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Stability of Continuous-Time Vehicles Formations with Time Delays in Undirected Communication Network

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Abstract: This study mainly focuses on stability analysis of vehicles formations with time delays in the communication network. The network model with time delays of swarm vehicles for continuous-time systems is introduced. The vehicles exchange information according to a pre-specified (undirected) communication graph. The feedback control is based only on relative information about vehicle states shared via the communication links. Asymptotical stability of vehicles formations for both delay-independent and delay-dependent cases is analyzed. The sufficient conditions for vehicles formations stabilities are investigated based on tools from linear matrix inequality theory, algebraic graph theory, matrix theory and control theory. Finally, an illustrative example is used to show the validity of the theoretical results.

Key words: Formation stability, decentralized control, time delays, linear matrix inequality, graph Laplacian

INTRODUCTION

Recent years have seen the emergence of formations of swarm vehicles as a topic of significant interest to the control community. Swarm vehicles systems have appeared widely in many applications including mobile vehicles, formation flight of Unmanned Air Vehicles (UAVs), clusters of satellites, automated highway systems. The coordinated motion of multiple autonomous vehicles has received increasing attention recently (Fax and Murray, 2004; Jadbabaie *et al*., 2003; Leonard and Fiorelli, 2001; Leonard and Ogren, 2003; Tanner *et al*., 2003a).

The research on vehicles formations is motivated by the motion of aggregates of individuals in nature. Swarms of birds and shools of fish achieve coordinated motions without any central controlling mechanism (Breder, 1954; Okubo, 1986). A computer graphics model to simulate flock behaviors is presented (Reynolds, 1987). The famous boid model is proposed that individuals in the swarm vehicles interacting with each other are based on local information and obey the following three rules:

- **Collision avoidance:** Each boid avoids collisions with nearby flockmates
- **Velocity matching:** Each boid attempts to match velocity with nearby flockmates
- **Flock centering:** Each boid attempt to stay close to nearby flockmates

A special version of the model introduced by Reynolds is the Vicsek model proposed by Vicsek *et al.* (1995). Some very interesting simulation results are provided by Vicsek *et al.* (1995). The results show that all vehicles eventually move in the same direction based on local information without any central control or leaders. Flocking behaviors have been analyzed in detail (Jadbabaie *et al*., 2003; Saber and Murray, 2003, 2004; Moreau, 2005; Ren and Beard, 2005; Saber *et al*., 2007). A theoretical explanation for Vicsek model is presented by Jadbabaie *et al.* (2003). Moreover, convergence results for case of leader following are also provided. Consensus problems for networks of dynamic vehicles with fixed and switching topologies are discussed by Saber and Murray (2003, 2004). A theoretical framework for design and analysis of distributed flocking algorithms is presented in the view of control engineering (Saber, 2006). Stability analysis of swarm vehicles are considered by Moreau (2005), Saber (2006) and Fax and Murray (2004).

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The motion of vehicles modeled as double integrators is investigated by Tanner *et al*. (2003b) and Saber (2006). Their goal is for the vehicles to achieve a common velocity while avoiding collisions. The control laws are related to graph Laplacians for an associated undirected graph with nonlinear terms resulting from artificial potential functions. Rather than reaching for pre-specific formation, the vehicles converge to an equilibrium formation that minimizes all individual potentials. A linear feedback control law is introduced to make the vehicles modeled as general second order systems reach a predefined formation by Lafferriere *et al*. (2004, 2005). It is very clear that time delays always exist in communication network composed of vehicles. It is very important to design a feedback control law in order to make the vehicles formations stable with coupling delays.

This study mainly focus on stability analysis of vehicles formations with time delays in the communication network. The vehicles which are modeled as general linear dynamics exchange information based on local interactions. Two cases of delay-independent and delay-dependent asymptotical stabilities are considered. The sufficient conditions for vehicles formations stabilities are investigated based on tools from linear matrix inequality theory, algebraic graph theory, matrix theory and control theory.

PRELIMINARIES

Here, basic concepts and notations in graph theory are stated (Diestel and Theory, 2000; Godsil and Gordone, 2000). The models of vehicles are also described.

