algebraic graph theory.

- **Definition 2:** (Dynamic Graphs): Dynamic graphs G (t) = (V, ε(t)) are undirected graph that consist of a set of vertices V = {1,...N} indexed by the set of agents and a time varying set of links $\varepsilon(t) = \{(i, j) \in V \times V\}$

The neighbor set of node i is defined by $N_i(t) \triangleq \{j \in V : (i,j) \in \varepsilon(t)\}$. The adjacency matrix A (t) = $[a_{ij}(t)]$ of an undirected graph G (t) is defined as $a_{ii}(t) = 0$ and $a_{ij}(t) = a_{ji}(t) > 0$ (i, j = 1...N) if $j \in N_i$. The Laplacian matrix of the weighted digraph is defined as L (t) = $[l_{ij}(t)]$, where, $l_{ii}(t) = \Sigma_j a_{ij}(t)$ and $l_{ij}(t) = -a_{ij}(t)$ where, $i \neq j$. For an undirected graph, the Laplacian matrix is symmetric positive semi-definite.

The graph G (t) = (V, ε (t)) is said to be jointly connected across [t,T] if and only if, for any $(i, j) \in V$, there is a path between i and j in the collection of graphs {G (t), G (t+Δt),...G (t)}. If for a given finite time t_1, there exists $t_2 \geq t_1$, such that G (t) is jointly connected over $[t_1, t_2]$, we say that G (t) is jointly after t_1.

CONNECTEDNESS PRESERVING MULTI-AGENT COORDINATION

At present, models of collective motion are mainly on the basis of local rules i.e., neighboring interactions rule. These models are often adopted to realize the complex control of multi-agent system such as consensus, rendezvous, synchrony, cooperation and so on (Jadbabaie *et al*., 2003; Ren and Beard, 2005; Tanner *et al*., 2007; Saber *et al*., 2007; Jie and Morse, 2003). In fact, interaction pattern between agents may not be confined to local interactions. In the study (Watts and Strogatz, 1998), it has been demonstrated that adding few long-range connections in a locally connected network leads to the so-called small-world effect: the average path length is decreased significantly, while the clustering property of the networks is preserved. This may help information exchange between far nodes. This motivates some research on coordination control by introducing a few long-range connections (Buscarino *et al*., 2005).

In this study, we build a small-world-type network by randomly adding a few long range connections between the distant and disconnected agents and present a coordinated control strategy which makes the multi-agent system achieve successfully flocking motion with arbitrary initial positions and velocities. The arbitrary initial states can be divided into connected system and disconnected system based on their communication network. For the initial connected system, it has been demonstrated that stable flocking motion can be achieved by adopting the strategy of connectedness preserving (Zavlanos *et al*., 2007). Of course, our coordinated control strategy can also solve flocking problem with the initial connected system. For simplicity, we will only discuss the initial disconnected system.

We firstly establish one leading subgroup that is connected and design novel inter-agent potentials that force agents to remain within the leading subgroup for all time. Then, within the leading subgroup, we choose randomly a virtual leader for the distant agent. The virtual leader is used for navigation of the distant agent and will steer the distant agents towards the leading group, which will make all agents connected together finally.

The leading subgroup is the main body of multi-agent system which satisfies:

- **Definition 3:** (Leading subgroup)For any $t \in [t_k, t_{k+1}]$, $k \in Z^+$, the set of $SG_L(t)$ is determined as $SG_L(t) = \{i \in V: i = \max SG_j(t), j = 1, 2,...m(t)\}$,where $SG_j(t)$, j = 1,2...m (t), $1 \leq m(t) \leq N$ are the subgroups of the group at time t, such that $\forall j = 1, 2, \cdots m(t)$, the subgroup $SG_j(t)$ is connected and $\cup_{j=1}^{m} SG_j(t) = V$

The control law of the agent I u_i can be defined as:

$$u_i = h_i(t)[-k_p \nabla_{p_i} \sigma(p_{iL}) - (q_i - q_L)] + \sum_{j \in N_i} f(p_i - p_j) - k_v \sum_{j \in N_i} (q_i - q_j) \quad (2)$$

where, $k_p, k_v > 0$ is constant and

$$h_i(t) = \begin{cases} 1, \text{if} & i \notin SG_L(t) \\ 0, \text{if} & i \in SG_L(t) \end{cases}$$

f (.) is the attraction/repulsion function that only depends on the relative distance between two agents; σ (.) represents the interaction potential between agent i and the virtual leader.

