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Image Segmentation with Partial Differential Equations

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Abstract: In many practical fields, image segmentation is the most important procedure. The purpose of image segmentation is to detect the objects in images. The methods based on statistic theory can work well on images with no noise or little noise. But the procedure of segmentation is difficult to be obtained and the accuracy of result often depends on some artificial parameters. A lot of physical phenomenon can be described by Partial Differential Equations (PDEs) and related procedure is easy to be displayed. With the applications of PDEs, it is convenient to accomplish segmentation and represent the procedure.

Key words: Image segmentation, partial differential equation, level set method

INTRODUCTION

Image segmentation is an important activity in the fields of image analysis, pattern recognition and computer vision and there are many approaches about it (Caselles *et al*., 1997; Chan and Vese, 2001; Renka, 2009; Sleigh, 1986). Segmentation is the important technique for detecting objects and analyzing images. Segmentation methods fall into several categories such as histogram analysis, region growing, edge detection and partial differential equations (PDE)-based methods, etc.

An image can be segmented by the histogram analysis method (Russ, 2003) based on the distribution of its intensity and some pre-defined thresholds. This method does not make use of spatial structural information and it is effective for simple images with small amounts of structure.

Edge-based segmentation methods (Jahne, 2002) first search the edges of objects in the image and then use this edge information to reconstruct complete boundaries for the principal objects in the image. It is based on the fact that the position of an edge given by an extreme of the first-order derivative or a zero crossing in the second-order derivative. The main shortcoming is that the true edges are often fragmented by noise in the image and it will lead to error.

Region growing (Gonzalez and Woods, 2002) is one of the most popular segmentation methods. It segments an image by splitting the image into smaller regions and merging them into larger ones with k-means, k-nearest neighborhood or some other clustering method. Merging or grouping criteria may be based on criteria such as homogeneity, proximity, color, gray level or texture.

The PDE based methods (Morel and Solimini, 1995) are the most convenient and effective methods for image segmentation. The main advantage is that the theory behind the concept and the solution techniques are well established in other fields such as physics and mechanics. The snake (Kass *et al*., 1987), the gradient vector flow (Xu and Prince, 1998) and the level set method (Osher and Fedkiw, 2003), are typical of the methods. Recently, a level set model without re-initialization (Li *et al*., 2005), Sobolev gradient method (Renka, 2009), p-Laplace model (Zhou and Mu, 2010) are proposed.

The level set method is first introduced by Osher and Sethian (1988). It is simple and adaptable for computing and analyzing the motion of an interface in two or three dimensions. The moving interface or fronts are described by the zero level set in conventional level set methods. The level set method has been applied in a wide range of successful applications, including problems in fluid mechanics, combustion, solids modeling, computer animation, material science and image processing over the years (Han *et al*., 2003). These interfaces may easily develop sharp corners, break apart and merge together in a robust and stable way.

FUNDAMENTAL MATHEMATICAL FORMULATIONS

Many functional and PDEs have been applied in image segmentation. One general image segmentation

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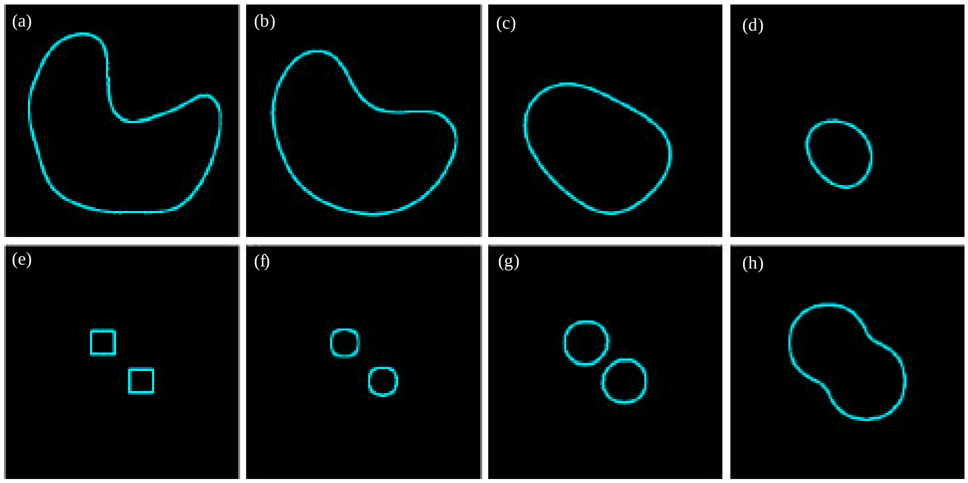

Fig. 1: (a-h) Mean curvature motion and a topology change

model was proposed by Mumford and Shah (1989). The related functional is denoted as following:

$$\varepsilon(u,\Gamma) = \int_{\Omega-\Gamma} |u - u_0|^2 dx + \beta|\Gamma| + v\int_{\Omega-\Gamma} |\nabla u|^2 dx \qquad (1)$$

where, β and v are pre-defined.