Algebraic graph theory and matrix theory: Let $G = (V, E, A)$ be a weighted directed graph of order n with the set of vehicles $V = \{v_1,...,v_n\}$, set of edges $E \subseteq V\times V$ and a weighted adjacency matrix $A = [w_{ij}]$ with nonnegative adjacency elements w_{ij}. The vehicle indices belong to a finite index set $I = \{1, 2,...,n\}$. An edge of G is denoted by $e_{ij} = (v_i, v_j)$. The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij} \in E \Leftrightarrow w_{ij}>0$. Moreover, we assume $w_{ii} = 0$ for all $i\in I$. The set of neighbors of vehicle v_i is denoted by $N_i = \{v_j \in V: (v_i, v_j)\in E\}$. The in-degree and out-degree of vehicle v_i are, respectively, defined as follows:

$$\deg_{in}(v_i) = \sum_{j=1}^{n} w_{ji}, \deg_{out}(v_i) = \sum_{j=1}^{n} w_{ij}$$

For a graph with 0-1 adjacency elements, $\deg_{out}(v_i) = |N_i|$. The degree matrix of the digraph G is a diagonal matrix $D = [d_{ij}]$ where, $d_{ij} = 0$ for all $i \neq j$ and d_{ii}. The graph Laplacian associated with the digraph G is defined as:

$$L(G) = D\text{-}A$$

For undirected graph, the adjacency matrix is symmetric, i.e., $w_{ij} = w_{ji}$. Its in-degree and out-degree are equal, i.e., $\deg_{in}(v_i)$. Then the Laplacian matrix is symmetric and defined by:

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{n} w_{ik}, j = i \\ -w_{ij}, \quad j \neq i \end{cases} \quad (1)$$

It is noted that zero is a eigenvalue of L and the associated eigenvector is $1_{n\times 1}$. If graph G is strongly connected, 0 is an isolated eigenvalue of L and all other eigenvalues of G are real-valued and are strictly positive.

MATRIX THEORY

Here, some results that are useful for analysis are introduced (Horn and Johnson, 1985; Boyd *et al*., 1994; Yu, 2002).

Definition 1: The Kronecker product of $P = [p_{ij}]$ and $Q = [Q_{ij}]$ is denoted by $P\otimes Q$ and is defined to be the $P\otimes Q = [p_{ij}Q]$.

Lemma 1: Given $A\in\mathbb{R}^{N\times N}$ with eigenvalues $\lambda_1,...,\lambda_N$ in any prescribed order, there is a unitary matrix $T\in\mathbb{R}^{N\times N}$ such that $T^{-1}AT = U = [u_{ij}]$ is upper triangular, with diagonal entries $u_{ij} = \lambda_j$, $i = 1,...,N$.

Lemma 2: If $A\in\mathbb{R}^{N\times N}$ and $B\in\mathbb{R}^{N\times N}$ are nonsingular, then so is $A\otimes B$ and $(A\otimes B)^{-1} = A^{-1}\otimes B^{-1}$.

Lemma 3: Let $X\in\mathbb{R}^{r\times s}$ and $B\in\mathbb{R}^{N\times N}$, then

$$(I_N\otimes X)(Y\otimes I_r)(I_N\otimes X)$$

Lemma 4: If there is a unitary matrix $T\in\mathbb{R}^{N\times N}$ such that $T^{-1}AT = U = [u_{ij}]$ is upper triangular, then

$$(T\otimes I_n)^{-1}(L\otimes I_n)(T\otimes I_n) = U\otimes I_n$$

Lemma 5: Assume that $a\in\mathbb{R}^{N_a}$, $b\in\mathbb{R}^{n_b}$ and matrices $X\in\mathbb{R}^{n_a\times n_a}, Y\in\mathbb{R}^{n_a\times n_b}, Z\in\mathbb{R}^{n_b\times n_b}$ and $N\in\mathbb{R}^{n_a\times n_b}$, if

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$$

holds, then

$$-2a^T Nb \leq \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T-N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Lemma 6: The LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$$

where, $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ depend affinely on x, is equivalent to

$$R(x)>0,\ Q(x)-S(x)R(x)^{-1}S^T(x)>0$$

MODEL DESCRIPTION

It is assumed that given N vehicles with the same dynamics:

$$\dot{x}_i = A_{veh}x_i + B_{veh}u_i \quad i=1,\cdots,N \quad x_i \in \mathbb{R}^{2n} \tag{2}$$

where, the entries of x_i represent n configuration variables for vehicle i and derivatives and u_i represent control inputs. The matrices A_{veh} and B_{veh} have the form:

$$A_{veh} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & a_{26} & \cdots & a_{2(2n)} \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & a_{46} & \cdots & a_{4(2n)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

$$B_{veh} = I_n \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The form of the first and third rows of A_{veh} is determined by the fact that the odd-numbered entries of x_i represent the position variable and the even-numbered entries represent velocity variable and that the control input affect the acceleration. We will use the notation:

$$x_p = \left((x_{1p}),\cdots,(x_{Np})\right)^T, x_v = \left((x_{1v}),\cdots,(x_{Nv})\right)^T$$