If the agent i isn't a member of the leading subgroup SG_L (t) and is disconnected with the main body of multi-agent system, we will introduce long rang interactions by choosing randomly an agent from the leading subgroup SG_L (t) as the virtual leader, h_i (t) = 1. Otherwise, we can design control law only based on local interactions, h_i (t) = 0. That is, in the multi-agent system, we utilize small-world effect by introduction of long range connections from the main body to the separate agent, to make the main body affect the distant agent, thereby influencing all the other agents nearby them. Thus, the distant agents don't separate from the main body of the multi-agent system and finally form a flock.

The attraction/repulsion function is responsible for cohesion and separation of the agents, while preserving all the connections between agents within the connected subgroup. In this study, we choose:

$$f(y) = -y[\frac{a}{(R^2 - \|y\|^2)^2} - b\exp(-\frac{\|y\|^2}{c})]$$

as force function. Potential function σ (.) is mainly used to steer the disconnected agent i to the leading subgroup SG_L (t) and to avoid collisions between agent i and its virtual leader. Then, we choose:

$$\sigma(p_{iL}) = \frac{L^2}{\|p_{iL}\|^2} + \log\|p_{iL}\|^2$$

as navigation function.

STABILITY ANALYSIS

Based on the above model, the interaction and neighboring relation between agents are time-varying, which will give rise to a switching dynamic system. Let the set TT = {T_1, T_2...} denote the switching time sequence. At every moment in the set TT each agent determines its neighbor set and the topology of dynamic graph G (t) changes, but during the dwell time T_{k+1}-T_k the topology of dynamic graphs G (t) is fixed.

For the convenience of the following analysis, we assume that the first m ($1<m\leq N$) agents are the member of the leading subgroup, that is, h_i = 0 for agent i, i = 1, 2,...m and h_i = 1 for agent i, i = m+1, m+2,...N. We divide all the agents into two types, the agent within the leading subgroup is called a type 1 agent; otherwise, it is called a type 2 agent.

For a type 1 agent, we can define a Lyapunov function such that:

$$Q_1(p,q) = \frac{1}{2}\sum_{i=1}^{m}[\sum_{j\in N_i} J(p_i - p_j) + q_i{}^T q_i] \quad (3)$$

where, $\nabla J(p_i - p_j) = -f(p_i - p_j)$. For a type 2 agent, define a Lyapunov function such that:

$$Q_2(p,q) = \frac{1}{2}\sum_{i=m+1}^{n}[U_i(p) + (q_i - q_L)^T(q_i - q_L)] \quad (4)$$

where, $U_i(p) = \sum_{j\in N_i} J(p_i - p_j) + 2k_p\sigma(p_i)$

Clearly, Q_2 is a positive semi-definite function. Thus, the total energy function Q = Q_1+Q_2 is a positive semi-definite function.

Theorem: Consider a group of N agents with dynamics (Eq. 1) each steered by control law (2), all agent velocities become asymptotically the same and collisions among agents are avoided and for any $T\in[t_0,+\infty]$, the group is jointly connected across [T,+∞].

Proof: The motion of the type 1 agent is independent and unaffected by the type 2 agent, but the motion of the type 2 agent is influenced by the type 1 agent and their neighbors. The derivative of Q is $\dot{Q} = \dot{Q}_1 + \dot{Q}_2$.