A two-phase segmentation is proposed by Chan and Vese (2001) and the evolution equation can be denoted by

$$\phi_t = \delta_\varepsilon(\phi)(-(u_0 - c_1)^2 + (u_0 - c_2)^2 - v + \beta\nabla(\frac{\nabla\phi}{|\nabla\phi|})) \qquad (2)$$

where, c_1, c_2 and β are pre-defined.

As follows, a p-Laplace equation (Zhou and Mu, 2010) is applied to image segmentation.

$$\frac{\partial u}{\partial t} = \mathrm{div}(\frac{\nabla u}{|\nabla u|^{2-p}}) + \lambda f(u) \qquad (3)$$

where, p>1 and λ is fixed.

Besides these mentioned above, there are also many other PDEs have been used in related applications.

In traditional level set method, curves or surfaces (active contours) are represented in implicit form as the zero level set of a high dimensional continuous function which called level set function. The evolution of the function ϕ can be governed by a Hamilton-Jacobi equation (Tian and Mu, 2009):

$$\begin{cases} \frac{\partial\phi}{\partial t} + F|\nabla\phi| = 0, \\ \phi(x,0) = \phi_0(x), \end{cases} \quad x \in \Omega \qquad (4)$$

where, the function F is called the speed function. For image segmentation, the function F depends on the image data I and the level set function ϕ. The well-known mean curvature motion is shown in the first row of Fig.1 a-h. Another example describes a topology change in the evolution is shown in the second row.

Recently, a new variational formulation of level set method without re-initialization is proposed by Li *et al.* (2005). In fact, the standard re-initialization method is to solve the re-initialization equation

$$\frac{\partial\phi}{\partial t} = \mathrm{sign}(\phi_0)(1 - |\nabla\phi|)$$

where, ϕ_0 is the function to be re-initialization and sign (ϕ) is the sign function. It is a simple version of Eq. 4.

LEVEL SET EVOLUTION BASED ON PARTIAL DIFFERENTIAL EQUATION

In order to get the level set equation, variational level set methods are traditional to be used. Based on this idea, the segmentation governed by a partial differential equation can be implemented.

Figure 2 a-d display the level set evolution for image segmentation of the first example, the image is 134×161 and the zero-contour of the initial level set function is shown in the first figure. It is encloses the object. The typical zero-contours and final zero-contours are shown in Fig. 3a-d.

As for the second numerical example, the evolution of the level sets and the two separately objects that we want to segment are shown in Fig. 3a-d. The image is

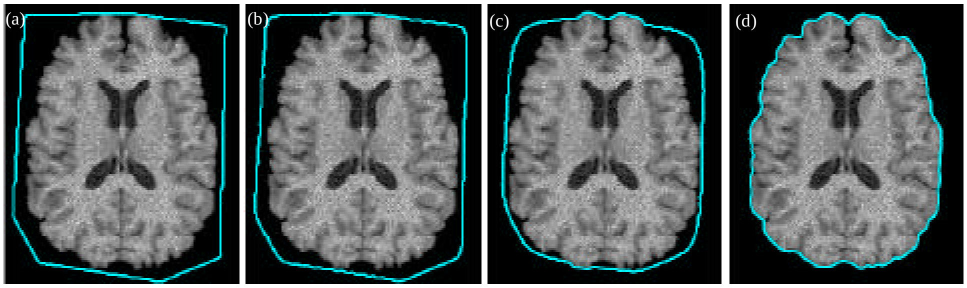

Fig. 2: (a-d) Extraction of one object with different initial contours

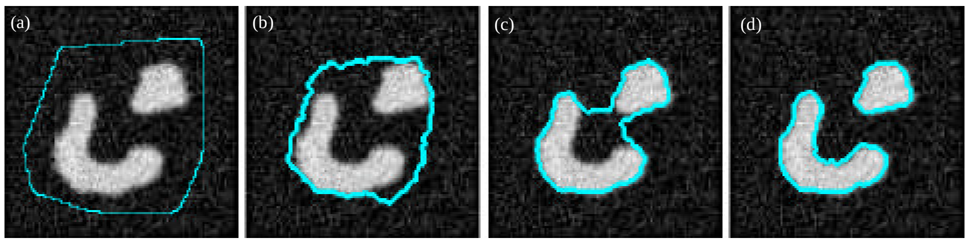

Fig. 3: (a-d) Extraction of two objects with serious pollution

84×84 and it has been polluted seriously. The figure on the left shows the initial contour. Due to the level set evolution, the algorithm allows the automatical change of topology. This example shows the PDE-based method can deal with the image with pollution and complex initial contour.

CONCLUSION

This study discarded image segmentation and the partial differential equation. Many PDEs can be applied to image segmentation and a lot of PDE-based methods or models have been proposed. Those are convenient and effective for image segmentation.

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