It is assumed that each vehicle is allowed to see only some of its neighbors and applies the same linear feedback as the others, that is, u_i is determined by relative information and is a linear feedback control law. Consider the feedback matrix:

$$F_{veh} = \begin{pmatrix} f & g & 0 & 0 & \cdots \\ 0 & 0 & f & g & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix}$$

As an example, consider vehicles in $\mathbb{R}^3$ so that vehicle i has position (x_{ipx}, x_{ipx}, x_{ipz}), velocity (v_{ivx}, v_{ivx}, v_{ivz}). Assume vehicle i see only vehicles k and j. With the linear feedback the equations of motion for vehicle 1 become:

$$\begin{aligned} x_{ipx} &= v_{ivx} \\ v_{ivx} &= a_{22}v_{ivx} + a_{24}v_{ivy} + a_{26}v_{ivz} + u_{ix} \\ x_{ipy} &= v_{ivy} \\ v_{ivy} &= a_{42}v_{ivx} + a_{44}v_{ivy} + a_{46}v_{ivz} + u_{iy} \\ x_{ipz} &= v_{ivz} \\ v_{ivz} &= a_{62}v_{ivx} + a_{64}v_{ivy} + a_{66}v_{ivz} + u_{iz} \\ u_{ix} &= f\times(l_{ii}x_{ipx} - l_{ij}x_{jpx} - l_{ik}x_{kpx}) + g\times(l_{ii}x_{ivx} - l_{ij}x_{jvx} - l_{ik}x_{kvx}) \\ u_{iy} &= f\times(l_{ii}x_{ipy} - l_{ij}x_{jpy} - l_{ik}x_{kpy}) + g\times(l_{ii}x_{iv} - l_{ij}x_{jvy} - l_{ik}x_{kvy}) \\ u_{iz} &= f\times(l_{ii}x_{ipz} - l_{ij}x_{jpz} - l_{ik}x_{kpz}) + g\times(l_{ii}x_{ivz} - l_{ij}x_{jvz} - l_{ik}x_{kvz}) \end{aligned}$$

where, f and g are feedback coefficients, l_{ij} is defined in (1). So, we can rewrite (Eq. 2) as:

$$\dot{x}_i = A_{veh}x_i + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}x_j, \quad i=1,\cdots,N \tag{3}$$

In reality, there usually are some time delays in communication network because of finite speeds of transmission and traffic congestions. Assume the time delays are the same in the network. We introduce the following dynamical model.

$$\dot{x}_i(t) = A_{veh}x_i(t) + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}x_j(t-\tau), \quad i=1,\cdots,N \tag{4}$$

Now, let us consider vehicles formations.

Definition 1: A formation is a vector

$$h = h_p \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{R}^{2nN}$$

The N vehicles are in formation h at time t if there are vectors q, $\omega \in \mathbb{R}^n$ such that $x_{ip}(t)-(h_{ip}) = q$ and $x_{iv}(t) = \omega$, for i = 1,...,N. The vehicles converge to formation h if there exist $\mathbb{R}^n$-valued functions $q(\bullet)$, $\omega(\bullet)$ such that:

$$x_{ip}(t) - (h_{ip}) - q(t) \to 0 \text{ and } x_{iv}(t) - \omega(t) \to 0 \text{ for } i=1,\cdots,N$$

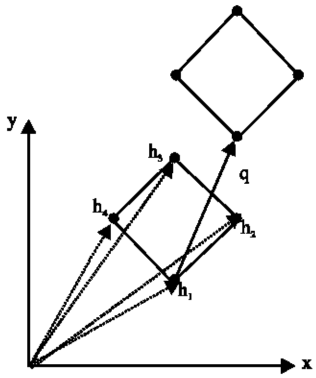

Fig. 1: Vehicles in formation

Definition 2 is given by Lafferriere *et al.* (2005).