For the type 1 agent, h_i = 0, the control law is

$$u_i = \sum_{j\in N_i} f(p_i - p_j) - k_v\sum_{j\in N_i}(q_i - q_j)$$

After some manipulation, the derivative of Q_1 is given by,

$$\begin{aligned}\dot{Q}_1 &= -\sum_{i=1}^{m} q_i^T(\sum_{j\in N_i} f(p_i - p_j)) + \sum_{i=1}^{m} q_i^T u_i \\ &= \sum_{i=1}^{m} q_i^T[-\sum_{j\in N_i} f(p_i - p_j)] + \sum_{i=1}^{m} q_i^T[\sum_{j\in N_i} f(p_i - p_j) - k_v\sum_{j\in N_i}(q_i - q_j)] \\ &= -k_v\sum_{i=1}^{m} q_i^T\sum_{j\in N_i}(q_i - q_j) = -q^T(L_1(t)\otimes I_2)q \leq 0\end{aligned}$$

where, q = $[q_1, q_2,...q_m]^T$ and L_1 (t) is the Laplacian matrix of the type 1 agents.

For the type 2 agent, h_i = 1, the control law is as follow:

$$\begin{aligned}u_i = {} & [-k_p\nabla_{p_i}\sigma(p_{iL}) - (q_i - q_L)] + \\ & \sum_{j\in N_i} f(p_i - p_j) - k_v\sum_{j\in N_i}(q_i - q_j)\end{aligned} \quad (5)$$

Let

$$\tilde{p}_i = p_i - p_L, \tilde{q}_i = q_i - q_L$$

and

$$\tilde{p} = [\tilde{p}_{m+1}^T, \tilde{p}_{m+2}^T, \cdots \tilde{p}_n^T]^T, \tilde{q} = [\tilde{q}_{m+1}^T, \tilde{q}_{m+2}^T, \cdots \tilde{q}_n^T]^T$$

Then, the Eq. 5 and energy function (4) can be rewritten as:

$$u_i = [-k_p \nabla_{p_i} \sigma(\tilde{p}_i) - \tilde{q}_i] + \sum_{j \in N_i} f(\tilde{p}_i - \tilde{p}_j) - k_v \sum_{j \in N_i} (\tilde{q}_i - \tilde{q}_j)$$

$$Q_2(p,q) = \frac{1}{2} \sum_{i=m+1}^{n} [U_i(\tilde{p}) + \tilde{q}_i^T \tilde{q}_i]$$

where,

$$U_i(\tilde{p}) = \sum_{j \in N_i} J(\tilde{p}_i - \tilde{p}_j) + 2k_p \sigma(\tilde{p}_i)$$

After some manipulation, the derivative of Q_2 is as follow:

$$\begin{aligned}\dot{Q}_2(p,q) &= k_p \sum_{i=1}^{N-m} \tilde{q}_i^T \nabla_{\tilde{p}_i} \sigma(\tilde{p}_i) + \sum_{i=1}^{N-m} \tilde{q}_i^T (\sum_{j \in N_i} f(\tilde{p}_i - \tilde{p}_j)) + \sum_{i=1}^{N-m} \tilde{q}_i^T u_i \\ &= \sum_{i=1}^{N-m} \tilde{q}_i^T [k_p \nabla_{\tilde{p}_i} \sigma(\tilde{p}_i) - \sum_{j \in N_i} f(\tilde{p}_i - \tilde{p}_j)] \\ &+ \sum_{i=1}^{N} \tilde{q}_i^T [-k_p \nabla_{\tilde{p}_i} \sigma(\tilde{p}_i) + \sum_{j \in N_i} f(\tilde{p}_i - \tilde{p}_j) - k_v \sum_{j \in N_i} (\tilde{q}_i - \tilde{q}_j) - \tilde{q}_i] \\ &= -\sum_{i=1}^{N-m} \tilde{q}_i^T [\tilde{q}_i + k_v \sum_{j \in N_i} (\tilde{q}_i - \tilde{q}_j)] = -\tilde{q}^T (I_{N-m} + k_v L_2(t)) \tilde{q} \le 0\end{aligned}$$

where, L_2 (t) is the Laplacian matrix of the type 2 agents. Since, L_1 (t), L_2 (t) and I_{N-M} are all positive semi-definite matrices, $\dot{Q}_1 \le 0 \dot{Q}_2 \le 0, \dot{Q} \le 0$ which implies that Q (t) is a nonincreasing function of time t. Hence, the set of all $(\tilde{p},\tilde{q}), \Omega = \{(\tilde{p},\tilde{q}) \mid Q \le Q_0\}$ is an invariant set and the energy function Q is bounded. Similarly, all $U_i(\tilde{p})$ are bounded. Let $\bar{\Omega}$ be the largest invariant set in Ω. On $\bar{\Omega}$, $\dot{Q} = 0$. By the LaSalle Invariance Principle, all trajectories of the agents that start from Ω converge to the largest invariant set inside the region $\bar{\Omega} = \{(p,q) \mid \dot{Q}_1 = 0, \dot{Q}_2 = 0\}$. The $\dot{Q}_1 = 0$ and $\dot{Q}_2 = 0$ is respectively equivalent to $q_1 = q_2 = ...q_m$ and $\tilde{q}_1 = \tilde{q}_2 = \cdots = \tilde{q}_{N-m} = 0$ which occurs only when $q_1 = q_2 = ... = q_{N-M} = q_L$. Thus, for the agents within the leading subgroup, their velocity will become asymptotically the same; and for other agents, their velocity will become asymptotically the same as their virtual leaders, but their virtual leaders are just the member of the leading subgroup. Hence, all agents will achieve the same velocity.

In the following, we need to carry out the connectivity persevering analysis and prove that the group is jointly connected across [T, +∞], for any finite $T \ge t_0$. Note that f (.) grows unbounded when $\|p_{ij}\| \to R$, hence the agents within the connected subgroup will always maintain connectivity. That is, the leading group SG_L (t) is always connected, the type 1 agents asymptotically converges to a configuration p* that minimizes all agent potentials, i.e. $\nabla J(p^*) = 0$. For the type 2 agent, $\forall t \in [t_0, +\infty]$, we see that, in steady state, $q_1 = q_2 = ... = q_{N-M} = q_L$ and $\dot{q}_1 = \dot{q}_2 = \cdots = \dot{q}_{N-m} = \dot{q}_L$, which implies that $u_i = \dot{q}_L$. Thus, from Eq. 2,we have

$$\nabla_{p_{ij}} (\sum_{j \in N_i} J_{ij}) + k_v \nabla_{p_{iL}} \sigma_{iL} = 0$$

Thus, the configuration converges asymptotically to a configuration that minimizes all inter-agent and leader-agent potentials. That is, the distance of leader-agent $\|p_i - p_L\|$ is finite for all t>0 and attains a fixed configuration that minimizes leader-agent potentials σ_{iL}. Since, the virtual leader is just the member of the leading subgroup, all agents will converge asymptotically to a configuration which make all agents are connected together.

Here, we need to point out that the above control strategy can be extended to the leader following coordinated control by redefining the leading subgroup. Moreover, it is robust of connection failures between followers and the leader. The leading subgroup is the subgroup that includes the leader. With some long range connections, the leading subgroup can influence others subgroup agents, but not be influenced by any external agents in others subgroup. In addition, the inter-agent force f (.) will grow unbounded, when $\|p_{ij}\| \to R$. Hence, the leading subgroup is always connected and the leader-agents' connections will be preserving. Even if some malicious attacks make all leader-agents' connections broken, the leader becomes disconnected and isolated. This worst case is instantaneous, because long range connections between leader and agents will be added in next time.

SIMULATION

Here, we give a simulation to verify the above results. We consider ten agents with random positions and velocities in the workspace. In Fig. 1a-e, agents are denoted with dots, while links between the agents are indicated by solid lines. Fig. 1 shows the evolving topologies of the systems at different time and Fig. 2 shows the moving trajectories of the system. Note that by introduction few long-range interactions, the initial disconnected system evolves under the control law (2) and gradually forms a connected swarm system. At last, asymptotic stable flocking of the group is achieved.