Figure 1 shows the interpretation of the vectors in the definition. Now the vehicles model with vehicles formations in h become:

$$\dot x_i(t) = A_{veh} x_i(t) + \sum_{j=1}^{N} l_{ij} B_{veh} F_{veh} (x_j(t-\tau) - h_j) \tag{5}$$

STABILITY OF VEHICLES FORMATIONS

According to definition 1, vehicles achieve in formation h if:

$$x_1(t) - h_1 = x_2(t) - h_2 = \cdots = x_N(t) - h_N = s(t) \text{ as } t \to \infty$$

We note the fact that

$$x_i(t)\text{-}h_i = s(t)$$

Then,

$$\begin{aligned} \dot s(t) &= A_{veh}(s(t) + h_i) + \sum_{j=1}^{N} l_{ij} B_{veh} F_{veh} (s(t-\tau)) \\ &= A_{veh} s(t) \end{aligned} \tag{6}$$

where, we use the properties:

$$A_{veh} h_i = 0$$
$$\sum_{j=1}^{N} l_{ij} B_{veh} F_{veh} (s(t-\tau)) = 0$$

Theorem 1: Consider a network of agents with equal communication time-delay τ>0 in all links. Assume the network topology G is fixed, undirected and connected. Eigenvalues of graph Laplacian can be ordered sequentially in an ascending order as:

$$0 = \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$$

If the following N-1 linear time-varying delayed differential equations are asymptotically stable about their zero solutions:

$$\dot v = A_{veh} v(t) + \lambda_i B_{veh} F_{veh} v(t-\tau), \qquad i = 2,3,\cdots,N$$

Then the vehicles converge to formation h.

Proof: Let x_i (t)-h_i = s(t)+e_i(t), we rewrite (Eq. 5) as:

$$\begin{aligned} &\dot s(t) + \dot e_i(t) + \dot h_i = A_{veh}(s(t) + e_i(t) + h_i) \\ &+ \sum_{j=1}^{N} l_{ij} B_{veh} F_{veh} (s(t-\tau) + e_i(t-\tau)) \end{aligned}$$

Note that $\dot h_i = 0$ and (Eq. 6), the equation can be:

$$\begin{aligned} \dot e_i(t) &= A_{veh} e_i(t) + \sum_{j=1}^{N} l_{ij} B_{veh} F_{veh} e_i(t-\tau) \\ &= A_{veh} e_i(t) + B_{veh} F_{veh} (e_1(t-\tau), \cdots, e_N(t-\tau))(l_{i1}, \cdots, l_{iN})^T \end{aligned}$$

Collecting the equations for all $e_i(t)$ into one system we get:

$$\dot e(t) = I_N \otimes A_{veh} e(t) + L \otimes B_{veh} F_{veh} e(t-\tau) \tag{7}$$

Let U be a matrix such that $\tilde L = U^{-1} L U$ is upper triangular. Then let $\tilde e(t) = U \otimes I_{2n} e(t)$ note that Lemma 1, 2, 3 and 4, (Eq. 7) can be equivalent to:

$$\dot{\tilde e}(t) = I_N \otimes A_{veh} \tilde e(t) + \tilde L \otimes B_{veh} F_{veh} \tilde e(t-\tau)$$

Because $\tilde L$ is upper triangular, its diagonal blocks are of the form:

$$\dot{\tilde e}_i(t) = A_{veh} \tilde e_i(t) + \lambda_i B_{veh} F_{veh} \tilde e_i(t-\tau), \qquad i = 1, \cdots, N$$

This equation can be rewritten as:

$$\dot v_i(t) = A_{veh} v_i(t) + \lambda_i B_{veh} F_{veh} v_i(t-\tau), \qquad i = 1, \cdots, N \tag{8}$$

Thus, we have transformed the stability of Eq. 5 to the stability of N linear delayed differential Eq. 8. We note that when λ_1 = 0, become identical with Eq. 6. Then if the following N-1 equations:

$$\dot v_i(t) = A_{veh} v_i(t) + \lambda_i B_{veh} F_{veh} v_i(t-\tau), \qquad i = 2, \cdots, N$$

are asymptotically stable, Eq. 5 is formation stable, namely, the vehicles converge to formation h. Thus, Theorem 1 is proved.

Then some sufficient conditions for vehicles formations are given. Two cases of time-independent and time-dependent delays are considered.

Theorem 2: If there exists two symmetric positive-definite matrices P, $S\in\mathbb{R}^{2n\times 2n}$, such that:

$$\begin{bmatrix} A_{veh}^T P + PA_{veh} + S & \lambda_i PB_{veh}F_{veh} \\ \lambda_i (B_{veh}F_{veh})^T P & -S \end{bmatrix} < 0 \quad (9)$$

for i = 2,...,N, then (5) is formation stable, that is, vehicles converge to formation h.

Proof: If there exists symmetric positive-definite P and S such that matrix inequality (9) holds, then we define:

$$V(v_i(t)) = v_i^T(t)Pv_i(t) + \int_{t-\tau}^{t} v_i^T(\alpha)Sv_i(\alpha)d\alpha$$

where, P>0 and S>0, then $V(v_i(t))$ is positive-definite. The derivative of $V(v_i(t))$ along the solution of (Eq. 8) is:

$$\begin{aligned} \dot V(v_i(t)) &= \dot v_i^T(t)Pv_i(t) + v_i^T(t)P\dot v_i(t) \\ &\quad + v_i^T(t)Sv_i(t) - v_i^T(t-\tau)Sv_i(t-\tau) \\ &= v_i^T(t)[PA_{veh} + A_{veh}^T P + S]v_i(t) \\ &\quad + 2v_i^T(t)\lambda_i P(B_{veh}F_{veh})v_i(t-\tau) \\ &\quad - v_i^T(t-\tau)Sv_i(t-\tau) \end{aligned}$$

Then

$$\dot V(v_i(t)) = \begin{bmatrix} v_i(t) \\ v_i(t-\tau) \end{bmatrix}^T \begin{bmatrix} A_{veh}^T P + PA_{veh} + S & \lambda_i PB_{veh}F_{veh} \\ \lambda_i (B_{veh}F_{veh})^T P & -Q \end{bmatrix} \begin{bmatrix} v_i(t) \\ v_i(t-\tau) \end{bmatrix}$$

If matrix inequality 9 holds, then we can see $\dot V(v_i(t)) < 0$.

According to Lyapunov stability theory, the states of Eq. 5 is asymptotically stable. Thus, proof of Theorem 2 is finished.

Remark 1: The delay-independent stability condition can be used to test whether a system is asymptotically stable for an arbitrary delay or not. It is obvious that such condition is conservative and is sufficient but not necessary, that is, if there is no solutions such that (Eq. 9) holds, we can not judge if a system is stable. When Theorem 2 does not work , we introduce Theorem 3 for delay-dependent stability.

Theorem 3: Given that the time-invariant delay $\tau\in[0, d]$ for some bounded d. If there exist symmetric matrices P>0, S, X,Z and Y such that:

$$\begin{bmatrix} M & \lambda_i PB_{veh}F_{veh} - Y & dA_{veh}^T Z \\ \lambda_i (B_{veh}F_{veh})^T P - Y^T & -S & d\lambda_i (B_{veh}F_{veh})^T Z \\ dZA_{veh} & d\lambda_i ZB_{veh}F_{veh} & -dZ \end{bmatrix} < 0 \quad (10)$$

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0 \quad (11)$$

for i = 2,...,N, then Eq. 5 is formation stable, that is, vehicles converge to formation h, where:

$$M = PA_{veh} + A_{veh}^T P + dX + Y^T + Y + S$$

Proof: Given that there exists symmetric positive-definite matrix P, symmetric matrices S,X,Z and matrix Y such that (10) and (11) hold. We note that:

$$\dot v_i(t) = (A_{veh} + \lambda_i B_{veh}F_{veh})v_i(t) - \lambda_i B_{veh}F_{veh}\int_{t-\tau}^{t} \dot v_i(\alpha)d\alpha \quad (12)$$

Then we select the following Lyapunov function:

$$V = V_1 + V_2 + V_3$$

Where:

$$\begin{aligned} V_1 &= v_i^T(t)Pv_i(t) \\ V_2 &= \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot v_i^T(\alpha)Z\dot v_i(\alpha)d\alpha d\beta \\ V_3 &= \int_{t-\tau}^{t} v_i^T(\alpha)Sv_i(\alpha)d\alpha \end{aligned}$$