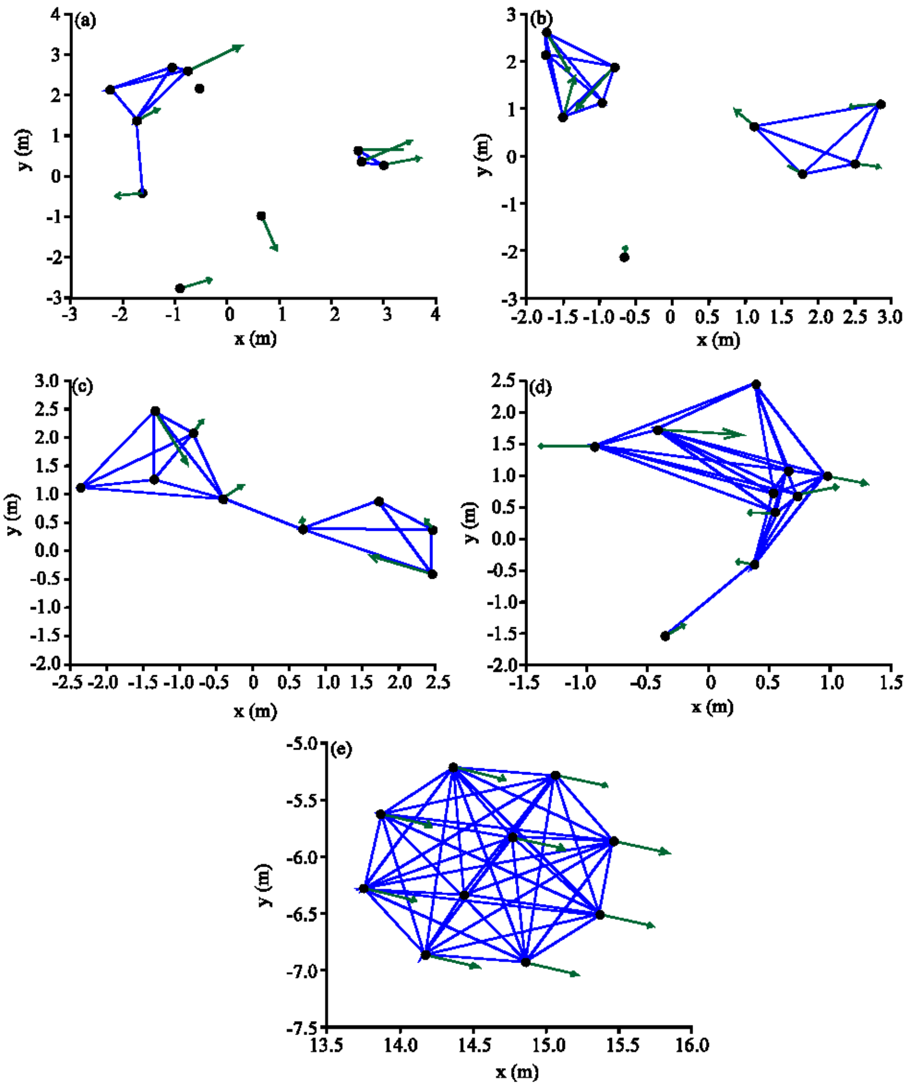

Fig. 1: The evolving topologies of all agents corresponding to initial disconnected system with different time (sec): (a) t = 0, (b) t= 1, (c) t = 2.5, (d) t = 4 and (e) t = 6

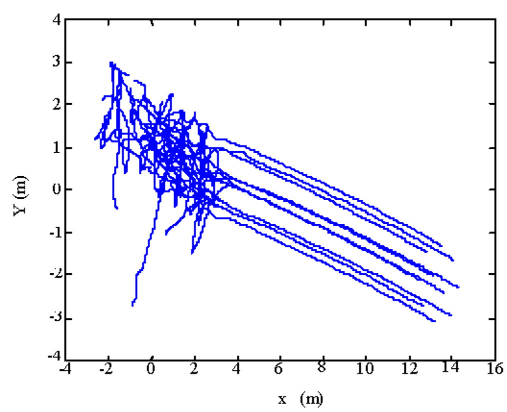

Fig. 2: The trajectories of the initial disconnected system

CONCLUSION

In this study, the flocking motion control has been investigated by introduction of few long-range connections. For the connected agents, force functions are used to maintain to the original connectivity and for the far and disconnected agents, few long-range connections are used to steer them to the main group. The conclusion indicates that introduction of long-range connections can increase group coordination and make the flocking motion achieved under most of the conditions.

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