The derivative of V along Eq. 12 is:

$$\dot V = \dot V_1 + \dot V_2 + \dot V_3$$

Where:

$$\begin{aligned} \dot V_1 &= \dot v_i^T(t)Pv_i(t) + v_i^T(t)P\dot v_i(t) \\ &= v_i^T(t)[PA_{veh} + \lambda_i PB_{veh}F_{veh} + A_{veh}^T P + \lambda_i (B_{veh}F_{veh})^T P]v_i(t) \\ &\quad - \int_{t-\tau}^{t} 2v_i^T(t)\lambda_i PB_{veh}F_{veh}\dot v_i(\alpha)d\alpha \end{aligned}$$

Let $a = \lambda_i B_{veh}F_{veh}\dot v_i(\alpha), b = Pv_i(t)$ applying Lemma 5, then:

$$\begin{aligned} \dot V_1 &\leq v_i^T(t)[PA_{veh} + A_{veh}^T P + dX + Y^T + Y]v_i(t) \\ &\quad + 2v_i^T(t)(\lambda_i PB_{veh}F_{veh} - Y)v_i(t-\tau) \\ &\quad + \int_{t-\tau}^{t} \dot v_i^T(\alpha)Z\dot v_i(\alpha)d\alpha \end{aligned}$$

And:

$$\begin{aligned} \dot V_2 &= \tau\dot v_i^T(t)Z\dot v_i(t) - \int_{t-\tau}^{t} \dot v_i^T(\alpha)Z\dot v_i(\alpha)d\alpha \\ &= \tau\dot v_i^T(t)Z\dot v_i(t) \\ &\quad - \int_{t-\tau}^{t} \dot v_i^T(\alpha)Z\dot v_i(\alpha)d\alpha \\ \dot V_3 &= v_i^T(t)Sv_i(t) - v_i^T(t-\tau)Sv_i(t-\tau) \end{aligned}$$

Then:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \le \begin{bmatrix} v_i(t) \\ v_i(t-\tau) \end{bmatrix}^T \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} v_i(t) \\ v_i(t-\tau) \end{bmatrix}$$

Where:

$$\Phi_{11} = PA_{veh} + A_{veh}^T P + dX + Y^T + Y + S + dA_{veh}^T ZA_{veh}$$
$$\Phi_{12} = \lambda_i PB_{veh}F_{veh} - Y + dA_{veh}^T Z\lambda_i B_{veh}F_{veh}$$
$$\Phi_{21} = \lambda_i (B_{veh}F_{veh})^T P - Y^T + d\lambda_i (B_{veh}F_{veh})^T ZA_{veh}$$
$$\Phi_{22} = -S + d\lambda_i^2 (B_{veh}F_{veh})^T ZB_{veh}F_{veh}$$

If it is satisfied that:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} < 0 \quad (13)$$

then, $\dot{V} < 0$, (5) are asymptotically stable, namely, the vehicles converge to formation d. According to Lemma 6, (Eq. 10) is equivalent to (Eq. 13).

Thus, the proof of Theorem 3 is completed

Remark 2: The delay-dependent stability is concerned with the value of the delay. According to Theorem 3, when the delay τ is less than the upper bound d, the system (Eq. 5) is stable. Moreover, we can obtain the upper bound by the following optimization problem.

$$\text{s.t.} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} < -\rho \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix}$$
$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$$

Where:

$$\Omega_{11} = A_{veh}^T ZA_{veh} + X$$
$$\Omega_{12} = A_{veh}^T Z\lambda_i B_{veh}F_{veh}$$
$$\Omega_{21} = \lambda_i (B_{veh}F_{veh})^T ZA_{veh}$$
$$\Omega_{22} = \lambda_i (B_{veh}F_{veh})^T Z\lambda_i B_{veh}F_{veh}$$
$$\Psi_{11} = PA_{veh} + A_{veh}^T P + Y^T + Y + S$$
$$\Psi_{12} = \lambda_i PB_{veh}F_{veh} - Y$$
$$\Psi_{21} = \lambda_i (B_{veh}F_{veh})^T P - Y^T$$
$$\Psi_{22} = -S$$

If there is a optimal solution ρ^* to this problem, The maximum delay can be obtained by:

$$d^* = (\rho^*)^{-1}$$

NUMERICAL SIMULATIONS

Consider a network composed of four vehicles each has the same dynamics as (2) depicts. The network topology structure is given in Fig. 2.

The Laplacian of the graph with 0-1 adjacency elements is:

$$L = \begin{bmatrix} 1 & -0.5 & 0 & -0.5 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 1 & -0.5 \\ -0.5 & 0 & -0.5 & 1 \end{bmatrix}$$

The eigenvalues of L are:

$$\lambda_i = 0,1,1,2$$

The feedback matrix

$$F_{veh} = \begin{bmatrix} f & g & 0 & 0 \\ 0 & 0 & f & g \end{bmatrix}$$

When $x_i \in \mathbb{R}^{2n}$ and n = 2,

$$A_{veh} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & 0 & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & 0 & a_{44} \end{bmatrix}$$

Let f = g = -1, a_{22}, = a_{24} = a_{42} = a_{44} = 0.

$$B_{veh} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Firstly, Theorem 2 is used to test if vehicles formations are stable. By MATLAB LMI Toolbox, there are no solutions such that (Eq. 9) holds. Then Theorem 3 can be used to decide the maximum delay such that Eq. 10 and 11 hold.

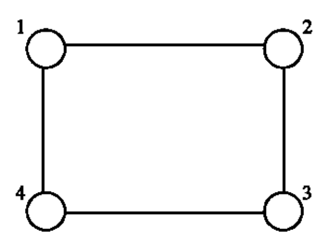

Fig. 2: Topology structure composed of four vehicles

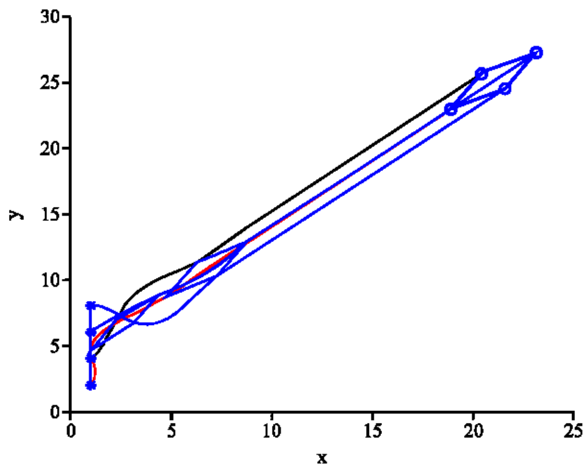

Fig. 3: Trajectories of vehicles

The following results are obtained. d* = 0.137. When the time delay is equal to $\tau = 0.13$:

$$P = \begin{bmatrix} 0.9593 & 0.4933 & 0 & 0 \\ 0.4933 & 0.8294 & 0 & 0 \\ 0 & 0 & 0.9593 & 0.4933 \\ 0 & 0 & 0.4933 & 0.8294 \end{bmatrix}$$

$$S = \begin{bmatrix} 1.7836 & 0.5818 & 0 & 0 \\ 0.5818 & 1.0662 & 0 & 0 \\ 0 & 0 & 1.7836 & 0.5818 \\ 0 & 0 & 0.5818 & 1.0662 \end{bmatrix}$$

$$X = \begin{bmatrix} 2.1490 & 1.5879 & 0 & 0 \\ 1.5879 & 2.9069 & 0 & 0 \\ 0 & 0 & 2.1490 & 1.5879 \\ 0 & 0 & 1.5879 & 2.9069 \end{bmatrix}$$

$$Y = \begin{bmatrix} -1.4493 & -0.9723 & 0 & 0 \\ -0.9723 & -1.7944 & 0 & 0 \\ 0 & 0 & -1.4493 & -0.9723 \\ 0 & 0 & -0.9723 & -1.7944 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1.0804 & 0.5899 & 0 & 0 \\ 0.5899 & 1.1394 & 0 & 0 \\ 0 & 0 & 1.0804 & 0.5899 \\ 0 & 0 & 0.5899 & 1.1394 \end{bmatrix}$$

Figure 3 shows that four vehicles with initial positions in a line marked by * eventually converge to a diamond formation marked by o with time delay $\tau = 0.13$.

CONCLUSION

This study provides a theoretical analysis for stability of continuous-time vehicles formations with time delays undirected graph. The delays in communication network are assumed to be same value. Two cases of delay-independent and delay-dependent vehicles formations are considered. The sufficient conditions are given to guarantee that with time delays the vehicles formation can asymptotically converge to predefined formation. Two theorems are concluded on the basis of linear matrix inequality theory. The conditions for the stability of vehicles formation are simple and easy to be solved as well as the results obtained by the theorems are conservative. In fact, the maximum delay solved by Theorem 3 is less than the practical maximum delay. In order to reduce the conservativeness, new methods and theories are necessary.

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